Röntgen Quantum Phase Shift: A Semiclassical Local Electrodynamical Effect?

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The Röntgen quantum phase shift is exhibited by the interference of point particles endowed with an electric dipole moment due to their motion relative to a source of the magnetic field. Here we show, using arguments involving the classical concepts of force and its impulse, that the Röntgen phase shift arises within a largely classical (semiclassical) theoretical framework. All the subtleties normally associated with the nonlocality of magnetic (Aharonov-Bohm-type) quantum phase phenomena are uncontroversially absent in the classical treatment.

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Quantum phase phenomena have been of much interest over the last decade or so, often referred to as topological or Berry phases [1]. The work on certain aspects of the quantum phase appears to have been motivated partly by the prospect of its applications in quantum information processing [2]. The prototypical example of a quantum phase phenomenon is the Aharonov-Bohm (AB) effect [3], which has been the subject of extensive discussion and the existence of which has been confirmed experimentally [4]. In the AB effect, charged particle states experience a shift in their interference pattern whenever a magnetic flux links the closed circuit defined by the particles' paths. Other well known quantum phase effects include the Aharonov-Casher effect [5] and the Matteucci-Pozzi effect [6], among others. Quantum phase effects have also been associated with certain types of laser light, particularly Laguerre-Gaussian light, which is endowed with quantized orbital angular momentum as well as photon spin [7]. The phase of light has been shown to influence atomic gross motion due to the exchange of angular momentum between light and matter [7–9].

Less well known is the so-called Röntgen effect. This arises whenever an electrically neutral system possessing an electric dipole moment is in motion in the presence of a magnetic field which may or may not be a static field. The corresponding interaction has been shown to arise naturally from canonical quantum electrodynamical theoretical developments incorporating the motion of the atomic center of mass as a dynamical variable [10]. Other effects that have been predicted to arise from the motion of neutral quantum systems concerns the rotational motion of a Bose-Einstein condensate which has been predicted to induce a magnetic monopole distribution or an electric monopole distribution [11,12].

It was Wilkens [13] who first drew attention to the significant point that the interaction energy associated with the Röntgen effect in the presence of the vacuum field was essential for preventing the appearance of spurious velocity dependent terms in the evaluated spontaneous emission rate for a moving atom. Recent work by Boussiakou *et a1*. [14] and Cresser and Barnett [15] has confirmed that the Röntgen interaction energy is an important ingredient in developing a quantum electrodynamical framework for the correct evaluation of the spontaneous emission rate of a moving atom, guaranteeing the consistency of quantum mechanics with special relativity in that context. In another paper Wilkens showed that a quantum phase can be associated with the Röntgen effect [16]. The emphasis on the quantum nature of the phase furthered the perception that phase phenomena involving charged particles or neutral atoms are entirely quantum mechanical effects.

In a recent report, Boyer [17] questioned the emphasis on the nonlocality feature exhibited by the magnetic Aharonov-Bohm effect, namely, that the electrons experience no force along their paths, as the magnetic flux is taken to be nonzero only in a small region of space away from field-free paths sampled by the electrons. The magnetic field at every point of the electron paths, albeit small, is nontheless nonzero, as pointed out earlier by Babiker and Loudon [18]. The point made in Ref. [18] is that since the magnetic flux lines of a solenoid of finite length are continuous and so close onto themselves, the total magnetic flux passing though the entire central plane perpendicular to the solenoid is zero. The flux on this plane enclosed by trajectories which, at least in some parts, are at finite distances from the solenoid is finite. Along such paths, the magnetic field is nonzero and an electron experiences a force at every point in its path.

In this Letter we show that, as a magnetic phase effect, the Röntgen quantum phase shift can be arrived at using arguments involving the concepts of force and its associated impulse due to the interaction of particles endowed with an electric dipole in motion relative to a localized source of magnetic field. Force arguments are, by definition, classical and, as will become evident, the Röntgen phase shift admits a semiclassical derivation.

Consider a physical situation, similar to that considered by Wilkens [16], in which the Röntgen effect is manifest. A straight line of magnetic charge of linear charge density Φ/ξ is aligned along the z axis, as shown in Fig. 1. Particles bearing an electric dipole moment of magnitude

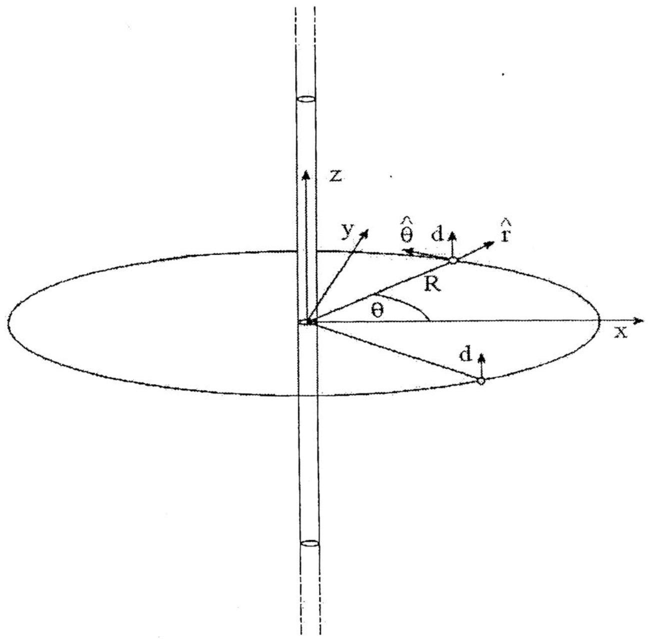

FIG. 1. The interference experiment (schematic) involving the Röntgen phase shift. The particles are endowed with electric dipole moments $\mathbf{d} = d\hat{\mathbf{z}}$ pointing in the z direction and have semicircular paths with a constant speed $v \ll c$, remaining at a constant radial distance R from the magnetic charge line, shown coinciding with the z axis. See the text for further clarification of this figure in connection with the Aharonov-Casher phase shift.

d pointing in the direction z (i.e., $\mathbf{d} = d\hat{\mathbf{z}}$) emerge at a constant speed $v \ll c$ from a single source situated at the point with Cartesian coordinates $(R, 0, 0)$ and are made to follow two semicircular paths, each of radius R in the xy plane. Their interference pattern is detectable in the vicinity of the Cartesian point $(-R, 0, 0)$, having each traversed a semicircular path πR.

In the particle rest frame the magnetic flux density $\mathbf{B}$ due to the magnetic line is perceived as an electric field. Since the motion is confined to semicircular paths that are entirely in the xy plane, it is convenient to use plane polar coordinates. At a general point in the motion, as shown in Fig. 1, the radial and azimuthal unit vectors are $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$. We can therefore write $\mathbf{v} = v\hat{\boldsymbol{\theta}}$ and $\mathbf{r} = R\hat{\mathbf{r}}$, and the electric field acting on the dipole is

$$\mathbf{E} = \mathbf{v} \times \mathbf{B} = \mathbf{v} \times \frac{\mu_0 \Phi}{2\pi\xi} \frac{\hat{\mathbf{r}}}{R} = -\left(\frac{\mu_0 \Phi v}{2\pi\xi R}\right)\hat{\mathbf{z}}. \quad (1)$$

The interaction energy is $U = -\mathbf{d} \cdot \mathbf{E}$. A dipole following the upper path (to be referred to as $+$) experiences a force $\mathbf{F}^+ = -\boldsymbol{\nabla} U$. Since $\mathbf{d} = d\hat{\mathbf{z}}$, this force points in the radial direction and has the form

$$\mathbf{F}^+ = \boldsymbol{\nabla}(\mathbf{d} \cdot \mathbf{E}) = \frac{\mu_0 \Phi v d}{2\pi\xi R^2}\hat{\mathbf{r}}. \quad (2)$$

Associated with this force is a cumulative impulse vector $\mathcal{I}^+$. At a general intermediate point in the upper path for which the polar coordinate is θ, as shown in Fig. 1, at time t this impulse vector is

$$\mathcal{I}^+(t) = \int_0^t \mathbf{F}^+(t')dt'. \quad (3)$$

Consider first the y component of this impulse vector. We have

$$\begin{aligned} \mathcal{I}_y^+(t) &= \int_0^t \mathbf{F}^+ \cdot \hat{\mathbf{y}} dt \\ &= \left(\frac{\mu_0 \Phi v d}{2\pi\xi R^2}\right) \int_0^t \sin\theta(t) dt \\ &= \left(\frac{\mu_0 \Phi d}{2\pi\xi R}\right)(1 - \cos\theta), \end{aligned} \quad (4)$$

where, in the last step, we have converted the t integral to a θ integral and made use of the relation $dt/d\theta = R/v$.

The effect of this impulse component is to change the speed of the particle in the y direction such that

$$M\frac{\Delta Y^+}{\Delta t} = \mathcal{I}_y^+, \quad (5)$$

where M is the particle mass. This point-by-point change of the speed, in turn, leads to a cumulative shift in the particle displacement along the y direction over the entire semicircular path, obtainable by integration. We have

$$\begin{aligned} Y^+ &= \frac{1}{M}\int_0^{\pi R/v} \mathcal{I}_y^+(t) dt \\ &= \left(\frac{\mu_0 \Phi d}{2\pi\xi M v}\right)\int_0^{\pi}(1 - \cos\theta)d\theta \\ &= \left(\frac{\mu_0 \Phi d}{2\xi M v}\right). \end{aligned} \quad (6)$$

The corresponding Y^- for the lower $(-)$ path is obtainable in an analogous manner. It has the same magnitude, but it is opposite in sign. The difference in displacement along the y direction between the $(+)$ and $(-)$ paths at the interference region is

$$\Delta Y = |Y^+ - Y^-| = \frac{\mu_0 \Phi d}{\xi M v}. \quad (7)$$

On turning next to consider the consequences of the x component of the impulse in Eq. (3) we find straightforwardly that it leads to a vanishing cumulative shift in the x direction, which is consistent with the symmetry of the problem. Thus when the particles interfere in the vicinity of the Cartesian point $(-R, 0, 0)$, their path difference is just ΔY, but the relevant quantity determining the Röntgen phase shift is

$$\Delta S_R = M v \Delta Y. \quad (8)$$

We therefore have

$$\Delta S_R = \frac{\mu_0 \Phi d}{\xi}. \quad (9)$$

When divided on both sides by $\hbar$, the result in Eq. (9) becomes identical to that obtained by Wilkens [16]. It is this last step that confers the Röntgen phase shift with its quantum signature, not withstanding the classical arguments used in the earlier steps.

Our main conclusion is that the Röntgen quantum phase shift can be arrived at using arguments firmly rooted in the classical concept of force. The issue of nonlocality that would commonly be associated with a magnetic quantum phase phenomenon is noncontroversially absent in our treatment since the phase shift emerges cumulatively as a result of point-by-point local force effects along the particle paths.

Finally, it is interesting to check whether the Aharonov-Casher effect can be treated along the lines above. In place of the magnetic monopole line, as in Fig. 1, we assume that we have a line of electric charge of density λ per unit length and the particles are endowed with a magnetic dipole $\boldsymbol{\mu} = \mu\hat{\mathbf{z}}$. The electric field due to the line of electric charge is

$$\mathbf{E}(x, y) = \frac{\lambda}{2\pi\epsilon_0 R}\hat{\mathbf{r}}. \quad (10)$$

In the particle frame this electric field is perceived as a magnetic field $\mathbf{B} = -\mathbf{v} \times \mathbf{E}/c^2$. The force would be $\mathbf{F} = \nabla(\boldsymbol{\mu} \cdot \mathbf{B})$, and, apart from the differences in the constant factors, we obtain the same expressions arising from the steps leading from Eq. (1) to the displacement in Eq. (7). Instead of the Röntgen phase shift (9), we obtain the Aharonov-Casher phase shift

$$\Delta S_{\mathrm{AC}} = \frac{\mu\lambda}{\epsilon_0 c^2}. \quad (11)$$

On dividing both sides by $\hbar$, this can be cast in the Aharonov-Casher form

$$\Delta S_{\mathrm{AC}}/\hbar = \frac{2\pi g \hbar \alpha}{mc\xi}, \quad (12)$$

where $\lambda = e/\xi$ and $\mu = ge\hbar/2m$, with g the gyromagnetic ratio, e the electronic charge, and $\alpha = e^2/4\pi\epsilon_0\hbar c$ the fine structure constant. Once again we see that the exact result normally predicted quantum mechanically [5] emerges from the above classical argument.

We have also verified by explicit evaluations that exactly the same results in Eqs. (9) and (12) emerge using the same sources of field, if, instead of the paths described in Fig. 1, we consider linear particle paths in the x-y plane, parallel to the y axis with one on each side of the z axis.

The derivations given here might lead one to suggest that the Röntgen and the Aharonov-Casher phase shifts are not pure quantum effects, contrary to what is commonly understood. However, the appearance of Planck's constant, albeit at the final stages of the derivations, does, in fact, confer the quantal nature to both phase shifts. In this manner they share their semiclassical connection with a number of treatments of well known effects, such as the Planck's blackbody radiation law and the photoelectric effect, in which contexts, too, Planck's constant assumes an auxiliary role in their derivations.

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