Symmetry of Spin Transport in Two-Terminal Waveguides with a Spin-Orbital Interaction and Magnetic Field Modulations

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We analyze symmetries of spin transport in two-terminal quantum waveguide structures with Rashba spin-orbit coupling and magnetic field modulations. Constraints, imposed by the device structure, on the spin polarization of the transmitted electron beam from the waveguide devices are derived. The results are expected to provide accuracy tests for experimental measurements and numerical calculations, as well as guidelines for spin-based device designs.

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Introduction.—The electrical generation of a spin-polarized current is one of the essential goals in semiconductor spintronics and has stimulated considerable theoretical and experimental efforts [1]. Most of them so far have focused on spin injection into nonmagnetic semiconductors from a polarized source made from, e.g., ferromagnetic metal (FM) or magnetic semiconductor materials, while some theoretical works have suggested methods of creating spin currents out of an unpolarized source. In the latter case various spin-related transport phenomena are utilized to generate an uneven spin distribution in the transmitted electrons. Of special interest is the relativistic spin-orbit interaction (SOI) [2,3] with tunable strength [4–7], which entangles spin states of charge carriers with their space motion. It has been shown theoretically that the SOI can be employed to generate spin polarization in a T-shaped structure [8] and even a pure spin current in a Y-shaped junction [9].

In this work we investigate spin-dependent electron transport and spin filtering behavior in a two-terminal quantum waveguide structure under a local SOI and magnetic field modulations. We focus our study on the analysis of symmetries. There is no doubt that symmetries are of great importance, since, once established, they can provide sample tests of experimental and numerical accuracies and greatly reduce the amount of data which has to be taken or calculated. The symmetry relations can also provide guidelines for the design of spin-based devices. In this work, we endeavor to establish a few basic relations, imposed by the symmetries, for the spin-dependent scattering parameters in the two-terminal multimode waveguide structure. But for the sake of completeness we first give a short description of the model and formalism employed.

Model and formalism.—We consider a multimode quantum waveguide defined by the lateral confining potential $V_c(x, y)$ on a near-surface two-dimensional electron gas (2DEG) in the (x, y) plane, subject to the modulations by a local magnetic field. The 2DEG is assumed to be formed in an asymmetric quantum well, where the SOI is contributed dominantly by the Rashba mechanism [2]. The Rashba strength can be tuned by one or more external gates [4–7]. The magnetic field is assumed to be flat in the y direction and to be inhomogeneous on the nanometer scale along the x axis, which can be created by, e.g., deposing patterned FM or superconducting materials on top of the waveguide [10,11]. The device is assumed to connect with two ideal leads with vanishing SOI and magnetic field. The model Hamiltonian describing such a system has the form

$$H = \left(\frac{\Pi^2}{2m^*} + V_c\right)\sigma_0 + \frac{g^*\mu_B \mathbf{B}\cdot\sigma}{2} + \frac{1}{2}(H_{SO}^R + H_{SO}^C + \text{H.c.}), \tag{1}$$

where $\Pi = \mathbf{p} + e\mathbf{A}$ is the canonical momentum with $\mathbf{A}$ the vector potential, m^*, $-e$, and g^* are, respectively, the effective mass, charge, and effective g factor of electrons, σ_0 is the 2×2 unit matrix, $\sigma = (\sigma_\mathbf{x}, \sigma_\mathbf{y}, \sigma_\mathbf{z})$ is the vector of the Pauli matrices, and μ_B the Bohr magneton. Two SOI terms are included in Eq. (1),

$$H_{SO}^R = \frac{\alpha}{\hbar}\hat{\mathbf{z}}\cdot(\sigma \times \Pi) \text{ and } H_{SO}^C = \gamma \nabla V_c \cdot (\sigma \times \Pi), \tag{2}$$

which arise from the interfacial electric field and the confining potential, respectively [12,13]. Here $\hat{\mathbf{z}}$ is the normal of the 2DEG plane and γ is the in-plane SOI constant. A symmetrized form for the SOI terms is adopted in Eq. (1) to account for the situation that the Rashba strength α and/or the in-plane electric field $\nabla V_c/e$ is a function of position [13,14]. The velocity operator can be derived directly from $\mathbf{v} = \partial H/\partial \mathbf{p}$, whose x component in the leads has a simple form, $v_x = (p_x/m^*)\sigma_0$.

The spin quantum axis is chosen to be along $\hat{\mathbf{z}}$. In lead λ ($\lambda = L$ for the left lead or R for the right lead), the eigenwave function ψ can be constructed as a linear combination of the propagating modes in the same lead,

$$\psi_\lambda = \sum_{n\sigma}(a_{n\sigma}^\lambda \phi_{n\sigma}^{\lambda+} + b_{n\sigma}^\lambda \phi_{n\sigma}^{\lambda-}). \tag{3}$$

Here $\phi_{n\sigma}^{\lambda+}$ ($\phi_{n\sigma}^{\lambda-}$) is the nth right-going (left-going) propagating mode in the lead λ, with a spin state vector $\chi(\sigma) = (1, 0)^T$ [for $\sigma = \uparrow (+1)$] or $(0, 1)^T$ [for $\sigma = \downarrow (-1)$]. The number of propagating modes in each lead is determined by the electron energy E and the lead parameters. All propagating modes are normalized with their velocities, so that the outgoing amplitudes, $\{b^L_{n_L\sigma_L}\}$ and $\{a^R_{n_R\sigma_R}\}$, are related with the ingoing ones, $\{a^L_{n'_L\sigma'_L}\}$ and $\{b^R_{n'_R\sigma'_R}\}$, through a scattering matrix S,

$$\begin{pmatrix} b^L_{n_L\sigma_L} \\ a^R_{n_R\sigma_R} \end{pmatrix} = \begin{pmatrix} r_{n_L\sigma_L,n'_L\sigma'_L} & t'_{n_L\sigma_L,n'_R\sigma'_R} \\ t_{n_R\sigma_R,n'_L\sigma'_L} & r'_{n_R\sigma_R,n'_R\sigma'_R} \end{pmatrix} \begin{pmatrix} a^L_{n'_L\sigma'_L} \\ b^R_{n'_R\sigma'_R} \end{pmatrix}. \quad (4)$$

Here $t_{n_R\sigma_R,n'_L\sigma'_L}$ represents the transmission amplitude for electrons incident from the state $(n'_L\sigma'_L)$ in the left lead scattered into an outgoing state $(n_R\sigma_R)$ in the right lead. The other three blocks in the matrix S have similar meanings. In writing Eq. (4) we adopted the Einstein's sum rule. However, the summation should only be taken over propagating modes in the leads. The current conservation leads to the unitary condition of the S matrix, $S^+S = 1$, which gives

$$\begin{pmatrix} a^L_{n_L\sigma_L} \\ b^R_{n_R\sigma_R} \end{pmatrix} = \begin{pmatrix} r^*_{n'_L\sigma'_L,n_L\sigma_L} & t^*_{n'_R\sigma'_R,n_L\sigma_L} \\ t'^*_{n'_L\sigma'_L,n_R\sigma_R} & r'^*_{n'_R\sigma'_R,n_R\sigma_R} \end{pmatrix} \begin{pmatrix} b^L_{n'_L\sigma'_L} \\ a^R_{n'_R\sigma'_R} \end{pmatrix}. \quad (5)$$

Assuming that electrons are incident from the left lead, the spin-dependent two-terminal conductance at zero temperature is given by summing over the transmission probabilities [15],

$$G_{\sigma_R\sigma_L} = \frac{e^2}{h}\sum_{n_L,n_R} |\, t_{n_R\sigma_R,n_L\sigma_L} \,|^2 . \quad (6)$$

The total (charge) conductance, $G = \sum_{\sigma_L,\sigma_R} G_{\sigma_R\sigma_L}$, can be written as

$$G = -\frac{e}{h}\sum_{n_L\sigma_L}\int dy\, \mathrm{Re}[\psi^+_{R,n_L\sigma_L}(\mathbf{r})(-ev_x)\psi_{R,n_L\sigma_L}(\mathbf{r})], \quad (7)$$

where $\psi_{R,n_L\sigma_L}(\mathbf{r})$ is the outgoing state corresponding to electrons incident from the mode $(n_L\sigma_L)$ in the left lead. The spin conductance with respect to a given direction $\mathbf{n}$ can be defined in a similar way [16]

$$\begin{aligned} G^s_{\mathbf{n}} &= -\frac{e}{h}\sum_{n_L\sigma_L}\int dy\, \mathrm{Re}[\psi^+_{R,n_L\sigma_L}(\mathbf{r})v^s_x(\mathbf{n})\psi_{R,n_L\sigma_L}(\mathbf{r})] \\ &= -\frac{e}{4\pi}\sum_{n_L\sigma_L,n_R\sigma_R\sigma'_R} t^*_{n_R\sigma'_R,n_L\sigma_L} t_{n_R\sigma_R,n_L\sigma_L} \\ &\quad \times \chi^+(\sigma'_R)(\sigma\cdot\mathbf{n})\chi(\sigma_R), \end{aligned} \quad (8)$$

where $v^s_x(\mathbf{n}) = [v_x(\frac{\hbar}{2}\sigma\cdot\mathbf{n}) + (\frac{\hbar}{2}\sigma\cdot\mathbf{n})v_x]/2$.

The spin polarization of the generated current in the right lead can be viewed as the ratio between the normalized spin conductance, $(G^s_{\mathbf{x}}, G^s_{\mathbf{y}}, G^s_{\mathbf{z}})/(-e/4\pi)$, and the normalized total conductance, $G/(e^2/h)$. It can also be experimentally measured as the quantum-mechanical averages of the spin angular momentum of electrons,

$$\frac{\hbar}{2}\mathbf{P} = \mathrm{Tr}\left(\hat{\rho}_s\frac{\hbar}{2}\sigma\right),$$

where $\hat{\rho}_s$ is the spin density operator of the transmission current in the right lead [17]. The two kinds of definitions are equivalent and give the following expressions:

$$\begin{aligned} P_x + iP_y &= \frac{2e^2/h}{G}\sum_{n_R,n_L\sigma_L} t_{n_R\downarrow,n_L\sigma_L} t^*_{n_R\uparrow,n_L\sigma_L}, \\ P_z &= \frac{(G_{\uparrow\uparrow} + G_{\uparrow\downarrow}) - (G_{\downarrow\downarrow} + G_{\downarrow\uparrow})}{G}. \end{aligned} \quad (9)$$

It can be seen that P_x (P_y) may be finite only when spin-flipping scattering occurs.

It should be stressed that the polarization has a vector nature: along a given direction $\mathbf{n}$ it is given by $\mathrm{Tr}(\hat{\rho}_s\sigma\cdot\mathbf{n}) = \mathbf{P}\cdot\mathbf{n}$. When $\mathbf{n}$ coincides with the spin quantum axis for which $\hat{\rho}_s$ is diagonal, the polarization has only one nonzero component and can be expressed as $P = (G_\uparrow - G_\downarrow)/(G_\uparrow + G_\downarrow)$. This definition has been adopted in most studies of spin-conserved transport.

Symmetries.—Symmetry of spin-independent electrical conduction has been extensively studied in the last two decades [15,18,19] and still is a subject of current interest [20]. For spin-dependent electron transport, symmetry analysis has been made for quasi-one-dimensional systems [8,21] and, in most cases, without including any magnetic field. Here we consider multichannel cases and the situations where a local magnetic field modulation is present in the device. It is the point of this work to establish a set of basic symmetry relations for spin transport in general two-terminal spintronic systems.

We begin with the case of zero magnetic field. In this case the time-reversal operator, $T = -i\sigma_yK$ (K is the complex conjugation), commutes with the system Hamiltonian H defined in Eq. (1). The transformed state, $T\psi$, is thus an eigenstate of the same Hamiltonian. The time-reversal operation changes $\phi^{\lambda\pm}_{n\sigma}$ to $\sigma\phi^{\lambda\mp}_{n\bar{\sigma}}$, where $\bar{\sigma} = -\sigma$. In the transformed state $T\psi_\lambda$, the right-going and left-going components for the same mode $(n_\lambda\sigma_\lambda)$ are $\sigma_\lambda b^{\lambda*}_{n_\lambda\bar{\sigma}_\lambda}$ and $\sigma_\lambda a^{\lambda*}_{n_\lambda\bar{\sigma}_\lambda}$, respectively. The combination of these facts and Eq. (4) results in

$$\begin{pmatrix} \sigma_L a^{L*}_{n_L\bar{\sigma}_L} \\ \sigma_R b^{R*}_{n_R\bar{\sigma}_R} \end{pmatrix} = \begin{pmatrix} r_{n_L\sigma_L,n'_L\sigma'_L} & t'_{n_L\sigma_L,n'_R\sigma'_R} \\ t_{n_R\sigma_R,n'_L\sigma'_L} & r'_{n_R\sigma_R,n'_R\sigma'_R} \end{pmatrix} \begin{pmatrix} \sigma'_L b^{L*}_{n'_L\bar{\sigma}'_L} \\ \sigma'_R a^{R*}_{n'_R\bar{\sigma}'_R} \end{pmatrix}. \quad (10)$$

Comparison of Eqs. (10) and (5) leads to

$$\begin{aligned} t'_{n_L\sigma_L,n_R\sigma_R} &= \sigma_L\sigma_R t_{n_R\bar{\sigma}_R,n_L\bar{\sigma}_L}, \\ r'_{n_R\bar{\sigma}_R,n'_R\bar{\sigma}'_R} &= \sigma_R\sigma'_R r'_{n'_R\sigma'_R,n_R\sigma_R}. \end{aligned} \quad (11)$$

Combined with the orthogonality and normalization conditions of the S matrix, Eq. (11) indicates an important conclusion: *no spin polarization in the transmitted flux can ever occur when the outgoing lead supports only one open* *channel.* This conclusion does not depend on the detail of the middle structure of the device and is valid for any two-terminal ballistic transport. Note that this result was previously obtained only for more restrictive cases where both the input and output leads were assumed to be within the single-channel regime [8,13].

When the system has additional symmetries, more constraints will be imposed on the scattering matrix. When both $V_c(\mathbf{r})$ and $\alpha(\mathbf{r})$ are invariant under the reflection transformation R_x (R_y) with respect to the x (y) axis, the system has a symmetry related with the operator $\sigma_x R_x$ ($\sigma_y R_y$). As a result, the transformed state $\sigma_x R_x \psi$ ($\sigma_y R_y \psi$) is an eigenstate of the Hamiltonian (1).

In the situation $V_c(x, y) = V_c(x, -y)$ and $\alpha(x, y) = \alpha(x, -y)$, the transverse part of the propagating state $\phi_{n\sigma}^{\lambda\pm}$ has a parity $(-1)^{n+1}$ and the operation $\sigma_y R_y$ changes $\phi_{n\sigma}^{\lambda\pm}$ to $i\sigma(-1)^{n+1}\phi_{n\bar{\sigma}}^{\lambda\pm}$. Thus, in the transformed state $\sigma_y R_y \psi_\lambda$, the right-going and left-going components for the same mode $n_\lambda \sigma_\lambda$ are $i\bar{\sigma}_\lambda(-1)^{n_\lambda} a^\lambda_{n_\lambda \bar{\sigma}_\lambda}$ and $i\bar{\sigma}_\lambda(-1)^{n_\lambda} b^\lambda_{n_\lambda \bar{\sigma}_\lambda}$, respectively. This observation yields

$$\begin{pmatrix} \bar{\sigma}_L(-1)^{n_L} b^L_{n_L\bar{\sigma}_L} \\ \bar{\sigma}_R(-1)^{n_R} a^R_{n_R\bar{\sigma}_R} \end{pmatrix} = \begin{pmatrix} r_{n_L\sigma_L,n'_L\sigma'_L} & t'_{n_L\sigma_L,n'_R\sigma'_R} \\ t_{n_R\sigma_R,n'_L\sigma'_L} & r'_{n_R\sigma_R,n'_R\sigma'_R} \end{pmatrix} \times \begin{pmatrix} \bar{\sigma}'_L(-1)^{n'_L} a^L_{n'_L\bar{\sigma}'_L} \\ \bar{\sigma}'_R(-1)^{n'_R} b^R_{n'_R\bar{\sigma}'_R} \end{pmatrix}. \tag{12}$$

By comparing Eq. (12) with Eq. (4), we get

$$t_{n_R\bar{\sigma}_R,n_L\bar{\sigma}_L} = \sigma_L\sigma_R(-1)^{n_L+n_R} t_{n_R\sigma_R,n_L\sigma_L}, \tag{13}$$

which indicates

$$G_{\bar{\sigma}_R\bar{\sigma}_L} = G_{\sigma_R\sigma_L}, \qquad P_x = P_z = 0. \tag{14}$$

However, P_y can be finite in this situation.

For the case $V_c(x, y) = V_c(-x, y)$ and $\alpha(x, y) = \alpha(-x, y)$, the propagating modes $\phi_{n\sigma}^{\lambda\pm}$ can be chosen to satisfy $\phi_{n\sigma}^{R\pm}(x > 0) = \phi_{n\sigma}^{L\mp}(-x)$. As a result, the operation $\sigma_x R_x$ changes $\phi_{n\sigma}^{L\pm}$, a propagating mode in the left lead, to $\phi_{n\bar{\sigma}}^{R\mp}$, a propagating mode in the right lead, and vice versa. In the eigenstate $\sigma_x R_x \psi$, the incident wave amplitudes for the mode $\phi^{L+}_{n_L\sigma_L}$ ($\phi^{R-}_{n_R\sigma_R}$) are $b^R_{n_L\bar{\sigma}_L}$ ($a^L_{n_R\bar{\sigma}_R}$), while the outgoing wave amplitudes are $a^R_{n_L\bar{\sigma}_L}$ ($b^L_{n_R\bar{\sigma}_R}$) for the mode $\phi^{L-}_{n_L\sigma_L}$ ($\phi^{R+}_{n_R\sigma_R}$). From this fact we obtain

$$\begin{pmatrix} a^R_{n_L\bar{\sigma}_L} \\ b^L_{n_R\bar{\sigma}_R} \end{pmatrix} = \begin{pmatrix} r_{n_L\sigma_L,n'_L\sigma'_L} & t'_{n_L\sigma_L,n'_R\sigma'_R} \\ t_{n_R\sigma_R,n'_L\sigma'_L} & r'_{n_R\sigma_R,n'_R\sigma'_R} \end{pmatrix} \begin{pmatrix} b^R_{n'_L\bar{\sigma}'_L} \\ a^L_{n'_R\bar{\sigma}'_R} \end{pmatrix}, \tag{15}$$

which together with Eq. (4) results in

$$t'_{n_L\sigma_L,n_R\sigma_R} = t_{n_L\bar{\sigma}_L,n_R\bar{\sigma}_R}. \tag{16}$$

In comparison with the results derived from the time-reversal symmetry, one finds the following constraint relation of the scattering parameters, imposed by the symmetry $T\sigma_x R_x$ of the system:

$$t_{n_L\sigma_L,n_R\sigma_R} = \sigma_L\sigma_R t_{n_R\sigma_R,n_L\sigma_L}. \tag{17}$$

All results obtained above are for the case of zero magnetic field. In the following we focus on the general case, i.e., both the SOI and the magnetic field may be finite. The presence of a finite magnetic field breaks the time-reversal symmetry. In fact, the system under the magnetic field $\mathbf{B}$ is the time-reversal counterpart of that under $-\mathbf{B}$. The eigenstate of $H(\mathbf{B})$ is transformed by T into that of $H(-\mathbf{B})$, with the same eigenenergy. From these facts one can derive the following relation:

$$t'_{n_L\sigma_L,n_R\sigma_R}(-\mathbf{B}) = \sigma_L\sigma_R t_{n_R\bar{\sigma}_R,n_L\bar{\sigma}_L}(\mathbf{B}), \tag{18}$$

which is a generalization of Eq. (11). For the case of two-terminal transport the unitary condition of the S matrix leads to

$$\sum_{n_L\sigma_L,n_R\sigma_R} |t_{n_R\sigma_R,n_L\sigma_L}|^2 = \sum_{n_L\sigma_L,n_R\sigma_R} |t'_{n_L\sigma_L,n_R\sigma_R}|^2. \tag{19}$$

From Eqs. (18) and (19) one arrives at a general relation for the total conductance

$$G(-\mathbf{B}) = G(\mathbf{B}). \tag{20}$$

This is nothing but the Onsager-Casimir relation [22] and holds for the magnetic field with arbitrary shape and strength.

In most cases, the magnetic field involved in spin devices is present along the perpendicular direction. For such a magnetic field, one can show that under the operation σ_z, the Hamiltonian $H(\alpha)$ changes to $H(-\alpha)$. It follows that

$$t_{n_R\sigma_R,n_L\sigma_L}(-\alpha) = \sigma_L\sigma_R t_{n_R\sigma_R,n_L\sigma_L}(\alpha). \tag{21}$$

This implies that

$$G_{\sigma_2\sigma_1}(-\alpha) = G_{\sigma_2\sigma_1}(\alpha), \qquad P_{x,y}(-\alpha) = -P_{x,y}(\alpha), \qquad P_z(-\alpha) = P_z(\alpha). \tag{22}$$

Further symmetric properties of the scattering matrix and the transport quantities can be derived for the perpendicular magnetic field distribution and device structure with certain experimentally realizable symmetries. When the magnetic field is homogeneous along the y direction, i.e., $\mathbf{B} = B_z(x)\hat{\mathbf{z}}$, and the confining potential and the SOI strength are invariant under the operation R_y, one can show that $H(\mathbf{B})$ changes to $H(-\mathbf{B})$ under the operation $\sigma_y R_y$, resulting in

$$t_{n_R\sigma_R,n_L\sigma_L}(-\mathbf{B}) = \sigma_L\sigma_R(-1)^{n_L+n_R} t_{n_R\bar{\sigma}_R,n_L\bar{\sigma}_L}(\mathbf{B}). \tag{23}$$

This equation is a generalization of Eq. (13) and implies that

$$G_{\sigma_R\sigma_L}(-\mathbf{B}) = G_{\bar{\sigma}_R\bar{\sigma}_L}(\mathbf{B}), \qquad P_{x,z}(-\mathbf{B}) = -P_{x,z}(\mathbf{B}), \qquad P_y(-\mathbf{B}) = P_y(\mathbf{B}). \tag{24}$$

In addition, the conservation property of the system under the transformation of $\sigma_y R_y T$ gives

$$t'_{n_L\sigma_L,n_R\sigma_R}(\mathbf{B}) = (-1)^{n_L+n_R} t_{n_R\sigma_R,n_L\sigma_L}(\mathbf{B}). \qquad (25)$$

Further, we consider two experimentally frequently employed situations in which the magnetic field profile is either symmetric or antisymmetric, $B_z(-x) = \pm B_z(x)$, while the distribution of the SOI strength and confining potential are symmetric, $\alpha(-\mathbf{r}) = \alpha(\mathbf{r})$ and $V_c(-\mathbf{r}) = V_c(\mathbf{r})$. For the case that the magnetic field is symmetric, the system Hamiltonian is invariant under the operation $\sigma_x R_x T$. For this symmetry Eq. (17) holds, which implies

$$G_{\uparrow\downarrow}(\mathbf{B}, \alpha) = G_{\downarrow\uparrow}(\mathbf{B}, \alpha). \qquad (26)$$

In the antisymmetric case the system is conserved under the transformation $\sigma_x R_x$. For this symmetry Eq. (16) holds, which together with Eq. (25) results in

$$t_{n_L\bar{\sigma}_L,n_R\bar{\sigma}_R}(\mathbf{B}, \alpha) = (-1)^{n_L+n_R} t_{n_R\sigma_R,n_L\sigma_L}(\mathbf{B}, \alpha). \qquad (27)$$

This equation implies

$$G_{\uparrow\uparrow}(\mathbf{B}, \alpha) = G_{\downarrow\downarrow}(\mathbf{B}, \alpha). \qquad (28)$$

We now discuss a simple interesting case in which $\alpha = 0$ and the 2DEG waveguide is modulated only by an antisymmetric magnetic field. In this case only spin-conserved transmission may be nonzero, so that $P_{x,y} = 0$. In the limit of infinite width of the waveguide, it has been shown [23] that the operator TR_xR_y is conserved, which results in a vanishing spin polarization of the transmitted electrons. For the case of a finite width, Eq. (28) indicates that $P_z = 0$. Thus for the 2DEG waveguide modulated by an antisymmetrical magnetic field, there is still not any spin filtering effect in the presence of a uniform transverse confinement.

It is worthwhile to note that when the Rashba term in Eq. (1) is replaced by the Dresselhaus term [3] $H^D_{\mathrm{SO}} = \frac{\beta}{\hbar} \times (\sigma_x \Pi_x - \sigma_y \Pi_y)$, a set of relations corresponding to Eqs. (10)–(28) can be established with the same procedure.

As we mentioned before, the derived symmetry relations for the two-terminal spin-dependent electron transport systems can be used to check the accuracy of numerical calculations. As an example, we notice that in a recent study [24], a significant conductance variation upon switching the magnetization direction of the FM stripe in a spin device was predicted. The prediction is obviously wrong from the view of symmetry. The mistake made in the study can be traced to an incorrect treatment of the spin-flipped transmission.

Conclusions.—In summary, we have studied spin transport in a two-terminal quantum waveguide structure modulated by the Rashba SOI and/or a magnetic field. A set of basic symmetry relations have been derived for the spin-dependent scattering parameters. The results lead to a few constraints, imposed by the device structure, on the spin polarization of the transmitted electron beam from the waveguide devices. A natural next step would be a generalization of these results to spin transport in multiterminal multichannel systems along the line of Ref. [18].

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