# Ultrasoft X-Ray Bremsstrahlung Isochromatic Spectra from 300–2000 eV Electrons on Ar and Kr

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For the first time the bremsstrahlung effect was studied experimentally in the spectral region of 6.5–10 nm with 300–2000 eV electron scattering on Ar and Kr atoms. The isochromatic curves displayed maxima at electron energies of $\simeq 0.7$ keV (Ar) and $\simeq 1$ keV (Kr); their positions are almost independent of the radiation wavelength in the range studied. These experimental data cannot be treated in the framework of theory based on the first Born approximation. A phenomenological modification of the quasiclassical (soft-photon) approximation is proposed which gives a qualitative treatment of data.

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Bremsstrahlung (BS), which accompanies a charged particle passing through matter, is traditionally of great interest for radiation physics, plasma physics, biology, and medicine. In recent years the interaction of x-ray radiation with noble gases has been actively studied in connection with design of tabletop x-ray lasers [1] and investigation of synchrotron radiation interaction with clusters [2,3].

Historically, the experimental investigations of continuous x-ray radiation were long related to the design of thin and thinner anodes to study comprehensively the elemental process of photon radiation in charge particle-atom collision. The natural result of such tendencies was the use of gas target as an anode [4–6]. Using the gas targets made it possible to observe polarization bremsstrahlung (PBS) [7] under electron scattering on Xe atoms [8,9], to observe two-photon BS effect [10], and to measure absolute cross sections of BS of high-energy electrons on Ne, Ar, Kr, and Xe atoms [11].

A theory of BS under electron scattering on atom based on the first Born approximation (FBA) was given in works by Bethe, Heitler, and Sauter and reviewed in Ref. [12]. Integral BS cross section with exact account for the electron scattering phases in static atom potential approximation was calculated by Pratt with co-workers [13]. Although the absolute BS cross-section measurements [11] differ from the theoretical results (accounting also for PBS), the photon spectrum shape [5,6] accords very good with the theory [13]. It should, however, be mentioned that the experimental data were obtained for electrons with the energy high enough, 2.5–10 keV in Ref. [5] and 6 keV in Ref. [6]. No reliable experimental data on BS spectra are available for lower electron energies, which are of fundamental importance and special interest for numerous applications mentioned above.

The results reported in this Letter are intended to fill up this gap in experimental BS studies. The measured isochromatic curves, which describe the relationship between BS intensity of frequency $\omega$ and electron energy, $E$, are notably different from those predicted by FBA theory at low $E$, and coincide with the theory only at higher $E$. Unfortunately, comparison of our experimental data with the theory given in Ref. [13] is not possible currently, since the data [13] were obtained for the electron energy $E \geq 1$ keV. As it was mentioned in Ref. [13], at lower $E$ the atomic electrons cannot be considered only as a source of a static potential.

The analysis of the isochromatic curves, which were obtained in the previous experiments with a Xe atomic jet [14,15] for an angle $\simeq 90°$ between the directions of the electrons and analyzed photons, showed that the intensity increased with electron energy. Recall that according to the nonrelativistic theory, the BS cross section $\sigma_\gamma$ in scattering of a fast electron on a heavy particle with the charge $Z$ in the given geometry of observation can be given by the expression [16]:

$$\frac{d\sigma_\gamma}{d\omega} = \frac{2\alpha Z^2 mc^2 r_e^2}{E\omega}\left[2\frac{v'}{v} + \left(3 - \frac{v'^2}{v^2}\right)\log\frac{v+v'}{v-v'}\right], \quad (1)$$

where $r_e = e^2/mc^2 \simeq 2.8 \times 10^{-13}$ cm is the classical electron radius, $\alpha \simeq 1/137$, $v$, $v'$ are the electron velocities before and after collision, respectively.

Taking Eq. (1) into account, we conclude that the isochromatic cross section (ICCS) first increases from the threshold value of $E_0 = \hbar\omega$, as first observed by Korsunskii *et al.* [17] for solid targets at $\hbar\omega = 1.05$ MeV, and then falls off to $\sim 1/E$. The ICCS maximum is at

$$E_{\max} \simeq 1.53\hbar\omega. \quad (2)$$

As for our experiments [14], the increase in the ICCS for $\hbar\omega = 177.1$ eV was observed up to the maximum energy $E \simeq 600$ eV studied and was not changed by its falling off. In this Letter we report the first measured data on ICCS for Ar and Kr atoms with characteristic maxima on these curves [18].

 

The experimental unit is described comprehensively in Refs. [4,15]. It should be noted that the target was a spatially confined supersonic rare gas jet in a vacuum. Specially selected pressure and temperature of the gas at the nozzle entrance ensured an atomic composition of the jet with a concentration of atoms $\simeq 10^{16}$ cm$^{-3}$ in the region of the jet-electron intersection. The electron beam intersected the jet at right angle to its axis. The x-ray radiation was taken in the solid angle $d\Omega \simeq 1.7 \times 10^{-3}$ sr and resolved into the spectrum with resolution 1 Å by a spectrometer monochromator. The x-ray radiation intensity was measured within a wavelength range 6.5–10 nm by a proportional counter filled with methane up to pressure $1.5 \times 10^4$ Pa. The measurements were carried out under single collisions. The angle $\chi$ between the directions of scattered electrons and BS was 97°. The total experimental error was 10%.

The measured quantity was the intensity of radiation in a given frequency region $d\omega$ and at solid angle $d\Omega$ for different electron energies. It can be given by the expression:

$$\hbar\,\omega n_e n_a v V \frac{d\sigma_\gamma}{d\omega d\Omega}, \tag{3}$$

where $n_e$ and $n_a$ are the concentrations of electrons and atoms, respectively, $v$ is the electron velocity, and $V$ is the volume of jet excitation region.

The product $n_e v$ only differs by an electron charge value from the current density $j$. During the experiment the values of $j$ and $n_a$ were kept constant on changing the electron energy. Therefore, the experimental dependence of BS intensity on electron energy is the direct representation of changes of differential cross section of BS, $d\sigma_\gamma/(d\omega d\Omega)$, with electron energy.

The experimental dependencies of ICCS (in relative units) on scattered electron energy at electron-Ar atom collisions are shown in Fig. 1 for several radiation wavelengths. Similar data for Kr are illustrated in Fig. 2. As is evident, the ICCS for both atoms are generally similar and independent on photon energy. At the same time the positions of the maxima are different: $E_{\max}(\mathrm{Ar}) \simeq 700$ eV, while $E_{\max}(\mathrm{Kr}) \simeq 1$ keV.

A comparison of the experimental data with FBA is presented in Fig. 3 (Ar) for a wavelength of 9 nm ($\hbar\omega =$ 138 eV) and in Fig. 4 (Kr) for a wavelength of 8.5 nm ($\hbar\omega = 146$ eV). The atomic form factors, which determine the screening effect of the atomic electrons [12], were calculated using the GAUSSIAN-98 program and with the help of the data reported in Ref. [19]. Both calculation methods gave the same results. The calculations were performed both without of stripping effect [20–22] (dashed curves) and with account of outer shell stripping $3p^6$ for Ar and $4p^6$ for Kr (pointed curves). The profiles of ICCS turned out to be similar, although the absolute values of cross sections differ considerably. In the figures both the calculations (with and without stripping) were combined in order to make a better comparison with the experiment. The arrows in Figs. 3 and 4 indicate the positions of the cross-section maxima according to the formula (2), which corresponds to BS in pure Coulomb field with the charge $Z$ (complete stripping).

One can see that the drop-down parts of ICCS, which are located to the right from the maxima, are well described by FBA theory. At the same time, the growing ICCS parts, which are to the left from the maxima, as well as the

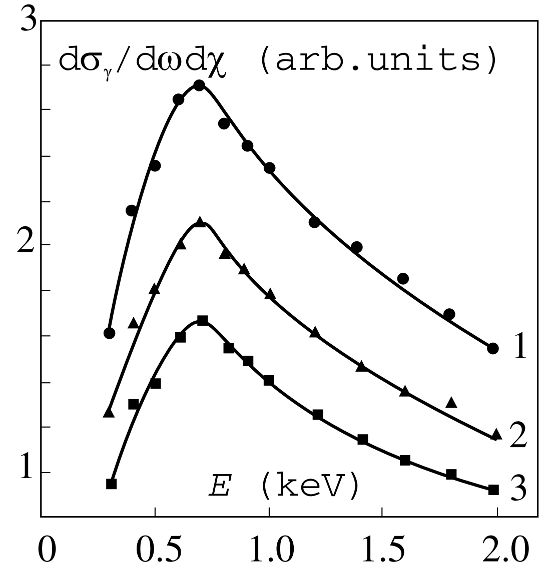


FIG. 1. The experimental dependencies of ICCS on the energy electron scattering on Ar atoms at 7 nm (1), 8 nm (2), and 9 nm (3).

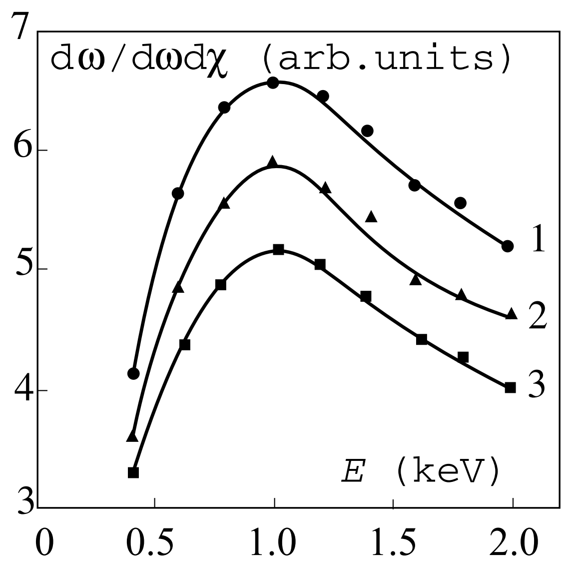


FIG. 2. The experimental dependencies of ICCS on the energy electron scattering on Kr atoms at 7.5 nm (1), 8 nm (2), and 8.5 nm (3).

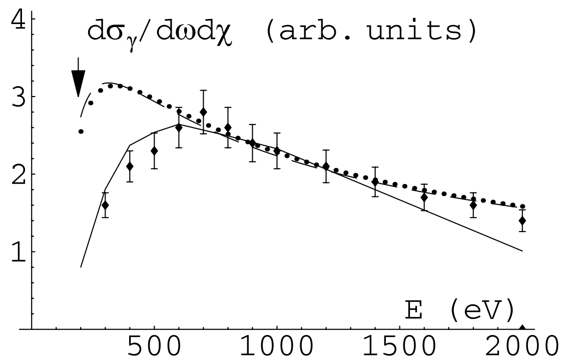


FIG. 3. The ICCS for Ar at $\hbar\omega = 138$ eV, $\chi = 97°$. The solid curve is the SPA with account of the phenomenological substitution (7), the dashed curve is the FBA theory without the account of stripping effect, and the pointed curve is the FBA theory with account of outer shell stripping. The arrow points to the maximum position by Eq. (2).

positions of these maxima themselves, are not described by FBA theory exactly enough. It should be mentioned that the observed difference from FBA cannot be attributed to PBS because at the frequencies studied PBS causes only a change in the atomic nucleus screening due to stripping effect. But the calculations show that stripping effect of external shells produces insignificant modifications in the shape of ICCS. At the same time, the screening effects due to the deeper shells are important; it is manifested by difference from the maxima positions predicted by FBA from those specified by the arrows in Figs. 3 and 4. The important role of screening near the end points of the photon spectra (corresponding to the low-$E$ parts of ICCS) was mentioned in Ref. [13]. It seems natural to us that the observed difference between theory and experiment is related to breaking of FBA.

To provide a theoretical treatment of observed specific features of BS spectra, we use soft-photon approximation (SPA) [23,24]. SPA was earlier used in BS theory in Ref. [25]. According to SPA the BS cross section of a soft photon

$$\hbar\,\omega \ll E \tag{4}$$

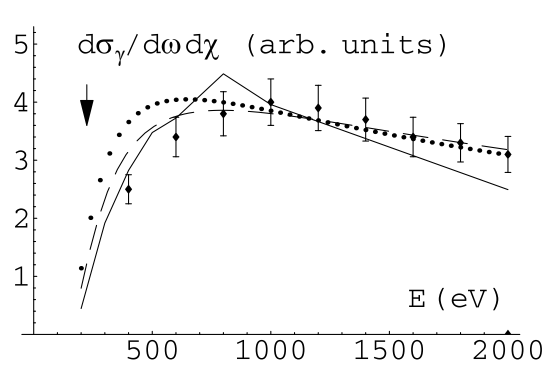


FIG. 4. The same, as in Fig. 3, for Kr at $\hbar\omega = 146$ eV.

under electron scattering by atom at angle $\theta$ is the product of ordinary elastic scattering cross section $d\sigma_{\rm el}(\theta)$ by the classical probability of BS in a frequency range $d\omega$ and solid angle $d\Omega$ with changing electron velocity $\mathbf{v} \to \mathbf{v}'$ [16]:

$$\frac{d\sigma_\gamma}{d\omega d\Omega} = \frac{\alpha[(\mathbf{v}-\mathbf{v}')\mathbf{k}]^2}{4\pi^2\omega^3} d\sigma_{\rm el}(\theta). \tag{5}$$

Here $\mathbf{k}$ is the wave vector of a photon, and the square brackets imply a vector product. If we align the axis $z$ along the vector $\mathbf{v}$, the following expression will be obtained after integration by azimuth angles of vectors $\mathbf{v}'$ and $\mathbf{k}$ and after the assumptions $v \simeq v'$:

$$\frac{d\sigma_\gamma}{\sin\chi d\chi d\omega} = \frac{2\pi\alpha E}{mc^2\omega}\int_0^\pi \sin\theta[\sin^2\theta(1+\cos^2\chi) + 2(1-\cos\theta)^2\sin^2\chi]\frac{d\sigma_{\rm el}(\theta)}{d\theta}d\theta. \tag{6}$$

Thus, to derive the differential cross section of BS, the differential cross section of elastic electron scattering by atom should be known. Unfortunately, the available experimental data on differential cross sections $\sigma_{\rm el}(\theta)$ cannot be used to recover the BS cross section. This is because the angle factor in the square brackets under the integral (6) takes a maximum value at $\theta \simeq \pi$, corresponding to the electron scattering backwards (Fig. 5). The differential cross section of ordinary fast electron scattering for such angles is small and is poorly studied experimentally (see, for example, [26]).

In our work the analysis of the ICCS by SPA was made by using the theoretical calculations of differential cross section of elastic scattering given in Ref. [27]. As the inequality (4) is poorly realized for low electron energies, we substituted

$$E \to (E - \hbar\omega)^2/E \tag{7}$$

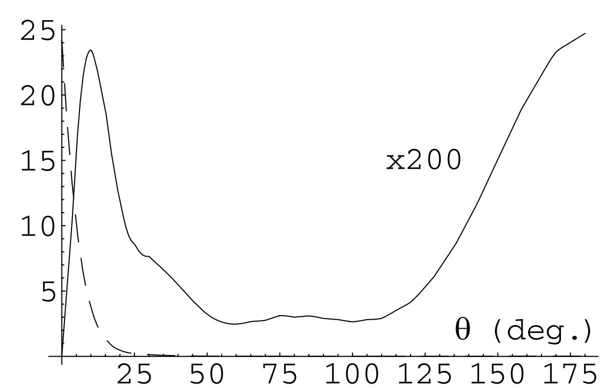


FIG. 5. The cross section of elastic scattering $d\sigma_{\rm el}/d\theta$, $2\pi$ Å$^2$ (dashed line) for Ar and the same quantity multiplied by the angular factor in the square brackets under the integral (6) (solid line) for electron energy $E = 600$ eV. The data from Ref. [27] are used in the calculations.

for the initial electron energy before the integral in Eq. (6). This substitution can be considered as a phenomenological refinement of SPA to a low electron energy region not contradictory to the condition (4). Unfortunately, the calculation accuracy in Ref. [27] is not so high for large angles of scattering and, besides, has a tendency to be at variance with experimental data for such angles [28]. The results of calculations by Eqs. (6) and (7) are shown in Figs. 3 and 4. Since the calculations in Ref. [27] were made with a large energy step, in particular, there are no data on cross sections for electron energies 1 keV $< E <$ 2 keV, the curves were plotted by the calculated points using the linear interpolation. Moreover, in case of Ar at $E = 2$ keV the cross section was assumed to be zero for scattering angles $\theta > 125°$, because the theoretical curves in this region display the chaotic behavior due to the inadequate calculation accuracy.

As seen, the ICCS calculated in terms of SPA (6) with the account of the phenomenological improvement (7) provide a qualitative correct description of the experimental data, the maximum positions for Ar and Kr included. The observed discrepancies between the calculated and measured data for high electron energies can seemingly be attributed to inadequate accuracy of the theoretical calculations of the cross section of electron-atom elastic scattering for large angles of scattering. So theoretical results obtained in the present work in framework of phenomenological modification of SPA should be considered as tentative.

Therefore the reported experimental data can stimulate the development of a consistent BS theory as well as theory of conventional electron scattering through large angles.

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