Thermodynamics of a Fermi Liquid beyond the Low-Energy Limit

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We consider the nonanalytic temperature dependences of the specific heat coefficient, $C(T)/T$, and spin susceptibility, $\chi_s(T)$, of 2D interacting fermions beyond the weak-coupling limit. We demonstrate within the Luttinger-Ward formalism that the leading temperature dependences of $C(T)/T$ and $\chi_s(T)$ are linear in T, and are described by the Fermi liquid theory. We show that these temperature dependences are universally determined by the states near the Fermi level and, for a generic interaction, are expressed via the spin and charge components of the exact backscattering amplitude of quasiparticles. We compare our theory to recent experiments on monolayers of He^3.

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The Landau Fermi liquid (FL) theory states that the low-energy properties of an interacting fermionic system are determined by fermions in the vicinity of the Fermi surface, and are similar to that of weakly interacting quasiparticles. At the lowest temperatures, when the decay of quasiparticles can be neglected, the specific heat $C(T) \propto T$ and spin susceptibility $\chi_s(T) = \text{const}$ of a FL differ from the corresponding quantities for the Fermi gas only via the renormalizations of the effective mass and g factor [1]. However, this low-temperature limit of the FL theory, considered by Landau, cannot tell whether the subleading terms in T are analytic or not, and whether they come only from low-energy states (and are therefore described by the FL theory) or from the states far away from the Fermi surface.

For noninteracting fermions, the subleading terms in $C(T)/T$ and $\chi_s(T)$ scale as T^2 and come from high-energy states. However, it was found in the 1960s that in 3D systems, the leading correction to $C(T)/T$ due to interaction with either phonons [2] or paramagnons [3] is nonanalytic in T ($T^2 \ln T$) and comes from the states in the vicinity of the Fermi surface. The same result was later shown to hold for the electron-electron interaction [4]. More recently, it was shown by various groups [5–10] that the temperature dependence of $C(T)/T$ is also nonanalytic in 2D and starts with a linear-in-T term. The same behavior was also found for the uniform spin susceptibility [6,8–10]. Two of us have shown [6] that, to second order in short-range interaction, these linear-in-T terms originate exclusively from the scattering of fermions with zero total momentum and either small or near $2k_F$ momentum transfers ("backscattering").

In this Letter, we consider the specific heat and spin susceptibility for a generic 2D Fermi liquid. The leading (constant) terms in $C(T)/T$ and $\chi_s(T)$ in a generic Fermi liquid are expressed via the two harmonics—$F_c^{(1)}$ and $F_s^{(0)}$—of the quasiparticle interaction function $F(\theta)$ or, equivalently, via the same harmonics of the scattering amplitude $A(\theta)$ (c and s refer to charge and spin components). We show that, although the linear-in-T terms in $C(T)/T$ and $\chi_s(T)$ are also universally expressed via the scattering amplitude, they are determined by $A(\theta)$ at a *particular angle* $\theta = \pi$, rather than by $A(\theta)$ averaged over the Fermi surface. As there is no simple relation between $A(\pi)$ and $F(\pi)$, these subleading terms cannot be simply expressed via the Landau function. The only exception is the Coulomb interaction in the high-density (small r_s) regime, when the $\mathcal{O}(T)$ term in $C(T)/T$ can be expressed via $F(\pi)$.

To shorten the presentation, we discuss in some detail the calculation for $C(T)$, and then present the result for $\chi_s(T)$, which can be obtained in a similar manner [11]. The most straightforward way to obtain $C(T)$ beyond the leading term in T is to find the thermodynamic potential $\Xi(T)$ within the Luttinger-Ward approach [12] and then use the relation $C(T) = -T\partial^2\Xi/\partial^2 T$. The thermodynamic potential Ξ is expressed as

$$\Xi = \Xi_0 - 2T\sum_{\omega_n}\int\frac{d^2k}{4\pi^2}\left[\ln[G_0G^{-1}] - \Sigma G + \sum_\nu \frac{1}{2\nu}\Sigma_\nu G\right], \tag{1}$$

where Ξ_0 is the thermodynamic potential of the free Fermi gas per unit area, $G_0 = (i\omega_n - \epsilon_k)^{-1}$, $G = (i\omega_n - \epsilon_k + \Sigma)^{-1}$, ω_n is the Matsubara frequency, Σ is the exact (to all orders in the interaction) self-energy, and Σ_ν is the skeleton self-energy of order ν. Both Σ_ν and the sum $\Sigma = \sum_\nu \Sigma_\nu$ are evaluated at *finite* T. Diagrams associated with the first two terms in (1) correspond to the self-energy insertions into the free thermodynamic potential $\Xi_0 = 2T\Sigma_{\omega_n}\int d^2k/(2\pi)^2 \ln G_0(\omega_n, k)$, diagrammatically represented by a loop [Fig. 1(1a)]. One can readily verify that such diagrams simply renormalize the constant term in $C(T)/T$. The nonanalytic temperature dependence of $C(T)/T$ comes from the third term in (1).

To understand the origin of the nonanalyticity in $\Xi(T)$, consider first a weak short-range interaction $U(q)$. To

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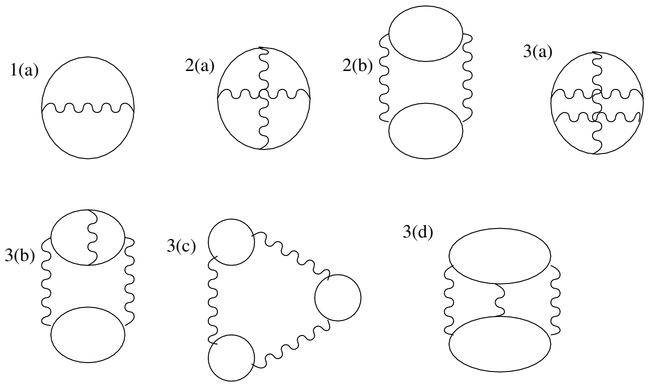

FIG. 1. Nontrivial second-order and third-order diagrams for the thermodynamic potential. For the Coulomb potential, diagrams (1a), (2b), and (3c) represent ring series.

second order in U, the skeleton term gives rise to diagrams (2a) and (2b) in Fig. 1. Assume momentarily that U is a constant. Then each of the two diagrams can be reexpressed as a product of two particle-hole bubbles $\Pi(q, \Omega_n)$, so that

$$\delta\Xi = -\frac{1}{2}TU^2\sum_{\Omega}\int\frac{d^2q}{4\pi^2}\Pi^2(q, \Omega_n), \qquad (2)$$

where $\delta\Xi \equiv \Xi - \Xi_0$.

It is intuitively clear that the nonanalyticity in $\delta\Xi$ should be related to that in $\Pi(q, \Omega_n)$. There are two regions of q where Π is nonanalytic. The first region is near $q = 0$, where $\Pi(q, \Omega_n) = -(m/2\pi)(1 - |\Omega_n|/[\Omega_n^2 + (v_F q)^2]^{1/2})$. For $v_F q \gg |\Omega_n|$, the Landau-damping term ($|\Omega_n|/q$) is nonanalytic in q. This nonanalyticity leads to a long-range tail of $\Pi(r, \Omega_n)$ in real space: $\Pi(r, \Omega_n) \propto |\Omega_n|/r$. The second region is near $2k_F$, where $\Pi(q, \Omega_n) = -(m/2\pi)[1 - (\bar{q} + \sqrt{\bar{q}^2 + \bar{\Omega}_n^2})^{1/2}]$, with $\bar{q} = (q - 2k_F)/2k_F$ and $\bar{\Omega} = \Omega/2k_F v_F$. The singularity at $\bar{q} = 0$ and $\Omega_n = 0$ is known as the Kohn anomaly. The nonanalyticity in $\delta\Xi(T)$ comes from the *dynamic* Kohn anomaly, which is the term $|\bar{\Omega}|/\sqrt{|\bar{q}|}$ in $\Pi(\bar{q}, \Omega_n = 0)$ for $-\bar{q} \gg |\Omega_n|/v_F$. This term leads to a long-range *dynamic* Friedel oscillation: $\Pi(r, \Omega_n) \propto |\Omega_n|\cos(2k_F r)/\sqrt{r}$.

Integrating in Eq. (2) over the two momentum regions where $\Pi(q, \Omega_n)$ is nonanalytic, we find that each of these two regions contributes a logarithmic singularity of the form $\Omega_n^2 \ln|\Omega_n|$, the prefactors being the same. This logarithmic singularity is the key effect. Had it been absent, the Matsubara sum of Ω_n^2 would have been controlled by high frequencies, of order E_F, and would have led to an analytic expansion $\delta\Xi = \text{const} + T^2 + \cdots$. The presence of the logarithm changes the story, as now the frequency sum contains a universal contribution from frequencies of order T. Using the Euler-Maclaurin summation formula, we obtain

$$\delta\Xi = -\frac{4\pi u^2 T^3}{v_F^2}S(M), \qquad S(M) \equiv \sum_{m=0}^{M} m^2 \ln\frac{M}{m} = \frac{1}{9}M^3 - \frac{1}{12}M - \frac{\zeta(3)}{4\pi^2} + \frac{1}{360M} + \cdots, \qquad (3)$$

where $u = mU/2\pi$, and $M \sim E_F/2\pi T \gg 1$. Those terms in (3) that depend on M yield a regular expansion for $\delta\Xi$ in powers of T^2, whereas the M-independent term [the third term in the second line of Eq. (3)] gives rise to a nonanalytic $\delta\Xi \propto T^3$, and hence to a T^2 term in $C(T)$.

Next, we take a more careful look at which four-fermion vertices actually contribute to the nonanalytic part of $C(T)$. For the $2k_F$ part, the answer follows immediately from the observation that, for a given direction of $\mathbf{q}$, the Kohn anomaly comes from the internal fermionic momenta near $\mathbf{q}/2$ and $-\mathbf{q}/2$ in both bubbles in diagrams (2a) and (2b) of Fig. 1. The relevant vertex then has the momentum structure $(\mathbf{k}, -\mathbf{k}; -\mathbf{k}, \mathbf{k})$, which corresponds to backscattering.

For the $q = 0$ part, the momentum structure is less obvious as, at the first glance, the internal momenta in the two bubbles in diagrams (2a) and (2b) of Fig. 1 are uncorrelated. However, the logarithmic singularity of the momentum integral comes only from the $|\Omega_n|/q$ term in each polarization bubble. One can show that this term comes from internal momenta that are nearly orthogonal to $\mathbf{q}$. Since the relevant momenta in the two bubbles are almost orthogonal to the same vector $\mathbf{q}$, they must be either nearly parallel or nearly antiparallel to each other. We verified that the contribution from the near parallel momenta, i.e., from forward scattering, vanishes and the full result comes from nearly antiparallel momenta. This implies that the $q = 0$ contribution to $\delta\Xi$ involves a vertex with the momentum structure $(\mathbf{k}, -\mathbf{k}; \mathbf{k}, -\mathbf{k})$. This vertex is also a part of the backscattering amplitude.

We can now extend our second-order analysis to a finite-range interaction $U(q)$. That only backscattering is relevant means that only $U(0)$ and $U(2k_F)$ contribute to the T term in $C(T)/T$. The contribution from diagram (2b) of Fig. 1 is proportional to $U^2(0) + U^2(2k_F)$, whereas that from the diagram (2a) is proportional to $U(0)U(2k_F)$. Collecting the prefactors, we obtain for the nonanalytic part of the specific heat $\Delta C \equiv C(T) - \gamma T$

$$\Delta C(T)/T = -(u_0^2 + u_{2k_F}^2 - u_0 u_{2k_F})\frac{3m\zeta(3)}{\pi}\frac{T}{E_F}, \qquad (4)$$

where $u_0 = mU(0)/2\pi$ and $u_{2k_F} = mU(2k_F)/2\pi$. This agrees with the result obtained in Ref. [6] by expressing $C(T)/T$ via the self-energy.

Consider now what happens when we add higher-order terms in U. They lead to two types of corrections: self-energy corrections to the fermionic lines in the two bubbles and corrections to the four-fermion vertices. The self-energy corrections are of the FL type: they account for the appearance of the quasiparticle Z factors and for the replacement of the bare fermionic mass m by m^*. Vertex corrections generate terms with more bubbles. A generic diagram of nth order has n bubbles. To obtain a T^3 contribution to $\Xi(T)$, we need to take dynamic, Ω_n/Q terms from two bubbles out of n and set $\Omega_n = 0$, $Q \to 0$ in the rest $n-2$ bubbles, because any extra power of Ω_n/Q eliminates the logarithmic singularity in the frequency integrand in $\delta\Xi(T)$. It is intuitively plausible that once two dynamic bubbles are chosen at the nth order, the rest of the nth order diagram constitutes the nth order correction to the static four-point vertex. If this conjecture is true, the series of the diagrams for the nonanalytic T^3 term in the thermodynamic potential can be reexpressed in terms of the two-bubble diagrams in which $U(0)$ and $U(2k_F)$ are replaced by exact *static* vertices $\Gamma(\mathbf{k}, -\mathbf{k}; \mathbf{k}, -\mathbf{k})$ and $\Gamma(\mathbf{k}, -\mathbf{k}; -\mathbf{k}, \mathbf{k})$. Accordingly, $\Delta C(T)/T$ is given by the same expression as in (4), but with Γ instead of U.

This conjecture, however, needs to be verified as different diagrams for the thermodynamic potentials contain different combinatorial factors, and it is *a priori* unclear whether these factors, combined with those counting the number of ways two dynamic bubbles can be chosen, give the right coefficients in the perturbative series for the static vertices. To verify that this is the case, we evaluated explicitly the T^3 term in the thermodynamic potential to third order in $U(q)$, and compared the result with that given by the two-bubble diagrams with the renormalized static vertices, evaluated independently. We found that the two expressions are identical. In what follows, we assume that this equivalence survives to all orders in $U(q)$.

The renormalization from $U(0)$ and $U(2k_F)$ to $\Gamma(\mathbf{k}, -\mathbf{k}; \mathbf{k}, -\mathbf{k})$ and $\Gamma(\mathbf{k}, -\mathbf{k}; -\mathbf{k}, \mathbf{k})$ includes static corrections coming from the states both away from and near to the Fermi surface [the latter produce powers of static $\Pi(\Omega_n = 0, Q \to 0) = -m/2\pi$ [1]]. In other words, $\Gamma(\mathbf{k}, -\mathbf{k}; \mathbf{k}, -\mathbf{k})$ and $\Gamma(\mathbf{k}, -\mathbf{k}; -\mathbf{k}, \mathbf{k})$ include all vertex corrections except for the terms coming from the dynamic part of the polarization bubble. In conventional notations [1], the fully renormalized $\Gamma(\mathbf{k}, -\mathbf{k}; \mathbf{k}, -\mathbf{k})$ and $\Gamma(\mathbf{k}, -\mathbf{k}; -\mathbf{k}, \mathbf{k})$ are then related to $\Gamma^k(\theta = \pi)$, which is the limit of $\Omega = 0$ and $q \to 0$ of $\Gamma(\mathbf{k}, \mathbf{p}, \mathbf{k}+\mathbf{q}, \mathbf{p}-\mathbf{q})$, where both $\mathbf{k}$ and $\mathbf{p}$ are on the Fermi surface and θ is the angle between these two vectors.

The matrix (in the spin space) $\hat{\Gamma}^k(\pi)$ is related to the quasiparticle scattering amplitude via $\hat{A}(\pi) = Z^2\hat{\Gamma}^k(\pi)$ [1]. Decomposing the scattering amplitude for a spin-invariant interaction into the charge and spin components, A_c and A_s, as

$$\hat{A}(\pi) = \frac{\pi v_F^*}{k_F}[A_c(\pi)\hat{I} + A_s(\pi)\hat{\sigma}\cdot\hat{\sigma}] \tag{5}$$

and comparing (5) with a similar decomposition for $\hat{\Gamma}^k(\pi)$

$$\hat{\Gamma}^k(\pi) = \Gamma^k(\mathbf{k}, -\mathbf{k}; \mathbf{k}, -\mathbf{k})\hat{I} - (1/2)\Gamma^k(\mathbf{k}, -\mathbf{k}; -\mathbf{k}, \mathbf{k}) \times (\hat{I} + \hat{\sigma}\cdot\hat{\sigma}), \tag{6}$$

we obtain

$$Z^2\Gamma^k(\mathbf{k}, -\mathbf{k}, \mathbf{k}, -\mathbf{k}) = \frac{\pi v_F^*}{k_F}[A_c(\pi) - A_s(\pi)];$$
$$Z^2\Gamma^k(\mathbf{k}, -\mathbf{k}; -\mathbf{k}, \mathbf{k}) = -2\frac{\pi v_F^*}{k_F}A_s(\pi). \tag{7}$$

Substituting $Z^2\Gamma^k(\mathbf{k}, -\mathbf{k}, \mathbf{k}, -\mathbf{k})$ instead of $U(0)$ and $Z^2\Gamma^k(\mathbf{k}, -\mathbf{k}; -\mathbf{k}, \mathbf{k})$ instead of $U(2k_F)$ into (4), we find the nonanalytic part of $C(T)$ in a generic Fermi liquid

$$\Delta C(T)/T = -\left(\frac{3\zeta(3)}{2\pi}\right)\left(\frac{m^*}{k_F}\right)^2[A_c^2(\pi) + 3A_s^2(\pi)]T. \tag{8}$$

A similar generalization of the second-order result for $\chi_s(T)$ [6] yields

$$\Delta\chi_s(T) = \frac{m^*}{4k_F^2}\chi_s(0)A_s^2(\pi)T, \tag{9}$$

where $\chi_s(0)$ is the spin susceptibility at $T = 0$.

Equations (8) and (9) are the two main results of this Letter. We see that the nonanalytic parts in $C(T)$ and $\chi_s(T)$ are parametrized by two scattering amplitudes, $A_c(\pi)$ and $A_s(\pi)$. These amplitudes are the new parameters in an extended version of the FL theory, which includes nonanalytic terms.

Two comments are in order here. First, the amplitudes $A_{c,s}(\pi)$ describe full vertices with zero total momentum, and therefore diverge at the Kohn-Luttinger pairing instability. Even above T_c, $A_{c,s}(\pi)$ depend on T logarithmically, $1/(a + b\ln T/E_F)$, because of the renormalizations in the Cooper channel [13]. This extra logarithmic dependence on T is likely to be small for T relevant to the experiments (see below), and we neglect it. Second, Eqs. (8) and (9) are valid for a circular Fermi surface (even if the quasiparticle dispersion differs from $k^2/2m$). For the anisotropic Fermi surface, the linear-in-T dependences of $\Delta C(T)/T$ and $\Delta\chi_s(T)$ survive as long as the Fermi surface has no inflection points, but the prefactors involve the curvature of the dispersion. For Fermi surfaces with inflection points, the powers of T change [14].

Notice also that $A_c(\pi)$ and $A_s(\pi)$ cannot be simply expressed in terms of the quasiparticle interaction function $F(\theta)$, which, we recall, is related to $Z^2\hat{\Gamma}^\omega(\theta) = (\pi v_F^*/k_F)[F_c(\theta)\hat{I} + F_s(\theta)\hat{\sigma}\cdot\hat{\sigma}]$. The harmonics $A_a^{(n)}$ and $F_a^{(n)}$ ($a = c, s$) are simply related [$A_a^{(n)} = F_a^{(n)}/(1 + F_a^{(n)})$ in 2D], but $A_a(\pi)$ is expressed only via infinite series of harmonics of the Landau function: $A_a(\pi) = \Sigma_{n=0}^{\infty}(-1)^n F_a^{(n)}/(1 + F_a^{(n)})$. A simple relation between $\hat{A}(\pi)$ and $\hat{F}(\pi)$ does exist if the interaction $U(q)$ is strongly peaked at $q = 0$. In this situation, only corrections due to $U(0)$ matter. These corrections come from ring diagrams and can be summed up exactly with the result $A_c(\pi) = F_c(\pi)/[1 + F_c(\pi)]$. If, in addition, $F_s(\theta) \ll 1$, the contribution to the specific heat from $F_c(\pi)$ dominates, and the singular term in the specific heat becomes

$$\Delta C(T)/T = -\frac{3m\zeta(3)}{4\pi}\frac{T}{E_F}\left(\frac{F_c(\pi)}{1 + F_c(\pi)}\right)^2. \tag{10}$$

This agrees with Ref. [15]. For the Coulomb interaction, $F_c(\pi) \to \infty$. The charge amplitude $F_c(\pi)$ drops out from (10), and the singular term in the specific heat becomes independent of r_s [7,15].

Scattering amplitudes in Eqs. (8) and (9) can be extracted from a measurement of $C(T)/T$ and $\chi_s(T)$ on the same system. To the best of our knowledge, a linear-in-T dependence of χ_s has not been measured yet. However, the linear temperature dependence of $C(T)/T$ has been observed in several experiments on fluid monolayers of He^3 adsorbed on graphite [16–18]. To a reasonable accuracy, the data can be fitted into a form $C/(NT/E_F^*) = \gamma(T/E_F^*)$, where N is the density per unit area in a fluid monolayer, $E_F^* = E_F(m/m^*)$ and $\gamma(x) \approx a - bx$ for small x [18]. Both a and b vary somewhat with N, but the variation is not dramatic, and to reasonable accuracy $a \sim 3$–3.3 and $b \sim 10$–14 [19]. According to Eqs. (3) and (8), $a = \pi^2/3 \approx 3.3$ and $b = 0.9[A_c^2(\pi) + 3A_s^2(\pi)]$. A fit to the data then yields $A_c^2(\pi) + 3A_s^2(\pi) \approx 11$–$15.5$ [19]. To estimate $A_c(\pi)$ and $A_s(\pi)$ separately, we assume that the scenario of "almost localized fermions" [20], which describes successfully the properties of bulk He^3, is applicable to the 2D case as well. In this scenario, the interaction in the charge channel is strong, whereas that in the spin channel is moderate. A strong interaction in the charge channel means that $F_c^{(n)} \gg 1$, in which case the consecutive terms in series for $A_c(\pi)$ almost cancel each other, and the result is likely to be small. A precise value for $A_c(\pi)$ depends on how $F_c^{(n)}$ decrease with n. However, in two model cases $F_c^{(n)} = g/(1+n^2)$ and $F_c^{(n)} = ge^{-n}$, we obtained almost identical results: $A_c^2(\pi) \approx 0.25$ in the limit of $g \gg 1$. This suggests that the observed value $A_c^2(\pi) + 3A_s^2(\pi) \approx 11$–$15.5$ is almost entirely due to the spin part of the amplitude. Neglecting $A_c^2(\pi)$, we obtain $|A_s(\pi)| \approx 1.9$–2.3. If the $n = 0$ harmonic of F_s dominates the result for $A_s(\pi)$, i.e., $A_s(\pi) \approx F_s^{(0)}/(1 + F_s^{(0)})$, then $F_s^{(0)} \approx -(0.66$–$0.7)$, which is consistent with the value of $F_s^{(0)} \approx -0.75$ in bulk He^3 [21].

Notice also that if $A_c(\pi)$ can be neglected compared to $A_s(\pi)$, $\Delta C(T)/T$ and $\delta\chi_s(T)$ contain only one unknown parameter $[A_s(\pi)]$. In this situation, the ratio $K = \Delta C(T)/(T\Delta\chi_s(T))$ is expressed only via the parameters describing the leading, analytic parts of $C(T)$ and χ_s: $K = -18\zeta(3)m^*/\pi\chi_s(0)$.

In addition, $A_s(\pi)$ determines the slope of the linear-in-T correction $\Delta\sigma(T)$ to the conductivity of a dirty 2D FL in the ballistic regime [22], which allows one to express $\delta\chi_s(T)$ in terms of $\Delta\sigma(T)$ as $\Delta\chi_s(T)E_F^*/\chi_s(0)T = [1 - \Delta\sigma(T)E_F^*/\sigma(0)T]^2/72$. These predictions are amenable to a direct experimental verification.

To summarize, in this Letter we showed that the 2D Fermi liquid theory describes not only the leading, constant terms in the specific heat coefficient $C(T)/T$ and the spin susceptibility $\chi_s(T)$, but also subleading, linear-in-T terms. We demonstrated that these terms come from backscattering (zero total momentum) processes, and are universally expressed via the spin and charge components of the scattering amplitude at the angle $\theta = \pi$. We extracted the spin component of the scattering amplitude from the experimental data on $C(T)$ for a monolayer of He^3.

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