Intrinsic Spin Hall Effect in the Two-Dimensional Hole Gas

B. Andrei Bernevig and Shou-Cheng Zhang
Department of Physics, Stanford University, Stanford, California 94305, USA
(Received 19 November 2004; published 29 June 2005)

We show that two types of spin-orbit coupling in the 2 dimensional hole gas, with and without inversion symmetry breaking, contribute to the intrinsic spin-Hall effect. Furthermore, the vertex correction due to impurity scattering vanishes in both cases, in sharp contrast to the case of usual Rashba coupling in the electron band. Recently, the spin-Hall effect in a hole doped GaAs semiconductor has been observed experimentally by Wunderlich *et al.* [Phys. Rev. Lett. **94**, 047204 (2005).]. From the fact that the lifetime broadening is smaller than the spin splitting, and the fact impurity vertex corrections vanish in this system, we argue that the observed spin-Hall effect should be in the intrinsic regime.

DOI: 10.1103/PhysRevLett.95.016801 PACS numbers: 72.25.Dc, 72.25.Hg, 73.43.−f, 85.75.−d

Recent theoretical work predicts dissipationless spin currents induced by an electric field in semiconductors with spin-orbit coupling [1–3]. The spin current is related to the electric field by the response equation

$$j_j^i = \sigma_s \epsilon_{ijk} E_k, \tag{1}$$

where j_j^i is the current of the ith component of the spin along the direction j and ϵ_{ijk} is the totally antisymmetric tensor in three dimensions. Because both the electric field and the spin current are even under time reversal, the spin current could be dissipationless or intrinsic, independent of the scattering rates. The response equation (1) was derived in [1] for p-doped semiconductors described by the Luttinger model of the spin-3/2 valence band, and in [2] for the 2-dimensional electron gas (2DEG) described by the Rashba model [2].

The spin-Hall effect predicted in [1,2] is fundamentally different from the extrinsic spin-Hall [4,5] effect due to Mott-type skew scattering by impurities. The intrinsic spin-Hall effect arises from the spin-orbit coupling of the host semiconductor band and has a finite value in the absence of impurities, while the extrinsic spin-Hall effect arises purely from the spin-orbit coupling to the impurity atoms; it is not a bulk effect like the ordinary Hall effect and its magnitude is typically many orders of magnitude smaller. The issue of impurity contributions to the spin-Hall effect has been intensively investigated theoretically. Remarkably, the authors of [6] calculated the vertex corrections due to impurity scattering in the Rashba model of the electron band in the context of the spin-Hall effect (other groups had also computed the vertex correction earlier [7,8]) and found that the vertex correction completely cancels the spin-Hall effect [9–11]. On the other hand, a number of numerical calculations show the spin-Hall effect independent of the disorder in the weak disorder limit [12,13]. The problem of the vertex correction does not occur in the Luttinger model of the hole band [14]. In fact, the vertex correction is identically zero, rendering the original prediction of [1] exact in the clean limit. Other work [15] claims that the spin accumulation at the edge of the sample due to spin current vanishes, due to the spin-torque term that shows up in the transport equations. Their analysis is entirely in the ground state of the system (not in the presence of electric field) and also assumes no spin relaxation which is crucial to obtaining edge spin accumulation.

Experimental observation of the spin-Hall effect has been recently reported in a electron doped sample [16] in a 2-dimensional hole gas (2DHG) [17]. In this Letter, we analyze the 2DHG experiment. In order to firmly establish the intrinsic spin-Hall effect, one needs to establish two things. First, the experimental system needs to be in the clean limit, which is the case of the 2DHG experiment [17]. Second, the effect must be robust to disorder in the clean limit. We show that the spin-Hall effect in the 2DHG arises from two contributions, one from the Luttinger Hamiltonian describing the splitting between the light and the heavy hole bands, and one from the structural inversion asymmetry (SIA) of the 2DHG band, with spin splitting scaling as k^3 [18,19]. This is different from the Rashba Hamiltonian of the 2DEG band, where the spin splitting scales with k. Remarkably, we find that the vertex correction due to impurity scattering vanishes for both types of spin-orbit couplings in the 2DHG band, in sharp contrast to the case of 2DEG. While the calculation details are complicated, the intuitive reason is simple: the two types of current vertices in the 2DHG have p- and d-wave symmetries, respectively, and vanish when averaged over s wave impurity scatterers. These two key facts establish a firm foundation to interpret the recent experiment [17] in terms of the intrinsic spin-Hall effect, where impurities play an unessential role. The present Letter also helps clarify the theoretical controversy about the disorder contribution to the spin-Hall effect, as it shows that the cancellation in the Rashba Hamiltonian for electrons is not generic, but rather the opposite.

The Hamiltonian for a 2DHG is a sum of both Luttinger and spin-$\vec{S} = 3/2$ SIA terms:

$$H = \left(\gamma_1 + \frac{5}{2}\gamma_2\right)\frac{k^2}{2m} - \frac{\gamma_2}{m}(\vec{k}\cdot\vec{S})^2 + \alpha(\vec{S}\times\vec{k})\cdot\hat{z}, \quad (2)$$

where the confinement of the well in the z direction quantizes the momentum on this axis. The crucial difference between the SIA term for 2DHG and the Rashba term for the 2DEG lies in the fact that S is a spin 3/2 matrix, describing both the light (LH) and the heavy (HH) holes. For the first heavy- and light-hole bands, the confinement in a well of thickness a is approximated by the relation $\langle k_z\rangle = 0$, $\langle k_z^2\rangle \approx (\pi\hbar/a)^2$. The energies are

$$E_{\pm}^{\mathrm{LH}(s=1);\mathrm{HH}(s=-1)} = \frac{\gamma_1}{2m}k^2 \pm \frac{1}{2}\alpha k + s_{\mathrm{LH,HH}}\sqrt{\alpha^2k^2 \pm \frac{\alpha\gamma_2}{m}k(k^2+\langle k_z^2\rangle) + \frac{\gamma_2^2}{m^2}(k^4+\langle k_z^2\rangle^2 - k^2\langle k_z^2\rangle)}, \quad (3)$$

where $s_{\mathrm{LH}} = 1$ and $s_{\mathrm{HH}} = -1$. The heavy- and light-hole bands are split at the Γ point by $\Delta = 2\gamma_2\langle k_z^2\rangle/m$ [20,21]. Depending on the confinement scale a the Luttinger term is dominant for a not too small, while the SIA term becomes dominant for infinitely thin wells, which correspond to high junction fields.

Expansion for small $k \ll \langle k_z\rangle$ shows the spin splitting of the HH bands is k^3 whereas the spin splitting of the LH bands is k, in agreement with [18,22]: $E_+^{\mathrm{HH}} - E_-^{\mathrm{HH}} = \frac{3}{8}\alpha(\alpha^2 - 4\frac{\gamma_2^2}{m^2}\langle k_z^2\rangle)k^3/\frac{\gamma_2^2}{m^2}\langle k_z^2\rangle^2 + \mathcal{O}(k^5)$ and $E_+^{\mathrm{LH}} - E_-^{\mathrm{LH}} = 2\alpha k + \mathcal{O}(k^3)$. Figure 1 gives a typical band structure for GaAs ($\gamma_1 = 6.92$, $\gamma_2 = 2.1$) similar to the sample used in [17]. The SIA splitting is calculated to be $\alpha \approx 10^5$ m/s. We can expand the second term in the anisotropic Luttinger Hamiltonian in terms of Clifford algebra of Dirac Γ matrices $\{\Gamma^a, \Gamma^b\} = 2\delta_{ab}I_{4\times4}$ $(a, b = 1, \ldots, 5)$ [3]. Since $\langle k_z\rangle = 0$ and $\langle k_z^2\rangle \neq 0$ we see that the effect of confinement renders the d^a's of [3]: $(\vec{k}\cdot\vec{S})^2 = d_a\Gamma^a$, $d_1 = 0$, $d_2 = 0$, $d_3 = -\sqrt{3}k_xk_y$, $d_4 = -\frac{\sqrt{3}}{2}(k_x^2 - k_y^2)$, and $d_5 = -\frac{1}{2}\times(2\langle k_z^2\rangle - k_x^2 - k_y^2)$.

Calculation with the full Hamiltonian (2) is analytically impossible, so we concentrate on limits which maintain analytic predictability. We first consider the case of a small junction field and neglect the SIA term:

$$H = \frac{\gamma_1}{2m}(k_x^2 + k_y^2 + \langle k_z^2\rangle) + \frac{\gamma_2}{m}d_a\Gamma^a. \quad (4)$$

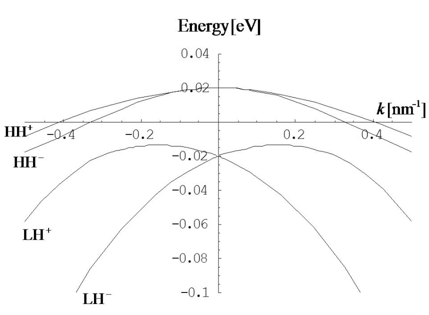

FIG. 1. Approximate band structure of the 2DHG ($\Delta = 40$ meV, spin splitting of the HH band at k_F roughly 5 meV). The confinement produces a Γ point gap between the LH and HH bands, whereas the SIA produces an inner splitting of the HH and LH bands into HH$^\pm$ and LH$^\pm$.

The energies are $E_{\mathrm{LH,HH}} = \frac{\gamma_1}{2m}(k^2 + \langle k_z^2\rangle) \pm d$, $(d = \sqrt{d_ad_a} = \sqrt{k^4 + \langle k_z^2\rangle^2 - \langle k_z^2\rangle k^2})$. In the experiment recently reported [17], the Γ-point gap is of order $\Delta E = [2\frac{\gamma_2}{m}\langle k_z^2\rangle \approx 2\frac{\gamma_2}{m}(\pi\hbar/a)^2] = 40$ meV, which corresponds to an $a = 8.3$ nm thick quantum well. Our simplistic prediction fares well with the quoted value 3–4 nm [17].

We want to compute the response function $Q_{ij}^l(i\nu_m) = -\frac{1}{V}\int_0^\beta\langle TJ_i^l(u)J_j\rangle e^{i\nu_m u}du$ of the spin current $J_i^l = \frac{1}{2}\{S^l, \frac{\partial H}{\partial k_j}\}$ to an electric current $J_j = \frac{\partial H}{\partial k_j}$. The spin conductance is defined as $\sigma_{ij}^l = \lim_{\omega\to0}\frac{Q_{ij}^l(\omega)}{-i\omega}$ and gives

$$\sigma_{ij}^l = \frac{\eta_{ab}^l}{V}\sum_k\frac{n_+^F - n_-^F}{d^3}\left[2\frac{m}{\gamma_2}d_b\frac{\partial d_a}{\partial k_j}\frac{\partial\varepsilon}{\partial k_i} + \epsilon_{abcde}d_e\frac{\partial d_c}{\partial k_i}\times\frac{\partial d_d}{\partial k_j}\right],$$

where η_{ab}^l is a tensor defined in [3], $n_\pm^F$ are the Fermi functions of the two bands, and $\varepsilon = \frac{\gamma_1}{2m}(k_x^2 + k_y^2 + \langle k_z^2\rangle)$ is the kinetic energy. The last term is the conserved spin conductance [3] (which represents the response of the spin projected onto the HH and LH bands [3]), whereas the first term is the contribution of the nonconserved part of the spin. Momentum integration yields the only nonzero components: $\sigma_{12}^3 = -\sigma_{21}^3$:

$$\sigma_{12}^3 = \frac{1}{4\pi}\left(\frac{3}{2}\frac{\gamma_1}{2\gamma_2}\left[\frac{2(k^2+\langle k_z^2\rangle)}{3\sqrt{k^4+\langle k_z^2\rangle^2-\langle k_z^2\rangle k^2}} - \ln[2\sqrt{k^4+\langle k_z^2\rangle^2-\langle k_z^2\rangle k^2} + 2k^2 - \langle k_z^2\rangle]\right] + \frac{2\langle k_z^2\rangle - k^2}{\sqrt{k^4+\langle k_z^2\rangle^2-\langle k_z^2\rangle k^2}}\right)_{k=k_{\mathrm{HH}}}^{k=k_{\mathrm{LH}}}, \quad (5)$$

where k_{LH}, k_{HH} are the Fermi momenta of the light- and heavy-hole bands. For the experimental data [17], the light-hole band is fully occupied, so $k_{\mathrm{LH}} = 0$ while $\sqrt{\langle k_z^2\rangle} = 3.7\times10^{-26}$ kg m/s and $k_{\mathrm{HH}} = 3\times10^{-26}$ kg m/s. The first two terms are due to the nonconserved spin and for GaAs, $\sigma_{1,2}^{3(\mathrm{noncons})} = \frac{0.7}{8\pi}$. The last term is the conserved spin conductance $\sigma_{12}^{3(\mathrm{cons})} = 0.6\times\frac{1}{4\pi}$. The total spin conductance is $\sigma_{12}^3 = \frac{1.9}{8\pi}$, in good agreement with the numerical estimate in [17]. For infinite confinement, $\sqrt{\langle k_z^2 \rangle} \rightarrow \infty$ the spin conductance from the Luttinger term vanishes as we enter the SIA regime.

We now investigate the effect of disorder on the Luttinger spin-Hall conductance by focusing on the vertex correction. The free Green function in our system is defined as $G_0(\mathbf{k}, i\omega_n) = [i\omega_n - H]^{-1} = (i\omega_n - \epsilon(\mathbf{k}) + \frac{\gamma_2}{m} d_a \Gamma_a)/((i\omega_n - \epsilon(\mathbf{k}))^2 - \gamma_2^2 d^2/m^2)$. We model the disorder as randomly distributed, spin-independent identical defects $V(\mathbf{r}) = u\Sigma_i \delta(\mathbf{r} - \mathbf{R}_i)$. In the Born approximation, the self-energy is related to the free Green function $\Sigma(i\omega_n) = n_{\mathrm{imp}} u^2 \int \frac{d\mathbf{k}}{(2\pi)^2} G_0(\mathbf{k}, i\omega_n)$ Since $\int d\mathbf{k} d_3 = \int d\mathbf{k} d_4 = 0$, the self-energy is an isotropic function of $\vec{k}$:

$$\Sigma(i\omega_n) = n_{\mathrm{imp}} u^2 \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{i\omega_n - \epsilon(\mathbf{k}) + \frac{\gamma_2}{m} d_5 \Gamma_5}{(i\omega_n - \epsilon(\mathbf{k}))^2 - \gamma_2^2 d^2/m^2}, \tag{6}$$

where $d_5 = -\frac{1}{2}(2\langle k_z^2 \rangle - k^2)$. This is different from the bulk Luttinger case, where the $d_5(k)$ integral over $\vec{k}$ vanishes as well, but the difference is not essential. The full impurity Green function is $G(\mathbf{k}, i\omega_n) = G_0(\mathbf{k}, i\omega_n + \Sigma(i\omega_n))$. The current vertex satisfies a Bethe-Salpeter equation similar to [7]. Similar to [14], since the Green function is an even function of the momentum components, while the charge current operator $V_j = \partial H/\partial k_j$ is momentum odd in the components k_j (because the Hamiltonian H is even in $\vec{k}$), it turns out that the free vertex cancels $\int d\mathbf{k} G(\mathbf{k}, i\omega_n) V_j(\mathbf{k}) G(\mathbf{k}, i\omega_n - i\nu_m) = 0$. Hence the vertex correction which is an iterative function of the free vertex vanishes as well [14]. This result holds even for the anisotropic Luttinger model, since it uses only the parity (P) invariance of the Hamiltonian. It is in fact a theorem that a P-invariant Hamiltonian has vanishing current vertex correction due to s-wave scattering, in the Born approximation. The effect of small SIA splitting ($\alpha k_F \ll \Delta$) on the Luttinger spin-Hall conductance enters perturbatively only to order α^2.

We now turn to the opposite case of strongly confined quantum wells, in which the SIA term is likely to dominate. The relatively large 5 meV measured splitting [17] of the HH band makes this regime significant for the experiment. We model the system by a Γ point gap Δ plus a spin 3/2 Rashba term $\alpha(\vec{k} \times \vec{S})\hat{z}$ [23]. We compute the spin conductance and expand it in terms of the ratio between the SIA spin splitting and the Γ point gap, $\frac{\alpha k_f}{\Delta} < 1$. The spin conductance gets a contribution from the HH band $\sigma_{12}^{3(\mathrm{HH})} = \frac{9}{8\pi}(1 + \frac{\alpha^2 m_{\mathrm{HH}}}{2\Delta})$. For infinitely thin quantum wells, $\Delta \rightarrow \infty$, the HH spin conductance is $9/8\pi$, which is the same as that obtained [19] by using the effective HH Hamiltonian Eq. (7). The second term is the first order finite thickness correction. If the Fermi level is low enough, there is also a light-hole band contribution of order $\sigma_{12}^{3(\mathrm{LH})} = \frac{1}{8\pi}(1 + \frac{3\alpha^2 m_{\mathrm{LH}}}{2\Delta})$.

Since working with spin 3/2 matrices is cumbersome and the LH states are fully filled [17], we project onto the HH states and use the truncated Hamiltonian [18,19]:

$$H = \frac{k^2}{2m} + \beta(k_-^3 \sigma_+ - k_+^3 \sigma_-) \equiv \frac{k^2}{2m} + \lambda_i(k)\sigma_i, \tag{7}$$

where $i = x, y$, $\lambda_1 = \beta k_y(3k_x^2 - k_y^2)$, and $\lambda_2 = \beta k_x(3k_y^2 - k_x^2)$, with β a constant. H above becomes exact in the limit of very confined quantum wells. The spin-Hall conductance in the disorder-free case is $9/8\pi$, as per the above analysis. We also calculated the spin polarization in the bulk due to an applied electric field and found it to be zero. Let $\lambda(k) = \sqrt{\lambda_i \lambda_i}$. The Fermi sphere is isotropic since the energy levels are $E_\pm = \frac{k^2}{2m} \pm \lambda$. The disorder-free Green function is $G_0(\mathbf{k}, i\omega_n) = \frac{1}{2}\sum_{s=\pm} \frac{1 + s\hat{\lambda}_i \sigma_i}{i\omega_n - E_s}$, where $\hat{\lambda}_i = \lambda_i/\lambda$. The self-energy for s-wave scattering of electrons becomes a state-independent constant (not a matrix) $\Sigma(i\omega_n) = n_{\mathrm{imp}} u^2 \int \frac{d\mathbf{k}}{(2\pi)^2} G_0(\mathbf{k}, i\omega_n)$, where n_{imp} is the density of impurities while u is the impurity potential strength. Since the spin-orbit coupling is small ($\ll E_F$), the density of states at zero order is a constant $D = m/2\pi\hbar^2$ (the term $\lambda = \beta k^3$ contributes with only a first order correction). The full Green function in the presence of impurities is $G(\mathbf{k}, i\omega_n) = G_0(\mathbf{k}, i\omega_n + \Sigma(i\omega_n))$. The spin-dependent part of the charge current operator $V_j(k) = \partial H/\partial k_j$ has d-wave symmetry [for example, the spin-dependent part of V_x is $6\beta k_x k_y \sigma_x + 3\beta(k_y^2 - k_x^2)\sigma_y$] and vanishes when integrated over the isotropic Fermi surface. This is the deep intuitive reason as to why the vertex correction cancels in this case, as we rigorously show below. By contrast, in the electron-band Rashba case, the spin-dependent part of the charge operator is a constant. The current vertex function $K_j(\mathbf{k}, i\omega_n, i\nu_m) = \langle G(\mathbf{k}, i\omega_n) V_j(k) G(\mathbf{k}, i\omega_n + i\nu_m)\rangle$ is a matrix function that does not commute with either the charge current operator or the Green function. $\langle \ldots \rangle$ is an impurity average. It satisfies the Bethe-Salpeter equation:

$$\begin{aligned} K_j(\mathbf{k}, i\omega_n, i\nu_m) = {} & G(\mathbf{k}, i\omega_n) V_j(\mathbf{k}) G(\mathbf{k}, i\omega_n - i\nu_m) \\ & + n_{\mathrm{imp}} u^2 G(\mathbf{k}, i\omega_n) \int \frac{d\mathbf{q}}{(2\pi)^2} \\ & \times K_j(\mathbf{q}, i\omega_n, i\nu_m) G(\mathbf{k}, i\omega_n - i\nu_m). \end{aligned} \tag{8}$$

Integrating both the righ-t and the left-hand side over the momentum $\mathbf{k}$, we see that the vertex correction $\Delta V_j(i\omega_n, i\nu_m) = \int \frac{d\mathbf{q}}{(2\pi)^2} K_j(\mathbf{q}, i\omega_n, i\nu_m)$ satisfies

$$\begin{aligned} \Delta V_j = {} & \int \frac{d\mathbf{k}}{(2\pi)^2} G(\mathbf{k}, i\omega_n) V_j(\mathbf{k}) G(\mathbf{k}, i\omega_n - i\nu_m) \\ & + n_{\mathrm{imp}} u^2 \int \frac{d\mathbf{k}}{(2\pi)^2} G(\mathbf{k}, i\omega_n) \Delta V_j G(\mathbf{k}, i\omega_n - i\nu_m). \end{aligned} \tag{9}$$

$\Delta V_j(i\omega_n, i\nu_m)$ is a 2×2 matrix, and we decompose it in the basis of the identity and the 3 Pauli matrices:

$$\Delta V_j(i\omega_n, i\nu_m) = \sum_{\mu=0}^{3} \Lambda_j^\mu(i\omega_n, i\nu_m)\sigma^\mu, \mu = 0, \ldots, 3, \quad (10)$$

where $\sigma^0 = I_{2\times 2}$, the identity matrix, and $\sigma^{1,2,3}$ are the 3 Pauli matrices. The $\Lambda_j^\mu(i\omega_n, i\nu_m)$ are scalars. By introducing the decomposition in the vertex equation, multiplying to the left of both sides of the equals by a σ_α matrix and taking the trace of the above equation, we obtain

$$2\Lambda_j^\nu = A_j^\nu(i\omega_n, i\nu_m) + \sum_{\mu=0}^{3} \Lambda_j^\mu M^{\nu\mu}(i\omega_n, i\nu_m)$$

$$M^{\nu\mu} = n_{\mathrm{imp}}u^2 \int \frac{d\mathbf{k}}{(2\pi)^2} \mathrm{Tr}[\sigma^\nu G(\mathbf{k}, i\omega_n)\sigma^\mu G(\mathbf{k}, i\omega_n - i\nu_m)]$$

$$A_j^\nu = \int \frac{d\mathbf{k}}{(2\pi)^2} \mathrm{Tr}[\sigma^\nu G(\mathbf{k}, i\omega_n) V_j(\mathbf{k}) G(\mathbf{k}, i\omega_n - i\nu_m)]. \quad (11)$$

By expanding and evaluating $M^{\nu\mu}$ [observing that $\int d\mathbf{k}\, \lambda^i(k)\lambda^j(k) \sim \delta_{ij}$ as well as $G_s^R G_s^A = \frac{2\pi\tau}{\hbar}\delta(\epsilon - E_s)$, where R, A stand for the retarded and advanced Green functions, and $\tau = \hbar^3/n_{\mathrm{imp}}u^2 m$] we observe that it is diagonal in μ, ν; that is $M^{\nu\mu} = \delta_{\nu\mu}$. Expanding the traces in Eq. (11), and since $\lambda_0(k) = \lambda_3(k) = 0$ it is easy to observe that (after azimuthal integration) $A_j^0(i\omega_n, i\nu_m) = A_j^3(i\omega_n, i\nu_m) = 0$ and hence the vertex corrections $\Lambda_j^0(i\omega_n, i\nu_m) = \Lambda_j^3(i\omega_n, i\nu_m) = 0$. We now have for the vertex correction $\Lambda_j^\nu(i\omega_n, i\nu_m) = A_j^\nu(i\omega_n, i\nu_m)$, $\nu = 1, 2$ and $j = x, y$, where after expanding the traces

$$A_j^\nu = \sum_{s,s'=\pm} \int \frac{d\mathbf{k}}{(2\pi)^2} \times \frac{(s+s')\frac{k_j}{m}\hat{\lambda}_\nu + 2ss'\hat{\lambda}_\nu \frac{\partial\lambda}{\partial k_j} + (1-ss')\frac{\partial\lambda_\nu}{\partial k_j}}{2(z - E_s)(z' - E_{s'})}, \quad (12)$$

with $z = i\omega_n + \Sigma(i\omega_n)$, $z' = i\omega_n - i\nu_m + \Sigma(i\omega_n - i\nu_m)$. We now compute this for $\nu = 1$, the case $\nu = 2$ being identical. Let $j = 1$ and we find for the numerator of the integrand in Eq. (12),

$$\frac{k_y k_x(3k_x^2 - k_y^2)}{k^3}\left[(s+s')\frac{1}{m} + 6ss'\beta k\right] + 6\beta k_x k_y(1 - ss').$$

This vanishes due to the integral over the azimuthal angle of $\mathbf{k}$ and hence $A_1^1(i\omega_n, i\nu_m) = 0$. For the case $\nu = 1$, $j = 2$, the numerator of Eq. (12) gives

$$\frac{k_y^2(3k_x^2 - k_y^2)}{k^3}\left[(s+s')\frac{1}{m} + 6ss'\beta k\right] + 3\beta(k_x^2 - k_y^2)(1 - ss'),$$

which also vanishes upon azimuthal angle integration $A_2^1(i\omega_n, i\nu_m) = 0$. In an identical way all the components of the vertex correction tensor vanish.

We have analyzed the spin-Hall transport in the case of a 2DHG. For relative weak confinement the spin-Hall conductance is of Luttinger-type, equal to roughly $1.9e/8\pi$ for the parameters in [17]. For strongly confined quantum wells, the system is dominated by a structural inversion asymmetry term of spin-3/2 SIA-type. The spin conductance for this system is $9e/8\pi$ plus finite quantum-well size corrections. We perform the full vertex correction and show that it vanishes for both Luttinger and SIA cases, in striking contrast to the k-linear Rashba case, where the vertex correction is of the same magnitude and of opposite sign to the spin-orbit coupling strength. Coupled with the fact that the lifetime broadening is smaller than the spin splitting, we hence conclude that the spin-Hall effect observed in [17] should be in the intrinsic regime.

We thank J. Sinova and J. Wunderlich for discussions. B. A. B. acknowledges support from the SGF Program. This work is supported by the NSF Grant No. DMR-0342832 and by the U.S. Department of Energy, Office of Basic Energy Sciences under Contract No. DE-AC03-76SF00515.

[1] S. Murakami, N. Nagaosa, and S. Zhang, Science **301**, 1348 (2003).
[2] J. Sinova *et al.*, Phys. Rev. Lett. **92**, 126603 (2004).
[3] S. Murakami, N. Nagaosa, and S. Zhang, Phys. Rev. B **69**, 235206 (2004).
[4] V. P. M. I. D'yakonov, Phys. Lett. **35A**, 459 (1971).
[5] J. Hirsch, Phys. Rev. Lett. **83**, 1834 (1999).
[6] J. Inoue, G. Bauer, and L. Molenkamp, Phys. Rev. B **70**, 041303 (2004).
[7] R. Raimondi *et al.*, Phys. Rev. B **64**, 235110 (2001).
[8] P. Schwab and R. Raimondi, Eur. Phys. J. B **25**, 483 (2002).
[9] E. Mishchenko, A. Shytov, and B. Halperin, Phys. Rev. Lett. **93**, 226602 (2004).
[10] R. Raimondi and P. Schwab, Phys. Rev. B **71**, 033311 (2005).
[11] A. Khaetskii, cond-mat/0408136.
[12] K. Nomura *et al.*, Phys. Rev. B **71**, 041304 (2005).
[13] B. Nikolic, L. Zarbo, and S. Sauma, cond-mat/0408693 [Phys. Rev. B (to be published)].
[14] S. Murakami, Phys. Rev. B **69**, 241202(R) (2004).
[15] S. Zhang and Y. Yang, Phys. Rev. Lett. **94**, 066602 (2005).
[16] Y. Kato *et al.*, Science **306**, 1910 (2004).
[17] J. Wunderlich *et al.*, Phys. Rev. Lett. **94**, 047204 (2005).
[18] R. Winkler, Phys. Rev. B **62**, 4245 (2000).
[19] J. Schliemann and D. Loss, Phys. Rev. B **71**, 085308 (2005).
[20] D. Arovas and Y. Geller, Phys. Rev. B **57**, 12302 (1998).
[21] M. G. Pala *et al.*, Phys. Rev. B **69**, 045304 (2004).
[22] R. Winkler *et al.*, Phys. Rev. B **65**, 155303 (2002).
[23] B. Bernevig and S. Zhang (to be published).