Effective Nucleon Masses in Symmetric and Asymmetric Nuclear Matter

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(Received 22 February 2005; published 7 July 2005)

The momentum and isospin dependence of the in-medium nucleon mass are studied. Two definitions of the effective mass, i.e., the Dirac mass m_D^* and the nonrelativistic mass m_{NR}^* which parametrizes the energy spectrum, are compared. Both masses are determined from relativistic Dirac-Brueckner-Hartree-Fock calculations. The nonrelativistic mass shows a distinct peak around the Fermi momentum. The proton-neutron mass splitting in isospin asymmetric matter is $m_{D,n}^* < m_{D,p}^*$ and opposite for the nonrelativistic mass, i.e., $m_{NR,n}^* > m_{NR,p}^*$, which is consistent with nonrelativistic approaches.

DOI: 10.1103/PhysRevLett.95.022302 PACS numbers: 21.65.+f, 14.20.Dh, 21.30.−x, 24.10.Cn

The introduction of an effective mass is a common concept to characterize the quasiparticle properties of a particle inside a strongly interacting medium. It is also a well established fact that the effective nucleon mass in nuclear matter or finite nuclei deviates substantially from its vacuum value [1–3]. However, there exist different definitions of the effective nucleon mass, which are often compared and sometimes even mixed up: the nonrelativistic effective mass m_{NR}^* and the relativistic Dirac mass m_D^*. These two definitions are based on completely different physical concepts. The nonrelativistic mass parametrizes the momentum dependence of the single particle potential. It is the result of a quadratic parametrization of the single particle spectrum. On the other hand, the relativistic Dirac mass is defined through the scalar part of the nucleon self-energy in the Dirac field equation which is absorbed into the effective mass $m_D^* = M + \mathrm{Re}\Sigma_s(k, k_F)$. This Dirac mass is a smooth function of the momentum. In contrast, the nonrelativistic effective mass—as a model independent result—shows a narrow enhancement near the Fermi surface due to an enhanced level density [1–3].

Although related, these different definitions of the effective mass have to be used with care when relativistic and nonrelativistic approaches are compared on the basis of effective masses. While the Dirac mass is a genuine relativistic quantity, the nonrelativistic mass m_{NR}^* can be determined from both relativistic as well as nonrelativistic approaches. A heavily discussed topic is in this context the proton-neutron mass splitting in isospin asymmetric nuclear matter. This question is of importance for the forthcoming new generation of radioactive beam facilities which are devoted to the investigation of the isospin dependence of the nuclear forces at its extremes. However, presently the predictions for the isospin dependence of the effective masses differ substantially [4].

Brueckner-Hartree-Fock (BHF) calculations [5,6] predict a proton-neutron mass splitting of $m_{NR,n}^* > m_{NR,p}^*$ in isospin asymmetric nuclear matter. This stands in contrast to relativistic mean-field (RMF) theory. When only a vector isovector ρ meson is included, Dirac phenomenology predicts equal masses $m_{D,n}^* = m_{D,p}^*$, while the inclusion of the scalar isovector δ meson, i.e., $\rho + \delta$, leads to $m_{D,n}^* < m_{D,p}^*$ [4,7]. When the nonrelativistic mass is derived from RMF theory, it shows the same behavior as the Dirac mass, namely, $m_{NR,n}^* < m_{NR,p}^*$ [4].

Relativistic *ab initio* calculations based on realistic nucleon-nucleon interactions, such as the Dirac-Brueckner-Hartree-Fock (DBHF) approach, are the proper tool to answer this question. However, results from DBHF calculations are still controversial. They depend strongly on approximation schemes and techniques used to determine the Lorentz and the isovector structure of the nucleon self-energy.

In one approach, originally proposed by Brockmann and Machleidt [8]—we call it the fit method in the following—one extracts the scalar and vector self-energy components directly from the single particle potential. Thus, mean values for the self-energy components are obtained where the explicit momentum dependence has already been averaged out. In symmetric nuclear matter, this method is relatively reliable, but the extrapolation to asymmetric matter introduces two new parameters in order to fix the isovector dependences of the self-energy components. This makes the procedure ambiguous, as has been demonstrated in [9]. Calculations based on this method predict a mass splitting of $m_{D,n}^* > m_{D,p}^*$ [10]. On the other hand, the components of the self-energies can directly be determined from the projection onto Lorentz invariant amplitudes. Projection techniques are complicated but more accurate. They have been used, e.g., in [11–13]. When projection techniques are used in DBHF calculations for asymmetric nuclear matter, a mass splitting of $m_{D,n}^* < m_{D,p}^*$ is found [9,14,15]. In the present work, we compare the Dirac and the nonrelativistic effective mass, both derived from the DBHF approach based on projection techniques, in symmetric and asymmetric nuclear matter.

In the relativistic Brueckner approach nucleons are dressed inside nuclear matter as a consequence of their two-body interactions with the surrounding particles. The in-medium interaction, i.e., the T matrix, is treated in the ladder approximation of the relativistic Bethe-Salpeter equation:

$$T = V + i\int VQGGT, \quad (1)$$

where V is the bare nucleon-nucleon interaction. The intermediate off-shell nucleons are described by a two-body propagator iGG. The Pauli operator Q prevents scattering to occupied states. The Green function G, which describes the propagation of dressed nucleons in the medium, fulfills the Dyson equation:

$$G = G_0 + G_0 \Sigma G. \quad (2)$$

G_0 denotes the free nucleon propagator, whereas the influence of the surrounding nucleons is expressed by the self-energy Σ. In the Brueckner formalism, this self-energy is determined by summing up the interactions with all the nucleons inside the Fermi sea F in Hartree-Fock approximation:

$$\Sigma = -i\int_F (\mathrm{Tr}[GT] - GT). \quad (3)$$

The coupled set of Eqs. (1)–(3) represents a self-consistency problem and has to be iterated until convergence is reached. The self-energy consists of scalar Σ_s and vector $\Sigma^\mu = (\Sigma_o, \mathbf{k}\Sigma_v)$ components:

$$\Sigma(k, k_F) = \Sigma_s(k, k_F) - \gamma_0 \Sigma_o(k, k_F) + \boldsymbol{\gamma} \cdot \mathbf{k}\Sigma_v(k, k_F). \quad (4)$$

The decomposition of the self-energy into the different Lorentz components (4) requires the knowledge of the Lorentz structure of the T matrix in (3). For this purpose the T matrix has to be projected onto covariant amplitudes. We use the subtracted T matrix representation scheme for the projection method described in detail in [13,15].

The effective Dirac mass is defined as

$$m_D^*(k, k_F) = \frac{M + \mathrm{Re}\Sigma_s(k, k_F)}{1 + \mathrm{Re}\Sigma_v(k, k_F)}; \quad (5)$$

i.e., it accounts for medium effects through the scalar part of the self-energy. The correction through the spatial Σ_v part is generally small [11,13,15].

The effective mass, which is usually considered in order to characterize the quasiparticle properties of the nucleon within nonrelativistic frameworks, is defined as

$$m_{NR}^* = |\mathbf{k}|[dE/d|\mathbf{k}|]^{-1}, \quad (6)$$

where E is the energy of the quasiparticle and $\mathbf{k}$ is its momentum. When evaluated at $k = k_F$, Eq. (6) yields the Landau mass related to the f_1 Landau parameter of a Fermi liquid [4,16]. In the quasiparticle approximation, i.e., the zero width limit of the in-medium spectral function, these two quantities are connected by the dispersion relation

$$E = \frac{\mathbf{k}^2}{2M} + \mathrm{Re}U(|\mathbf{k}|, k_F). \quad (7)$$

Equations (6) and (7) then yield the following expression for the effective mass:

$$m_{NR}^* = \left[\frac{1}{M} + \frac{1}{|\mathbf{k}|}\frac{d}{d|\mathbf{k}|}\mathrm{Re}U\right]^{-1}. \quad (8)$$

In a relativistic framework, m_{NR}^* is obtained from the corresponding Schrödinger equivalent single particle potential,

$$U(|\mathbf{k}|, k_F) = \Sigma_s - \frac{1}{M}(E\Sigma_o - \mathbf{k}^2\Sigma_v) + \frac{\Sigma_s^2 - \Sigma_\mu^2}{2M}. \quad (9)$$

An alternative would be to derive the effective mass from Eq. (6) via the relativistic single particle energy $E = (1 + \mathrm{Re}\Sigma_v)\sqrt{\mathbf{k}^2 + m_D^{*2}} - \mathrm{Re}\Sigma_o$. However, since the single particle energy contains relativistic corrections to the kinetic energy, a comparison to nonrelativistic approaches should be based on the Schrödinger equivalent potential (9) [16].

Thus, the nonrelativistic effective mass is based on a completely different physical idea than the Dirac mass, since it parametrizes the momentum dependence of the single particle potential. Hence, it is a measure of the nonlocality of the single particle potential U. The nonlocality of U can be due to nonlocalities in space, resulting in a momentum dependence, or in time, resulting in an energy dependence. In order to clearly separate both effects, one has to distinguish further between the so-called k mass and E mass [16]. The k mass is obtained from Eq. (8) at *fixed* energy, while the E mass is given by the derivative of U with respect to the energy at *fixed* momentum. A rigorous distinction between these two masses requires the knowledge of the off-shell behavior of the single particle potential U. As discussed, e.g., by Frick *et al.* [6], the spatial nonlocalities of U are mainly generated by exchange Fock terms, and the resulting k mass is a smooth function of the momentum. Nonlocalities in time are generated by Brueckner ladder correlations due to the scattering to intermediate states which are off shell. These are mainly short-range correlations which generate a strong momentum dependence with a characteristic enhancement of the E mass slightly above the Fermi surface [3,6,16]. The effective nonrelativistic mass defined by Eqs. (6) and (8) contains both nonlocalities in space and time and is given by the product of k mass and E mass [16]. It should therefore show such a typical peak structure around k_F. The peak reflects—as a model independent result—the increase of the level density due to the vanishing imaginary part of the optical potential at k_F, which is seen, e.g., in shell model calculations [2,3,16]. One has to account, however, for correlations beyond mean field or Hartree-Fock in order to reproduce this behavior.

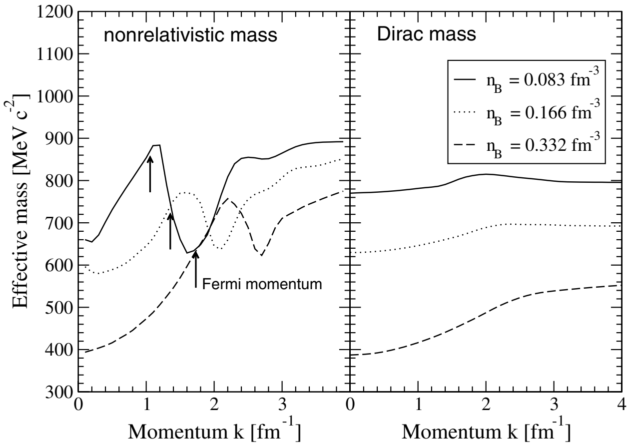

FIG. 1. The effective mass in isospin symmetric nuclear matter as a function of the momentum $k = |\mathbf{k}|$ at different densities.

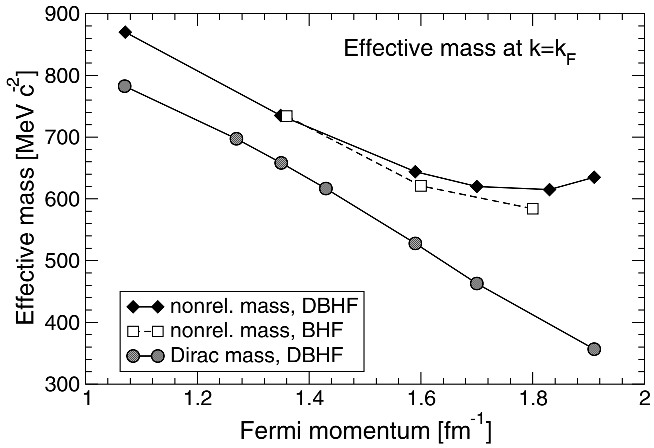

FIG. 2. The effective mass in isospin symmetric nuclear matter at $k = |\mathbf{k}| = k_F$ as a function of the Fermi momentum k_F.

The following results and discussions are based on the Bonn A nucleon-nucleon potential. However, they do not strongly depend on the particular choice of the interaction.

In Fig. 1 the nonrelativistic effective mass and the Dirac mass are shown as a function of momentum k at different Fermi momenta of $k_F = 1.07$, 1.35, and 1.7 fm^{-1}, which correspond to nuclear densities $n_B = 4k_F^3/6\pi^2 = 0.5n_0$, n_0, and $2n_0$, where $n_0 = 0.166\ \text{fm}^{-3}$ is the nuclear saturation density. The projection method reproduces a pronounced peak of the nonrelativistic mass slightly above k_F as it is also seen in nonrelativistic BHF calculations [16]. With increasing density, this peak is shifted to higher momenta and slightly broadened. The Dirac mass is a smooth function of k with a moderate momentum dependence. The latter is in agreement with the "reference spectrum approximation" used in the self-consistency scheme of the DBHF approach [15]. Both Dirac and nonrelativistic mass decrease in average with increasing nuclear density. For completeness it should be mentioned that, if m^*_{NR} is extracted directly from the single particle energy (6) instead from the potential (9), results are very similar. Differences occur only at high momenta, where relativistic corrections to the kinetic energy come into play.

In the relativistic framework the single particle potential and the corresponding peak structure of the nonrelativistic mass are the result of subtle cancellation effects of the scalar and vector self-energy components. This requires a very precise method in order to determine variations of the self-energies Σ, which are small compared to their absolute scale. The used projection techniques are the adequate tool for this purpose. Less precise methods yield only a small enhancement, i.e., a broad bump around k_F [11,16]. The extraction of mean self-energy components from a fit to the single particle potential is not able to resolve such a structure at all.

Figure 2 compares the density dependence of the two effective masses. Both the nonrelativistic and the Dirac mass are determined at the Fermi momentum $k = |\mathbf{k}| = k_F$ and shown as a function of k_F. Initially, the nonrelativistic mass decreases with increasing Fermi momentum k_F. However, at high values of the Fermi momentum k_F, it starts to rise again. The Dirac mass, in contrast, decreases continuously with increasing Fermi momentum k_F. In addition, results from nonrelativistic BHF calculations [17], based on the same Bonn A interaction, are also shown, and the agreement between the nonrelativistic and the relativistic Brueckner approach is quite good.

In Fig. 3 the neutron nonrelativistic and the Dirac mass are plotted for various values of the asymmetry parameter $\beta = (n_n - n_p)/n_B$ at fixed nuclear density $n_B = 0.166\ \text{fm}^{-3}$. An increase of β enhances the neutron density and thus has for the density of states the same effect as an increase of the density in symmetric matter. Another interesting issue is the proton-neutron mass splitting in asymmetric nuclear matter. Although the Dirac mass derived from the DBHF approach has a proton-neutron mass split-

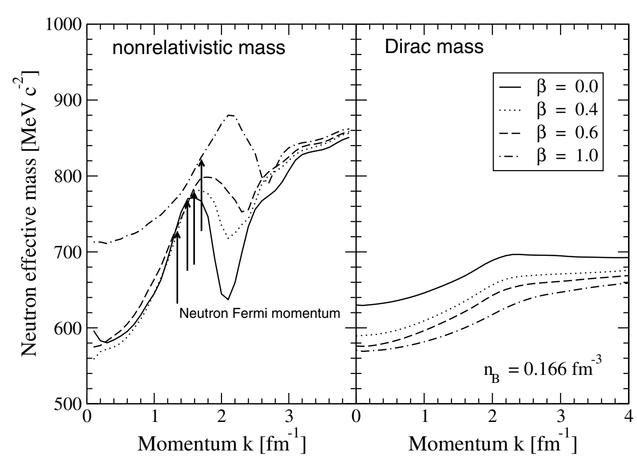

FIG. 3. Neutron effective mass as a function of the momentum $k = |\mathbf{k}|$ for various values of the asymmetry parameter β at fixed nuclear density $n_B = 0.166\ \text{fm}^{-3}$.

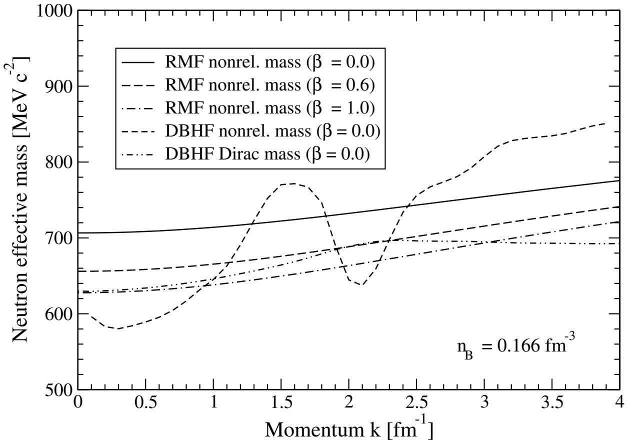

FIG. 4. Neutron effective mass obtained in the RMF approximation as a function of the momentum $k = |\mathbf{k}|$ at fixed nuclear density $n_B = 0.166\ \mathrm{fm}^{-3}$.

ting of $m^*_{D,n} < m^*_{D,p}$, as can be seen from Fig. 3, the nonrelativistic mass derived from the DBHF approach shows the opposite behavior; i.e., $m^*_{NR,n} > m^*_{NR,p}$, which is in agreement with the results from nonrelativistic BHF calculations [5,6]. In Fig. 3 only neutron masses are depicted, but the corresponding proton masses always behave opposite; i.e., a neutron mass which is decreasing (increasing) with asymmetry corresponds to a increasing (decreasing) proton mass.

Figure 4 demonstrates finally the influence of the explicit momentum dependence of the DBHF self-energy. In RMF theory the Dirac mass and the vector self-energy are momentum independent. The nonrelativistic mass is now determined from the RMF approximation to the single particle potential, i.e., neglecting the momentum dependence of the scalar Σ_s and vector fields Σ_o and Σ_v in Eqs. (5) and (9). The single particle energy is then given by $E_{\mathrm{RMF}} = (1 + \mathrm{Re}\Sigma_v(k_F))\sqrt{|\mathbf{k}|^2 + m_D^{*2}(k_F)} + \mathrm{Re}\Sigma_o(k_F)$. In Fig. 4 this "RMF" nonrelativistic mass is plotted for various values of the asymmetry parameter β at $n_B = 0.166\ \mathrm{fm}^{-3}$. For comparison the full DBHF nonrelativistic and Dirac masses for symmetric nuclear matter are shown as well. Because of the parabolic momentum dependence of E_{RMF}, the corresponding RMF mass has no bump or peak structure but is a continuously rising function with momentum. At $k = k_F$, it corresponds to the RMF Landau mass [16,18]. The RMF nonrelativistic mass decreases with an increasing asymmetry parameter. RMF theory predicts the same proton-neutron mass splitting for the Dirac and the nonrelativistic mass, i.e., $m^*_{D,n} < m^*_{D,p}$ and $m^*_{NR,n} < m^*_{NR,p}$. This is a general feature of the RMF approach [4]. Full DBHF theory is in agreement with the prediction of RMF theory concerning the Dirac mass. However, the mass splitting of the nonrelativistic mass is reversed due to the momentum dependence of the self-energies, respectively, the nonlocal structure of the single particle potential, which is neglected in RMF theory.

In summary, effective nucleon masses in isospin symmetric and asymmetric nuclear matter have been derived from the DBHF approach based on projection techniques. We compared the momentum and isospin dependence of the relativistic Dirac mass and the nonrelativistic mass which parametrizes the energy dependence of the single particle spectrum. First, the nonrelativistic effective mass shows a characteristic peak structure at momenta slightly above the Fermi momentum k_F, which indicates an enhanced level density near the Fermi surface. The Dirac mass is a smooth function of k with a weak momentum dependence. Second, the controversy between relativistic and nonrelativistic approaches concerning the proton-neutron mass splitting in asymmetric nuclear matter has been resolved. The Dirac mass shows a mass splitting of $m^*_{D,n} < m^*_{D,p}$, in line with RMF theory. However, the nonrelativistic mass derived from the DBHF approach has a reversed proton-neutron mass splitting $m^*_{NR,n} > m^*_{NR,p}$, which is in agreement with the results from nonrelativistic BHF calculations.

The authors would like to thank B. L. Friman and H. Müther for helpful discussions.

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