Realization of Hardy's Thought Experiment with Photons

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We present an experimental realization of Hardy's thought experiment [Phys. Rev. Lett. **68**, 2981 (1992)], using photons. The experiment consists of a pair of Mach-Zehnder interferometers that interact through photon bunching at a beam splitter. A striking contradiction is created between the predictions of quantum mechanics and local hidden variables. The contradiction relies on nonmaximally entangled position states of two particles. A Clauser-Horne–type inequality is derived and violated.

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Quantum mechanics poses a challenge to the notion that objects carry with them values of observables, such as position, that both determine the outcomes of measurements and that are local, i.e., uninfluenced by events that happen outside the object's backward light cone. It was first pointed out by Bell [1] that the predicted correlations between outcomes of measurements on two spatially separated systems prepared in an entangled quantum state, were too strong to be reproduced by any theory based on local ''hidden'' variables (LHVs). He formulated an inequality which places a bound on the correlations predicted by any such theory, opening the possibility of performing experimental tests whose realization [2] decided in favor of quantum mechanics. By considering three particles in an entangled quantum state Greenberger, Horne, and Zeilinger (GHZ) [3] later proposed a scheme in which quantum mechanics and LHV theories predict opposite measurement outcomes, leading to an even stronger contrast between the two. The predictions were verified with polarization entangled photons [4].

More recently, Hardy formulated a thought experiment [5] that involves only two spatially separated particles, like in the Bell case, but which leads to a strong contradiction between the LHV and quantum mechanical predictions in a similar spirit to the GHZ scheme. A crucial feature in Hardy's thought experiment is that the two particles are nonmaximally entangled. He later generalized his scheme in a way that could be tested with polarization entangled photons [6,7]. Here we present an experimental realization of Hardy's original thought experiment, using photons. The scheme follows essentially the proposal of Ref. [8]. It differs from the polarization-based scheme in that the variables used are paths taken by photons. This makes the contradiction particularly striking, since position is an external variable that translates intuitively to its classical equivalent.

The basic building block of Hardy's thought experiment is a Mach-Zehnder interferometer for quantum particles (Fig. 1, shaded area). The interferometer is tuned so that particles entering in arm e^- exit in arm c^-:

$$|e^-\rangle \rightarrow \frac{|v^-\rangle + i|w^-\rangle}{\sqrt{2}} \rightarrow i|c^-\rangle.$$

If the amplitude for the particle in one arm, say w^-, were to be obstructed by a second particle in w^+ that collides with it, only the v amplitude would reach the second beam splitter, and would split into arms c^+ and d^+ with equal amplitude. The detection of a particle in d^+ would thus indicate the presence of the obstructing particle without the latter being affected. For this reason, this scheme was named ''interaction-free measurement'' [9].

Hardy's original thought experiment (Fig. 1) has two interferometers, one for electrons and one for positrons, arranged in such a way that their w arms intersect. If both the electron and the positron take arms w in their respective interferometers, they will annihilate with certainty to pro-

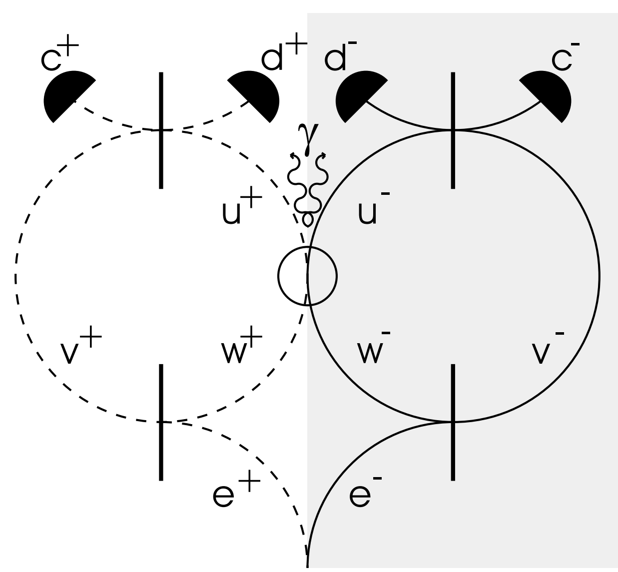

FIG. 1. Setup for Hardy's original thought experiment: electrons and positrons enter two interferometers (one on the left, unshaded, and one on the right, shaded) simultaneously. If both particles take arms w, they annihilate.

duce gamma radiation: $|w^+\rangle|w^-\rangle \to |\gamma\rangle$. Therefore the presence of either particle in its w arm will affect the other's interferometer output:

$$\begin{aligned}|e^+\rangle|e^-\rangle &\to \tfrac{1}{2}(|v^+\rangle + i|w^+\rangle)(|v^-\rangle + i|w^-\rangle)\\ &\to \tfrac{1}{2}[|v^+\rangle|v^-\rangle + i|u^+\rangle|v^-\rangle + i|v^+\rangle|u^-\rangle - |\gamma\rangle]\\ &\to \tfrac{1}{4}[-3|c^+\rangle|c^-\rangle + i|c^+\rangle|d^-\rangle + i|d^+\rangle|c^-\rangle\\ &\quad - |d^+\rangle|d^-\rangle - 2|\gamma\rangle]. \end{aligned} \tag{1}$$

The situation can be analyzed in terms of two simultaneous interaction-free measurements: From the point of view of the interferometer on the left, a click at d^+ implies the presence of the obstructing electron in u^-: $[d^+ \Rightarrow u^-]$. Similarly, for the interferometer on the right, a click at d^- implies the presence of the positron in u^+: $[d^- \Rightarrow u^+]$. Indeed, every time a click is recorded at $d^\pm$ the other particle is found in $u^\mp$. If we assume the particles are independent (described by LHVs), we conclude that they can never emerge simultaneously in d^+ and d^-. This would imply that they were in u^+ and u^-, which cannot occur because of the annihilation process.

A paradox then arises because sometimes [Eq. (1)] the particles *do* emerge simultaneously at d^+ and d^- (with probability $p = \frac{1}{16}$). Quantum mechanically, the $|d^+\rangle|d^-\rangle$ term arises, in fact, from the nonmaximally entangled nature [6] of the state just before the final beam splitters, $|v^+\rangle|v^-\rangle + i|u^+\rangle|v^-\rangle + i|v^+\rangle|u^-\rangle$.

It should be noted that if one were convinced (by experiment) that both the interferometers and the annihilation process work ideally, then the recording of a click at d^+ and d^- would immediately present a paradox. This feature is not present in the polarization-based schemes.

It is striking to analyze a single run of the experiment in which a click is recorded at d^+ and d^- from the point of view of different frames of reference [5,10]. A frame of reference can always be chosen in which one particle, say the positron, reaches a detector before the other reaches the final beam splitter. In that frame upon recording a click at d^+ one can make the prediction that the electron is in u^- with $p = 1$ since the state of the electron is projected onto $|u^-\rangle$. Alternatively, one can choose a frame moving in the opposite direction and, upon recording a d^- event, predict with certainty that the positron is in u^+. Thinking locally, one would then argue that each particle must have traveled in its w arm. However, by comparing results in the different frames one then runs into a contradiction because, had they come from w^+ and w^-, they would have annihilated. Of course, in the context of an experiment, one has to first build up confidence in the validity of the predictions $[d^\pm \Rightarrow u^\mp]$ and the annihilation in order for d^+d^- clicks to constitute evidence for a paradox.

Our scheme to implement the thought experiment uses indistinguishable photons as a substitute for the electron and positron, and photon bunching at a beam splitter [11] as the annihilating interaction. The central part of our setup is a set of seven beam splitters arranged as in Fig. 2. The two interferometers share a central beam splitter where the bunching occurs and can be identified as the sets of four beam splitters on the left (unshaded) and on the right (shaded). The outermost beam splitters balance the losses through the central beam splitter. The path lengths are tuned so that photons entering $e^\pm$, if not lost through the central or outer beam splitters, emerge exclusively in arm $c^\pm$. In the experiment, pairs of identical photons enter the interferometers simultaneously through e^+ and e^-: $|e^+\rangle|e^-\rangle \to \frac{1}{2}(|v^+\rangle + i|w^+\rangle)(|v^-\rangle + i|w^-\rangle)$. As in the electron-positron case, four terms can be identified corresponding to the four combinations of the paths the two photons can take. The $|w^+\rangle|w^-\rangle$ term bunches at the central beam splitter, $|w^+\rangle|w^-\rangle \to \frac{i}{\sqrt{2}}(|2u^+\rangle + |2u^-\rangle)$. This excludes the possibility of detecting a photon leaving each interferometer simultaneously. The absence of such a coincidence click plays an equivalent role to the electron-positron annihilation. The $|v^+\rangle|v^-\rangle$ term evolves into a superposition of states in which neither, one, or both photons are lost through the outermost beam splitters. The cases in which one or both are lost do not give rise to a coincidence click. The $|v^+\rangle|v^-\rangle$ term then simply picks up a reduction in amplitude and a change in phase from the reflections: $|v^+\rangle|v^-\rangle \to -\frac{1}{2}|v^+\rangle|v^-\rangle$. Finally, the $|v^\pm\rangle|w^\mp\rangle$ terms evolve into a superposition of states in which one photon is lost through the outermost beam splitter or one photon crosses over to the other's interferometer or the photons end up one in $v^\pm$ and the other in $u^\mp$. Postselection on coincidence counts gives the evolution:

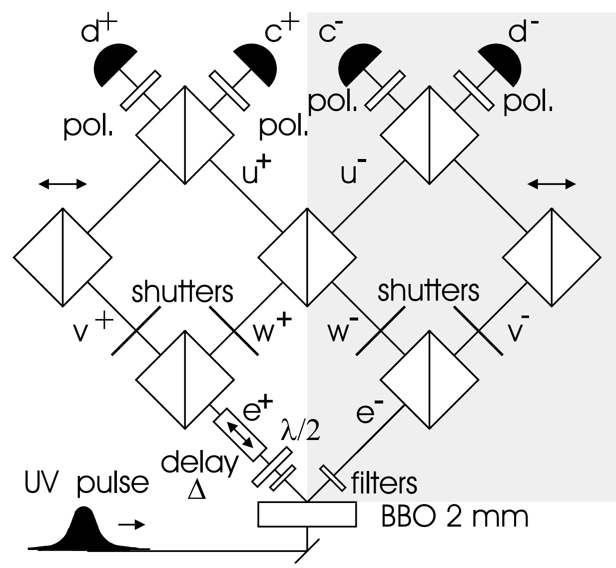

FIG. 2. Setup for the implementation of Hardy's thought experiment using photons. The two interferometers (left: unshaded; right: shaded) share a central beam splitter, where indistinguishable photons taking arms w^+ and w^- bunch. The outermost beam splitters balance the losses from one interferometer to the other. Postselection of the cases in which one photon emerges from each interferometer leads to a state identical to that of Hardy's thought experiment (Fig. 1).

$|v^{\pm}\rangle|w^{\mp}\rangle \to -\frac{1}{2}|v^{\pm}\rangle|u^{\mp}\rangle$. Combining these terms, we get the desired postselected state of the thought experiment:

$$\frac{1}{\sqrt{3}}[|v^{+}\rangle|v^{-}\rangle + i|u^{+}\rangle|v^{-}\rangle + i|u^{-}\rangle|v^{+}\rangle], \tag{2}$$

cf. second line of Eq. (1), dropping the $|\gamma\rangle$ term which does not give rise to a coincidence click. The paradox is the same: $[d^{\pm} \Rightarrow u^{\mp}, p(d^{+}d^{-}) = \frac{1}{16} > p(u^{+}u^{-}) = 0]$.

In practice, neither the bunching nor the implications $[d^{\pm} \Rightarrow u^{\mp}]$ will be perfect. It is therefore necessary to derive an inequality describing the predictions of LHV theories. An LHV theory simultaneously predicts the results for complementary c/d and u/v measurements for any given value of the hidden variables. The predicted results on one side must be independent of the measurement performed on the other side. Probabilities such as $p(u^{+}, c^{+}; u^{-}, c^{-})$ denote the fraction of all hidden variable values that give the results shown in the brackets. These probabilities are not directly measurable. However, measurable probabilities such as $p(u^{+}; u^{-})$ are derived from them by adding all possible outcomes for the complementary measurements on each side:

$$p(u^{+}; u^{-}) = p(u^{+}, d^{+}; u^{-}, d^{-}) + p(u^{+}, d^{+}; u^{-}, c^{-}) + p(u^{+}, c^{+}; u^{-}, d^{-}) + p(u^{+}, c^{+}; u^{-}, c^{-}).$$

Since probabilities are positive, this implies

$$p(u^{+}; u^{-}) \geq p(u^{+}, d^{+}; u^{-}, d^{-}). \tag{3}$$

The expression for $p(d^{+}; d^{-}) = p(u^{+}, d^{+}; u^{-}, d^{-}) + p(u^{+}, d^{+}; v^{-}, d^{-}) + p(v^{+}, d^{+}; u^{-}, d^{-}) + p(v^{+}, d^{+}; v^{-}, d^{-})$ allows us to rewrite Eq. (3) as:

$$p(d^{+}; d^{-}) \leq p(u^{+}; u^{-}) + p(u^{+}, d^{+}; v^{-}, d^{-}) + p(v^{+}, d^{+}; u^{-}, d^{-}) + p(v^{+}, d^{+}; v^{-}, d^{-}). \tag{4}$$

One can use the equalities (derived as above)

$$p(d^{+}; v^{-}) = p(u^{+}, d^{+}; v^{-}, c^{-}) + p(u^{+}, d^{+}; v^{-}, d^{-}) + p(v^{+}, d^{+}; v^{-}, c^{-}) + p(v^{+}, d^{+}; v^{-}, d^{-}),$$
$$p(v^{+}; d^{-}) = p(v^{+}, c^{+}; u^{-}, d^{-}) + p(v^{+}, c^{+}; v^{-}, d^{-}) + p(v^{+}, d^{+}; u^{-}, d^{-}) + p(v^{+}, d^{+}; v^{-}, d^{-}),$$

to bound the last three terms on the right-hand side:

$$p(d^{+}; v^{-}) + p(v^{+}; d^{-}) \geq p(u^{+}, d^{+}; v^{-}, d^{-}) + p(v^{+}, d^{+}; u^{-}, d^{-}) + p(v^{+}, d^{+}; v^{-}, d^{-}).$$

Using this inequality in Eq. (4) gives the final result:

$$p(d^{+}; d^{-}) \leq p(u^{+}; u^{-}) + p(d^{+}; v^{-}) + p(v^{+}; d^{-}), \tag{5}$$

which is similar to the Clauser-Horne inequality [12].

We now discuss the experimental requirements to violate this inequality. The quality of the bunching depends on the distinguishability of the photons emerging from the central beam splitter. The parts of their wave packets that are distinguishable do not bunch and are either both reflected, both transmitted, or end up on the same side. The case in which they are both reflected is equivalent to each particle remaining in its own interferometer and leads to a $c^{+}c^{-}$ click. The case in which they are both transmitted, however, leads to the photons each emerging randomly from the last beam splitters giving an equal amount of $c^{+}c^{-}$, $c^{+}d^{-}$, $d^{+}c^{-}$, and $d^{+}d^{-}$ clicks. The implications $[d^{\pm} \Rightarrow u^{\mp}]$ remain unaltered since the only way a d^{+} or a d^{-} click can arise is from the amplitude in which both photons swap interferometers, which can occur only if the photons emerge in u^{+} and u^{-}. Consequently, the quality of the implications depends only on the quality of the interferometers, which can be made high by careful alignment. By contrast, the quality of the annihilation poses a significant restriction. A $u^{+}u^{-}$ event arising from the distinguishable amplitudes both swapping interferometer only leads to a $d^{+}d^{-}$ click with $p = \frac{1}{4}$, contributing 4 times more $u^{+}u^{-}$ than $d^{+}d^{-}$ events. Even assuming perfectly working interferometers $[p(d^{\pm}; v^{\mp}) = 0]$, this leads to the requirement that the probability that the photons are distinguishable $p(\text{disting.})$ be less than $\frac{1}{8} = 12.5\%$ for the inequality (5) to be violated.

In our experiment (Fig. 2) a Ti:sapphire mode-locked laser produces light pulses of 120 fs duration, centered at a wavelength of 780 nm, at a repetition rate of 82 MHz. The light is frequency doubled using a β-barium borate (β-BBO) crystal. The frequency doubled light arrives at a second, 2 mm thick β-BBO crystal where it is down-converted [13] to produce pairs of near-degenerate photons having orthogonal polarization. To make the photons less distinguishable, two 3 nm bandwidth filters were placed in arms e^{+} and e^{-}, together with a half-wave plate at 45° in arm e^{+} to align the polarizations. For a detailed discussion on the effect of filters on bunching in this type of system, see [14]. The light then passes through the setup and is detected at c^{+}, c^{-}, d^{+}, and d^{-}. The outer beam splitters are mounted on piezoelectrically driven translation stages used to tune the length of the v arms.

We measure simultaneous clicks between detectors on the left and on the right ($c^{+}c^{-}$, $c^{+}d^{-}$, $c^{-}d^{+}$, $d^{+}d^{-}$), using coincidence logic to distinguish genuine two-photon events. To measure in the c/d or u/v bases, we close the shutters in arms v and w as appropriate. For example, if the w^{-} arm is blocked, then the photons reaching c^{-} and d^{-} must have come from arm v^{-}. Thus to measure, say, the number of $d^{+}v^{-}$ events, $N(d^{+}v^{-})$, we measure $N(d^{+}c^{-}) + N(d^{+}d^{-})$ with the shutter in arm w^{-} closed.

For a fair measurement of the contradiction Eq. (5), the right-hand side should not be underestimated by the measurement technique. To guarantee this, we measured the rates $N(d^{+}d^{-})$, $N(c^{+}d^{-})$, $N(d^{+}c^{-})$, and $N(c^{+}c^{-})$ under

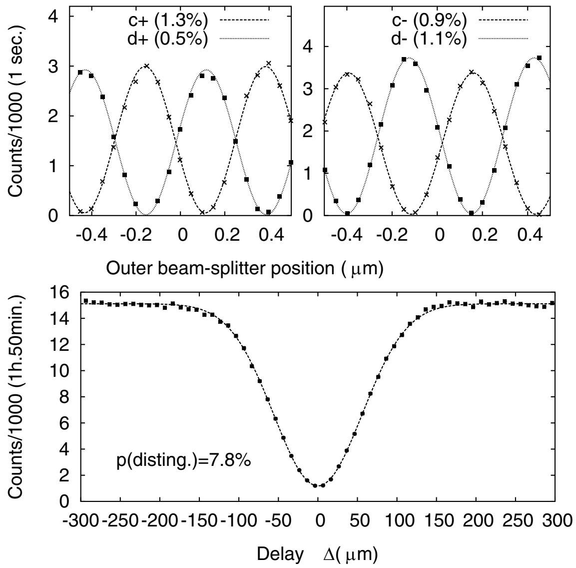

FIG. 3. Top: Interference fringes that characterize the output of each interferometer at the time of the experiment. The probabilities that a photon emerges in the d arm deduced from the fits are given in the plots. Bottom: $N(u^+u^-)$ as a function of delay Δ. The probability that the photons are distinguishable p(disting.) at zero delay is given by $p(\text{disting.}) = N(u^+u^-)_{\Delta=0}/N(u^+u^-)_{\Delta\gg 0} = 8\%$.

all combinations of blocking arms $v^\pm$ and $w^\pm$ and ensured the efficiencies were all within 10% of each other, with the d^+d^- efficiency less than the others in all configurations.

Figure 3 shows the quality of the interferometers and bunching at the time of the experiment. The probability of a d^+d^- click from two photons accidentally emerging in the d arms is less than 0.55%. The quality of the bunching is above the threshold required to measure a violation [$p(\text{disting.}) = 8\% < 12.5\%$].

Figure 4 shows a comparison between the measured $N(d^+d^-)$ and the LHV bound on it, given by the sum of the measured $N(d^+v^-)$, $N(d^-v^+)$, and $N(u^+u^-)$. We find a violation of the LHV inequality Eq. (5) by 12 standard deviations. The violation is consistent with the quantum mechanical predictions based on the probability of bunching and the detection efficiencies in our setup.

In conclusion, we have implemented Hardy's thought experiment consisting of two interacting Mach-Zehnder interferometers, demonstrating the contradiction between quantum mechanics and LHV theories in a striking way. It should be noted that the concept of creating entanglement by influencing a single photon interferometer with another photon also plays a crucial role in optical approaches to quantum logic gates [15].

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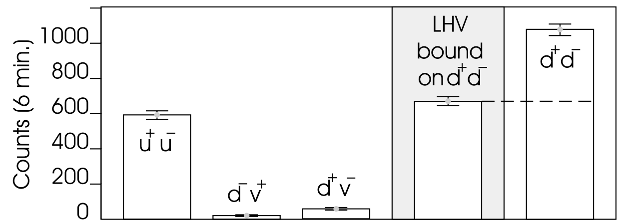

FIG. 4. The three measurements on the left $N(u^+u^-)$, $N(d^+v^-)$, and $N(v^+d^-)$ lead to an LHV bound on $N(d^+d^-)$ [Eq. (5)], shown in the shaded area. The measured $N(d^+d^-)$ violates this by 12 standard deviations.

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