Constraining the Spectrum of Supernova Neutrinos from ν-Process Induced Light Element Synthesis

Takashi Yoshida,[1,*] Toshitaka Kajino,[2,3] and Dieter H. Hartmann[4]

[1]*Astronomical Institute, Graduate School of Science, Tohoku University, Sendai 980-8578, Japan*
[2]*National Astronomical Observatory of Japan, and The Graduate University for Advanced Studies, Tokyo 181-8588, Japan*
[3]*Department of Astronomy, Graduate School of Science, University of Tokyo, Tokyo 113-0033, Japan*
[4]*Department of Physics and Astronomy, Clemson University, Clemson, South Carolina 29634, USA*
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We constrain energy spectra of supernova neutrinos through the avoidance of an overproduction of the ^{11}B abundance during Galactic chemical evolution. In supernova nucleosynthesis calculations with a parametrized neutrino spectrum as a function of temperature of $\nu_{\mu,\tau}$ and $\bar{\nu}_{\mu,\tau}$ and total neutrino energy, we find a strong neutrino temperature dependence of the ^{11}B yield. When the yield is combined with observed abundances, the acceptable range of the $\nu_{\mu,\tau}$ and $\bar{\nu}_{\mu,\tau}$ temperature is found to be 4.8 to 6.6 MeV. Nonzero neutrino chemical potentials would reduce this temperature range by about 10% for a degeneracy parameter $\eta_\nu = \mu_\nu/kT_\nu$ smaller than 3.

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The light elements (Li-Be-B) are continuously produced by supernovae (SNe) [1–3], as well as interactions of Galactic cosmic rays (GCRs) with the interstellar medium (ISM), nucleosynthesis in asymptotic giant branch (AGB) stars, and novae during Galactic chemical evolution (GCE, i.e., the evolution in chemical composition of stars and interstellar gas during Galactic history) [4,5]. In the case of boron, cosmic ray induced spallation in the ISM and supernova ejecta dominate the production; ^{11}B is contributed through both channels, while ^{10}B production is probably exclusively due to GCRs. The contribution from supernovae to the production of ^{11}B can be calibrated with the isotopic ratio N(^{11}B)/N(^{10}B), measured with great precision in primitive meteorites.

The SN ν-process plays an important role for ^{11}B and ^{7}Li production [1]. The interaction of neutrinos, emitted in copious amounts during core collapse and the subsequent cooling phase of proto-neutron stars, with matter in the ejecta of SNe, contributes uniquely to GCE. Recent studies based on the theoretical yields derived by Woosley and Weaver (WW95) [2] suggest that the SN contribution to the ^{11}B abundance is significantly larger than that required to explain the boron evolution in the Galactic disk and the meteoritic ^{11}B/^{10}B ratio [4–6].

To match the abundance of ^{11}B established during GCE, we previously assumed neutrino energy spectra to resemble Fermi-Dirac (FD) distributions with zero chemical potentials $\mu_\nu = 0$ [1–3] and fixed neutrino temperatures of 6.0, 3.2, and 5.0 MeV for $\nu_{\mu,\tau}(\bar{\nu}_{\mu,\tau})$, ν_e, and $\bar{\nu}_e$, respectively [7]. The $\nu_{\mu,\tau}$ temperature of 6.0 MeV is significantly smaller than the 8.0 MeV used in the other previous studies of the ν-process [1,2,8]. This reduction is derived from an investigation of the dependence of the ^{11}B yield on the total neutrino energy E_ν and the decay time scale τ_ν of the neutrino flux. The yield is roughly proportional to E_ν and rather insensitive to τ_ν. The temperature dependence was not investigated very well.

Studies of supernova explosions with detailed neutrino transport (e.g., [9,10] and references therein) have indicated that emerging neutrino spectra do not closely follow FD distributions with $\mu_\nu = 0$. Since the high-energy tail of the energy distribution is predominantly important for the ν-process (e.g., [1]), the use of FD distribution with $\mu_\nu = 0$ may be justified as an approximation as long as the spectrum above $\varepsilon_\nu \approx 10$ MeV is a good match to the shapes obtained in detailed transport simulations [10–12]. However, if the ^{11}B yield depends strongly on the neutrino temperatures, which have not yet been clarified theoretically, the nonzero chemical potentials would change the resultant ^{11}B abundance in a different matter from what follows from FD distributions with $\mu_\nu = 0$. The purpose of this Letter is to investigate the neutrino temperature dependence of the SN ν-process in detail, and to find out how robustly lower neutrino temperatures may provide the means to avoid overproduction of the ^{11}B abundance in GCE and meteoritic ^{11}B/^{10}B ratio.

The adopted model for SN neutrinos is guided by numerical simulations from the literature, with a few additional simplifying assumptions. The neutrino luminosity is assumed to be uniformly partitioned among the neutrino flavors, and is assumed to decrease exponentially in time with a time scale $\tau_\nu = 3$ s [1]. The latter assumption is not critical, because the ejected masses of ^{11}B and ^{7}Li are insensitive to τ_ν [7]. We initially assume that the spectra indeed obey the FD form with $\mu_\nu = 0$.

Only the total neutrino energy E_ν and the temperatures $T_{\nu_{\mu,\tau}}$ are free parameters. The allowed range of the total neutrino energy E_ν is

$$1 \times 10^{53} \text{ ergs} \le E_\nu \le 6 \times 10^{53} \text{ ergs}, \quad (1)$$

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which includes the reduced range 2.4×10^{53} ergs $\leq E_\nu \leq 3.5 \times 10^{53}$ ergs, corresponding to the estimated range in gravitational binding energy of a neutron star with mass $\sim 1.4 M_\odot$ [13]. The considered range of the neutrino temperature $T_{\nu_{\mu,\tau}}$ is

$$4.0 \text{ MeV} \leq T_{\nu_{\mu,\tau}} \leq 9.0 \text{ MeV}. \qquad (2)$$

Temperatures of the ν_e and $\bar{\nu}_e$, T_{ν_e} and $T_{\bar{\nu}_e}$, are less important for the ν-process of the light elements, and we set their values to 3.2 and 5.0 MeV, respectively [7].

The SN model used in this work is identical to that described by [7]. We use progenitor model 14E1, with a mass at explosion of $16.2 M_\odot$ [14], corresponding to SN 1987A. The propagation of a shock wave during the SN explosion is followed with a spherically symmetric Lagrangian PPM code [15,16]. The explosion energy and the mass cut are set to 1×10^{51} ergs and $1.61 M_\odot$, respectively. Then, we calculate explosive nucleosynthesis by postprocessing as described in [7]. The reaction rates of the ν-process are derived by interpolating the logarithmic values of the cross sections listed in the tables of [17]. This setup determines the thermodynamic histories of the various mass shells that ultimately constitute the supernova ejecta (no fallback), and the ν-process yields within the ejecta are then determined through the cross sections as soon as the time- and energy-dependent neutrino flux is specified. We calculate the yields for a parameter grid with 126 points, with steps of 1×10^{53} ergs in E_ν and steps of 0.25 MeV in $T_{\nu_{\mu,\tau}}$. Shown in Fig. 1 is an example of the produced mass fractions for $E_\nu = 3 \times 10^{53}$ ergs and $T_{\nu_{\nu,\tau}} = 6$ MeV.

Ratios of the ejected masses of ^{11}B and ^{7}Li to those of WW95 [2], defined as the overproduction factor f_ν, are shown in Fig. 2 as contours in the E_ν vs $T_{\nu_{\mu,\tau}}$ plane. For the WW95 case of $T_{\nu_{\mu,\tau}} = 8$ MeV and $E_\nu = 3 \times 10^{3}$ ergs, we find ejected ^{11}B and ^{7}Li masses of $1.92 \times 10^{-6} M_\odot$ and $7.37 \times 10^{-7} M_\odot$, which are very close to the yields of $1.85 \times 10^{-6} M_\odot$ and $6.67 \times 10^{-7} M_\odot$ obtained with the S20A model of [2], respectively. The ^{11}B mass ratio changes between 0.038 [lower left corner of Fig. 2(a)] and 2.9 (upper right corner) in the assumed ranges of E_ν [Eq. (1)] and $T_{\nu_{\mu,\tau}}$ [Eq. (2)]. The mass ratio of ^{7}Li changes between 0.039 [lower left corner of Fig. 2(b)] and 3.3 (upper right corner). Note that dependence on the explosion energy and the mass cut is weak.

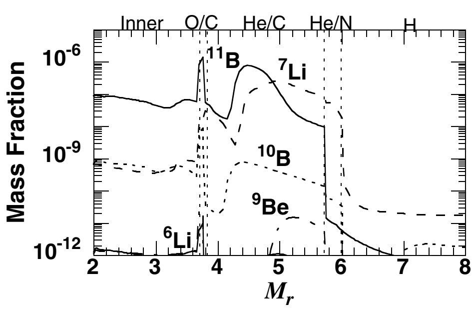

FIG. 1. Mass fraction distribution of the light elements in the $16.2 M_\odot$ model with $T_{\nu_{\mu,\tau}} = 6$ MeV. The horizontal axis is the interior mass in units of the solar mass.

We constrain the neutrino temperature $T_{\nu_{\mu,\tau}}$ by requiring that overproduction of ^{11}B in GCE must be avoided. The overproduction factor depends on details of the GCE model, and ranges between 0.18 [5] and 0.40 [4]. These values are obtained by combining the solar $^{11}\text{B}/^{10}\text{B}$ ratio with a measure of the relative cosmic-ray and supernova contribution to solar ^{11}B. They are shown in Fig. 2(a) as two solid lines. If we adopt the $1.4 M_\odot$ neutron star energy range mentioned above (2.4×10^{53} ergs $\leq E_\nu \leq 3.5 \times 10^{53}$ ergs [13]), we obtain the shaded region shown in Fig. 2(a), which implies that the neutrino temperature $T_{\nu_{\mu,\tau}}$ satisfies

$$4.8 \text{ MeV} \leq T_{\nu_{\mu,\tau}} \leq 6.6 \text{ MeV}. \qquad (3)$$

With the neutrino temperature and total energy constrained by GCE of ^{11}B, we can derive a corresponding constraint on the ^{7}Li yield. Figure 2(b) shows the shaded

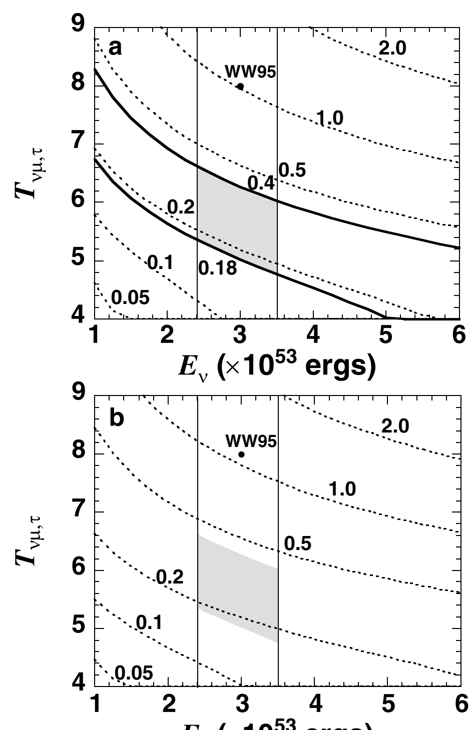

FIG. 2. Contour lines of the overproduction factor f_ν for (a) ^{11}B and (b) ^{7}Li in the parameter plane of total neutrino energy E_ν and neutrino temperature $T_{\nu_{\mu,\tau}}$ (see text for details). The region between the two solid vertical lines indicates the energy range relevant for a neutron star of mass $\sim 1.4 M_\odot$ [13]. The point labeled WW95 indicates the specific parameter values used in [2]. In panel (a), the region between the two solid contour lines is the range of ejected mass appropriate for GCE of ^{11}B. The shaded region is the part of parameter space in which both constraints (GCE yield of ^{11}B and neutron star binding energy) are simultaneously satisfied. A similar box is drawn in (b) for the case of ^{7}Li.

region corresponding to the E_ν-$T_{\nu_{\mu,\tau}}$ limits of the shaded box in Fig. 2(a). This region implies an ejected mass ratio of ^{7}Li between 0.19 and 0.43. If ^{11}B production is indeed dominated by the contributions from the ν-process, the analysis presented above implies a predicted range of yields for ^{7}Li, which in turn constrains the contribution to ^{7}Li production from AGB stars and novae.

We note that the smallest value of our allowed range for $T_{\nu_{\mu,\tau}}$ is in fact smaller than the assumed value of $T_{\bar{\nu}_e} = 5.0$ MeV. Since the neutrinospheres of ν_e and $\bar{\nu}_e$ are larger than those of $\nu_{\mu,\tau}$ and $\bar{\nu}_{\mu,\tau}$ due to charged current interactions, the average energy of ν_e and $\bar{\nu}_e$ are smaller than those of $\nu_{\mu,\tau}$ and $\bar{\nu}_{\mu,\tau}$ (e.g., [9]). Thus, if $T_{\nu_{\mu,\tau}}$ is indeed smaller than 5.0 MeV, T_{ν_e} and $T_{\bar{\nu}_e}$ should also be smaller than 5.0 MeV.

We also note that neutrino oscillations would raise the contribution of electron neutrinos to the ^{11}B and ^{7}Li production. If neutrino conversion between ν_e and $\nu_{\mu,\tau}$ occurs in the oxygen-rich layer [e.g., large mixing angle with θ_{13} large (LMA-L) case in [18,19]], the rates of charged current reactions such as $^4\mathrm{He}(\nu_e, e^- p)^3\mathrm{He}$ and $^{12}\mathrm{C}(\nu_e, e^- p)^{11}\mathrm{C}$ increase, keeping the rates of neutral-current reactions unchanged. The yields of ^{11}B and ^{7}Li would increase by this effect and, thus, lower neutrino temperature is favorable for avoiding overproduction of ^{11}B. Additional constraints derive from r-process nucleosynthesis in neutrino driven winds [7].

We use a specific stellar mass model of $\sim 20 M_\odot$ to demonstrate the sensitivity of the ^{11}B and ^{7}Li yields to E_ν and $T_{\nu_{\mu,\tau}}$. This sensitivity can also be applied to supernova models with different progenitor masses, because the dominant production processes for ^{11}B and ^{7}Li are the ν-process and α-capture reactions, which are insensitive to progenitor masses, specifically, the ^{4}He and ^{12}C abundances. In the He-rich layer, ^{7}Li is produced through the reaction sequences $^4\mathrm{He}(\nu, \nu' p)^3\mathrm{H}(\alpha, \gamma)^7\mathrm{Li}$ and $^4\mathrm{He}(\nu, \nu' n)^3\mathrm{He}(\alpha, \gamma)^7\mathrm{Be}(n, p)^7\mathrm{Li}$. Most of ^{11}B is produced through $^7\mathrm{Li}(\alpha, \gamma)^{11}\mathrm{B}$ and $^7\mathrm{Be}(\alpha, \gamma)^{11}\mathrm{C}(\beta^+)^{11}\mathrm{B}$, or the ν-process $^{12}\mathrm{C}(\nu, \nu' p)^{11}\mathrm{B}$ in the oxygen-rich layer [1,2,7]. The dependence on E_ν and $T_{\nu_{\mu,\tau}}$ of the ejected masses of ^{11}B and ^{7}Li solely relates to that of the ν-process reaction rates. The α-capture rates do not depend on the neutrino parameters, and the abundances of ^{4}He and ^{12}C are solely determined during the precollapse stage. Thus, the ejected masses of ^{11}B and ^{7}Li are proportional to the ν-process reaction rates in accordance with the values of E_ν and $T_{\nu_{\mu,\tau}}$. The neutrino spectrum might depend on progenitor mass, but the extent of this effect has not yet been established.

Many studies of ν-induced nucleosynthesis assume FD distributions with $\mu_\nu = 0$ [1–3,7,8]. However, simulations of neutrino transport in supernova explosions show that the energy spectra are better represented by FD distributions with nonzero chemical potential [10–12]. Therefore, we now consider the effect of nonzero chemical potentials within a semianalytic model. We assume that the energy dependence of the neutrino-matter interaction cross sections can be expressed as a simple power law $\sigma(\epsilon) = \sigma_0 \epsilon^\alpha$. The specific case of $\alpha = 2$ was discussed in [12]. Here we extend their discussion to a wider range of values for α. We assume that the energy spectra are exact FD distributions, specified by values of temperature T_ν and degeneracy parameter $\eta_\nu = \mu_\nu / kT_\nu$, where k is the Boltzmann constant. With the following definition of a moment function

$$F_q(\eta_\nu) = \frac{1}{2\pi(\hbar c)^3} \int_0^\infty \frac{x^q dx}{\exp(x - \eta_\nu) + 1}, \tag{4}$$

the neutrino number density $n_\nu(T_\nu, \eta_\nu)$ and energy density $\epsilon_\nu(T_\nu, \eta_\nu)$ can be expressed as $F_2(\eta_\nu)(kT_\nu)^3$ and $F_3(\eta_\nu) \times (kT_\nu)^4$, respectively. For a neutrino spectrum specified by T_ν and η_ν, the average cross section $\sigma(T_\nu, \eta_\nu)$ is then given by $[F_{\alpha+2}(\eta_\nu)/F_2(\eta_\nu)]\sigma_0 (kT_\nu)^\alpha$, and is related to the average cross section one would obtain from a spectrum with zero chemical potential $\sigma(T_\nu, 0)$ as $[F_{\alpha+2}(\eta_\nu)F_2(0)]/[F_{\alpha+2}(0)F_2(\eta_\nu)]\sigma(T_\nu, 0)$.

The reaction rate for any ν-process reaction under consideration at a given time t and at a distance r from the source is given by $\lambda(T_\nu, \eta_\nu; t) = \sigma(T_\nu, \eta_\nu)\phi(T_\nu, \eta_\nu; t)$, where the neutrino number flux is

$$\phi(T_\nu, \eta_\nu; t) = \frac{1}{4\pi r^2} \frac{E_\nu}{\frac{F_3(\eta_\nu)}{F_2(\eta_\nu)} kT_\nu} \frac{1}{\tau_\nu} \exp\left(-\frac{t - r/c}{\tau_\nu}\right). \tag{5}$$

Note that $\phi(T_\nu, \eta_\nu; t)$ is a function of not only T_ν but also η_ν because the average neutrino energy depends on T_ν and η_ν: the average energy per neutrino is $\langle \varepsilon_\nu \rangle = [F_3(\eta_\nu)/F_2(\eta_\nu)]kT_\nu$ and $F_3(0)/F_2(0) = 3.1514$ for $\eta_\nu = 0$. The reaction rate $\lambda(T_\nu, \eta_\nu; t)$ can then be expressed as

$$\lambda(T_\nu, \eta_\nu; t) = C_\alpha(\eta_\nu)\lambda(T_\nu, 0; t), \tag{6}$$

where $C_\alpha(\eta_\nu)$ is the scaling function

$$C_\alpha(\eta_\nu) = \frac{F_{\alpha+2}(\eta_\nu)}{F_{\alpha+2}(0)} \frac{F_3(0)}{F_3(\eta_\nu)}, \tag{7}$$

and both $\lambda(T_\nu, \eta_\nu; t)$ and $\lambda(T_\nu, 0; t)$ have the same T_ν dependence $\propto T_\nu^{\alpha-1}$.

We now apply this semianalytic model. First, we determine the effective power law indices for the total neutral-current cross sections on ^{56}Fe and ^{58}Fe from the calculations presented in [20]. We find indices of 3.7 and 3.8, respectively. This implies $\sigma(T_\nu, 3)/\sigma(T_\nu, 0) = 1.72(1.73)$ for $\alpha = 3.7(3.8)$, consistent with the values reported in [20]. We also evaluate the power law indices α of the cross sections of $^4\mathrm{He}(\nu, \nu' p)^3\mathrm{H}$ and $^{12}\mathrm{C}(\nu, \nu' p)^{11}\mathrm{B}$ by fitting the cross sections in [17]. For $^4\mathrm{He}(\nu, \nu' p)^3\mathrm{H}$ and $^{12}\mathrm{C}(\nu, \nu' p)^{11}\mathrm{B}$, we find $\alpha = 6.7$ and 5.9, respectively. These indices are much larger than the values obtained for reactions with the larger nuclear systems ^{56}Fe and ^{58}Fe, indicating a significant mass number dependence of α. We therefore evaluate $C_\alpha(\eta_\nu)$ for α ranging from 4 to 7.

Figure 3 shows $C_\alpha(\eta_\nu)$ as a function of η_ν for various values of α, indicating that the rates of the ν-process can

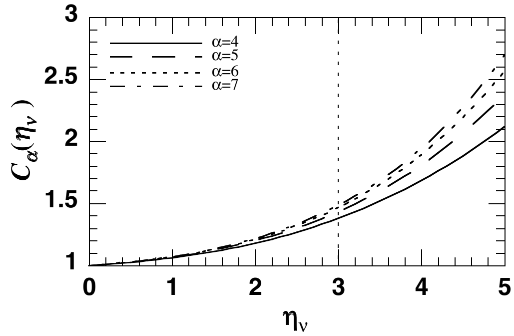

FIG. 3. The scaling function with nonzero chemical potential $C_\alpha(\eta_\nu)$ defined by Eq. (7). The particular values of α shown are 4, 5, 6, and 7.

vary substantially with the adopted values for these two key parameters. The production of ^{7}Li and ^{11}B is proportional to the reaction rates of ^{4}He$(\nu, \nu' p)^3$H and ^{12}C$(\nu, \nu' p)^{11}$B, which have similar values of α (see above), so that the ejected masses of ^{11}B and ^{7}Li in the case of $\eta_\nu = 3$ would be increased by about 50% in comparison to the yield obtained for $\eta_\nu = 0$.

When we allow for nonzero chemical potentials, the corresponding range of $T_{\nu_{\mu,\tau}}$ derived from the GCE constraint for ^{11}B changes. Consider the relation between the neutrino temperatures derived from a given yield obtained with either nonzero chemical potential T_ν or with zero chemical potential $T_{\nu 0}$ by enforcing $\lambda(T_\nu, \eta_\nu; t) = \lambda(T_{\nu 0}, 0; t)$. The ratio of these two temperatures is given as

$$\frac{T_\nu}{T_{\nu 0}} = C_\alpha(\eta_\nu)^{-1/(\alpha-1)}. \tag{8}$$

For nonzero chemical potentials, $T_\nu/T_{\nu 0}$ is a monotonically decreasing function of η_ν. In the case of $\eta_\nu = 3$ we find $T_\nu/T_{\nu 0} = 0.90$ for $\alpha = 4$ and 0.94 for $\alpha = 7$. This implies that the neutrino temperature satisfying the GCE production constraint of ^{11}B is reduced to 4.3 MeV $\leq T_{\nu_{\mu,\tau}}(\eta_\nu = 3) \leq$ 5.9 MeV, about 6%–10% smaller than the range inferred for $\eta_\nu = 0$ [Eq. (3)]. Likewise, T_{ν_e} and $T_{\bar{\nu}_e}$ would be reduced by a comparable fraction. In the case of negative η_ν, $T_\nu/T_{\nu 0}$ increases very weakly, e.g., $T_\nu/T_{\nu 0} = 1.015$ for $\eta_\nu = -3$.

In summary, the ejected masses of ^{11}B and ^{7}Li increase with $\nu_{\mu,\tau}$ and $\bar{\nu}_{\mu,\tau}$ temperature through the energy dependence of the cross sections of the ν-process: this dependence ($\propto T_\nu^{\alpha-1}$) is stronger than the dependence on the total neutrino energy. To reproduce the supernova contribution of ^{11}B within the framework of GCE, neutrino temperature is constrained to 4.8 MeV $\leq T_{\nu_{\mu,\tau}}(\eta_\nu = 0) \leq$ 6.6 MeV. Nonzero neutrino chemical potential leads to a larger light element yield. The ejected masses of ^{11}B and ^{7}Li would be increased by about 50% in the case of $\eta_\nu = 3$. For a given yield, the required neutrino temperatures are reduced correspondingly, but the change is less than 10%. The inferred temperature range provides a constraint on theoretical models of neutrino transport in supernovae and constrains their ^{7}Li yields, which imposes constraints on contributions from AGB stars and novae to Galactic ^{7}Li.

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*Electronic address: tyoshida@astr.tohoku.ac.jp

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