Beating the Photon-Number-Splitting Attack in Practical Quantum Cryptography

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We propose an efficient method to verify the upper bound of the fraction of counts caused by multiphoton pulses in practical quantum key distribution using weak coherent light, given whatever type of Eve's action. The protocol simply uses two coherent states for the signal pulses and vacuum for the decoy pulse. Our verified upper bound is sufficiently tight for quantum key distribution with a very lossy channel, in both the asymptotic and nonasymptotic case. So far our protocol is the *only* decoy-state protocol that works efficiently for currently existing setups.

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Unlike classic cryptography, quantum key distribution (QKD) [1–3] can help two remote parties to set up a secure key by the noncloning theorem [4]. Further, proofs for the unconditional security over noisy channel have been given [5–8]. The security of practical QKD with weak coherent states has also been shown [9,10]. However, there are still some limitations for QKD in practice, especially over long distance. In particular, large loss seems to be the main challenge to the long-distance QKD with weak coherent states. A dephased coherent state $|\mu e^{i\theta}\rangle$ is actually a mixed state of

$$\rho_u = \frac{1}{2\pi}\int_0^{2\pi} |\mu e^{i\theta}\rangle\langle\mu e^{i\theta}| d\theta = \sum_n P_n(\mu)|n\rangle\langle n| \quad (1)$$

and $P_n(\mu) = \frac{\mu^n e^{-\mu}}{n!}$. Here μ is a non-negative number. In practice, especially in doing long-distance QKD, the channel transmittance η can be rather small. If $\eta < (1 - e^{-\mu} - \mu e^{-\mu})/\mu$, an eavesdropper (Eve) in principle can have the full information of Bob's sifted key by the photon-number-splitting (PNS) attack [11]: Eve blocks all single-photon pulses and part of multiphoton pulses and separates each of the remained multiphoton pulses into two parts; therefore, each part contains at least one photon. To each split pulse, she keeps one part and sends the other part to Bob, through a lossless channel. As it has been shown [9,10], they can still distill some final key if the fraction of tagged bits (counts caused by multiphoton pulses) is not too large. However, the key distillation requires information of the value Δ, the upper bound of the fraction of tagged bits, or equivalently, the value Δ_1, the lower bound of the fraction of Bob's detected bits caused by single-photon pulses from Alice.

Given the separate result of key distillation [9,10] with tagged bits, verifying a tight bound for Δ is the first important thing in QKD. We shall show how to verify it efficiently. Since the calculation of Δ and Δ_1 are equivalent, here we only focus on the verification of Δ. As it was shown in Ref. [12], Eve may have many choices in doing the PNS attack; therefore, a simple-minded method does not give a faithful verification. A very important method with decoy states was then proposed by Hwang [13], where the *unconditional* verification of the multiphoton counting rate is given. Hwang's decoy-state method can faithfully estimate the upper bound of Δ through decoy pulses, given *whatever* type of PNS attack. (Remark: decoy-state method is not the only solution to the issue [14,15].) However, Hwang's initial protocol [13] does not give a sufficiently tight bound. For example, in the case of $\mu = 0.3$, by Hwang's method, the optimized verified upper bound of Δ is 60.4%. As it has been mentioned [13,16], decoy-state method can be combined with the result in [9,10] to distill unconditionally secure final key. With the value $\Delta =$ 60.4%, the key rate can be rather low in practice [9,10]. Following Hwang's work [13], decoy-state method was then studied by Lo and co-workers [16,17]. They proposed their main protocol using an infinite number of decoy states. In such a way the counting rates of each state $|n\rangle\langle n|$ can be calculated; therefore, an exact value of Δ can be given. However, such a protocol seems to be inefficient in practice, because it requires an infinite number of classes of different coherent states to work as the decoy states [18]. Prior to this, a review of PNS attack was given with some very shortly stated rough ideas for possible solution [17]. However, no explicitly demonstrated result was given there [17,18].

Here we present a new deocy-state protocol with explicit demonstrations. Our protocol uses only three different states and the verified bound values are sufficiently tight for long-distance QKD. The main idea here is to watch the counting rates of all three classes of states and treat them jointly with nontrivial inequalities. In the protocol, coherent states with average photon number μ, μ' are used for signal pulses and vacuum is used for the decoy pulse. Since both μ and μ' are in a reasonable range, pulses produced in both states can be used to distill the final key. Moreover, we have for the first time considered the nonasymptotic effects in the decoy-state method.

For simplicity, we denote those pulses produced in state $|\mu e^{i\theta}\rangle$, $|\mu' e^{i\theta}\rangle$, $|0\rangle$ as class Y_μ, $Y_{\mu'}$, and Y_0, respectively. The value θ in a coherent pulse is random. Alice mixes the

positions of all pulses; therefore, no one but Alice knows which pulse belongs to which class in the protocol. They observe the counting rates of each class and then verify the upper bounds of counts caused by multiphoton pulses from class Y_μ, $Y_{\mu'}$, respectively. If these values are too large, they abandon the protocol; otherwise they go on to do key distillations using pulses from each class of Y_μ *and* $Y_{\mu'}$ by the method given in [9,10].

We first define the *counting rate* of *any* state ρ: the probability that Bob's detector clicks whenever a state ρ is *sent out* by Alice. We *disregard* what state Bob may receive here. This *counting rate* is called the *yield* in other literatures [13,16]. We denote the counting rate (yield) of vacuum, class Y_0, Y_μ, $Y_{\mu'}$, by notations s_0, S_μ, $S_{\mu'}$, respectively. These three parameters are observed in the protocol itself: after all pulses are sent out, Bob announces which pulse has caused a click and which pulse has not caused a click. Since Alice knows which pulse belongs to which class, Alice can calculate the *counting rates* of each class of pulses. Therefore, we shall regard s_0, S_μ, $S_{\mu'}$ as known parameters in protocol. The value s_0 is the counting rate at Bob's side when Alice sends vacuum pulses. We shall also call s_0 the vacuum count or dark count rate.

Their task is to verify the upper bound of Δ, the fraction of multiphoton counts among all counts caused by pulses in class Y_μ, and also the upper bound of Δ', the fraction of multiphoton counts among all counts caused by pulses in class $Y_{\mu'}$. We shall show how they can deduce the upper values of Δ, Δ' from the values of $\{s_0, S_\mu, S_{\mu'}\}$. We shall focus on Δ first and later obtain Δ' based on the knowledge of Δ.

For convenience, we *always* assume

$$\mu' > \mu; \qquad \mu' e^{-\mu'} > \mu e^{-\mu} \tag{2}$$

in this Letter. A dephased coherent state $|\mu e^{i\theta}\rangle$ has the following convex form:

$$\rho_\mu = e^{-\mu}|0\rangle\langle 0| + \mu e^{-\mu}|1\rangle\langle 1| + c\rho_c \tag{3}$$

and $c = 1 - e^{-\mu} - \mu e^{-\mu} > 0$,

$$\rho_c = \frac{1}{c}\sum_{n=2}^{\infty} P_n(\mu)|n\rangle\langle n|. \tag{4}$$

Similarly, state $|\mu' e^{i\theta}\rangle$ after dephasing is

$$\rho_{\mu'} = e^{-\mu'}|0\rangle\langle 0| + \mu' e^{-\mu'}|1\rangle\langle 1| + c\frac{\mu'^2 e^{-\mu'}}{\mu^2 e^{-\mu}}\rho_c + d\rho_d \tag{5}$$

and $d = 1 - e^{-\mu'} - \mu' e^{-\mu'} - c\frac{\mu'^2 e^{-\mu'}}{\mu^2 e^{-\mu}} \geq 0$. ρ_d is a density operator. (We shall only use the fact that d is nonnegative and ρ_d *is* a density operator.) In deriving the above convex form, we have used the fact $P_n(\mu')/P_2(\mu') > P_n(\mu)/P_2(\mu)$ for all $n > 2$, given the conditions of Eq. (2). With these convex forms of density operators, it is equivalent to say that Alice sometimes sends nothing ($|0\rangle\langle 0|$), sometimes sends $|1\rangle\langle 1|$, sometimes sends ρ_c, and sometimes sends ρ_d, though Alice does not know which time she has sent out which one of these states. In each individual sending, she only knows to which class the pulse belongs. We shall use notations s_0, s_1, s_c, S_μ, $S_{\mu'}$, s_d for the *counting rates* of state $|0\rangle\langle 0|$, $|1\rangle\langle 1|$, ρ_c, ρ_μ, $\rho_{\mu'}$, ρ_d, respectively. Given any state ρ, nobody but Alice can tell whether it is from class Y_μ or $Y_{\mu'}$. Asymptotically, we have

$$s_\rho(\mu) = s_\rho(\mu') \tag{6}$$

and $s_\rho(\mu)$, $s_\rho(\mu')$ are *counting rates* for state ρ from class Y_μ and class $Y_{\mu'}$, respectively.

We shall use the safest assumption that Eve also controls the detection efficiency and dark count of Bob's detector. We only consider the overall transmittance including the channel, Bob's devices, and detection efficiency. By Eq. (3), we relate Δ with parameter s_c, the multiphoton counting rate in class Y_μ, by:

$$\Delta = c\frac{s_c}{S_\mu}. \tag{7}$$

To verify the upper bound of Δ for pulses from class Y_μ, we only need to verify the upper bound of s_c, the *counting rate* of mixed state ρ_c. The task is reduced to formulating s_c by $\{s_0, S_\mu, S_{\mu'}\}$, which are measured directly in the protocol itself. The coherent state $\rho_{\mu'}$ is convexed by ρ_c and other states. Given the condition of Eq. (2), the probability of ρ_c in state $\rho_{\mu'}$ is larger than that in ρ_μ. Using this fact we can make a preliminary estimation of s_c. From Eq. (5) we immediately obtain

$$S_{\mu'} = e^{-\mu'}s_0 + \mu' e^{-\mu'}s_1 + c\frac{\mu'^2 e^{-\mu'}}{\mu^2 e^{-\mu}}s_c + ds_d. \tag{8}$$

s_0 is known, s_1 and s_d are unknown, but they are never less than zero. Therefore, we have

$$cs_c \leq \frac{\mu^2 e^{-\mu}}{\mu'^2 e^{-\mu'}}(S_{\mu'} - e^{-\mu'}s_0 - \mu' e^{-\mu'}s_1). \tag{9}$$

We can obtain Hwang's main result [13] by

$$cs_c \leq \frac{\mu^2 e^{-\mu}}{\mu'^2 e^{-\mu'}}(S_{\mu'} - e^{-\mu'}s_0) \leq \frac{\mu^2 e^{-\mu}}{\mu'^2 e^{-\mu'}}S_{\mu'}. \tag{10}$$

Combining this with Eq. (7) we have $\Delta \leq \frac{\mu^2 e^{-\mu}S_{\mu'}}{\mu'^2 e^{-\mu'}S_\mu}$. This is just Eq. (12) in Ref. [13]. In the normal case that there is no Eve's attack, and Alice and Bob will find $S_{\mu'}/S_\mu = \frac{1-e^{-\eta\mu'}}{1-e^{-\eta\mu}} = \mu'/\mu$ in their protocol; therefore, they can verify $\Delta \leq \frac{\mu e^{-\mu}}{\mu' e^{-\mu'}}$, which is just Eq. (13) of Hwang's work [13]. Our derivation looks significantly simpler.

Having obtained the crude results above, we now show that the verification can be done more sophisticatedly and one can further tighten the bound significantly. In the

inequality (10), we have dropped terms s_1 and s_d, since we only have trivial knowledge about s_1 and s_d there, i.e., $s_1 \geq 0$ and $s_d \geq 0$. Therefore, inequality (9) has no advantage at that moment. However, after we have obtained the crude upper bound of s_c, we can have a larger-than-zero lower bound for s_1, provided that our crude upper bound for Δ given by Eq. (10) is not too large. From Eq. (3) we have

$$e^{-\mu}s_0 + \mu e^{-\mu}s_1 + cs_c = S_\mu. \tag{11}$$

With the crude upper bound for s_c given by Eq. (10), we have the nontrivial lower bound for s_1 now:

$$s_1 \geq S_\mu - e^{-\mu}s_0 - cs_c > 0. \tag{12}$$

Therefore, tight values for s_c and s_1 can be obtained by solving the simultaneous constraints of Eq. (11) and inequality (9). We have the following final bound after solving them:

$$\Delta \leq \frac{\mu}{\mu' - \mu}\left(\frac{\mu e^{-\mu}S_{\mu'}}{\mu' e^{-\mu'}S_\mu} - 1\right) + \frac{\mu e^{-\mu}s_0}{\mu' S_\mu}. \tag{13}$$

Here we have used Eq. (7). In the case of $s_0 \ll \eta$, if there is no Eve, $S'_\mu/S_\mu = \mu'/\mu$. Alice and Bob must be able to verify

$$\Delta = \frac{\mu(e^{\mu'-\mu} - 1)}{\mu' - \mu}\bigg|_{\mu'-\mu\to 0} = \mu \tag{14}$$

in the protocol. This is close to the real value of fraction of multiphoton counts: $1 - e^{-\mu}$, given that $\eta \ll 1$. In our derivation, all multiphoton counts from pulses in class Y_μ are due to only *one* mixed state, ρ_c. Therefore, we only need to calculate *one* unknown parameter, s_c. However, in Ref. [16], they have considered the contribution of each Fock state and there are an infinite number of unknown variables of $\{s_n\}$. Therefore, they need an *infinite* number of different coherent states in their main protocol [16] while we only need three.

With the upper bound of s_c (or Δ), pulses from class Y_μ can be used for key distillation by [9,10]. Given s_c, we can calculate the lower bound of s_1 through Eq. (12). Given the fact that pulses in both classes have the same value of s_1 and $1 - \Delta - \frac{s_0 e^{-\mu}}{S_\mu} = \frac{s_1 \mu e^{-\mu}}{S_\mu}$, we have

$$\Delta' \leq 1 - \left(1 - \Delta - \frac{e^{-\mu}s_0}{S_\mu}\right)\frac{S_\mu \mu'}{S_{\mu'}\mu}e^{\mu-\mu'} - \frac{e^{-\mu'}s_0}{S_{\mu'}}. \tag{15}$$

The values of μ, μ' should be chosen in a reasonable range, e.g., from 0.2 to 0.5. To maximize the key rate, one needs to consider the quantities of transmittance, quantum bit error rate (QBER), and vacuum counts jointly. The optimization is not studied in this Letter.

The results above are only for the asymptotic case. In practice, there are statistical fluctuations; i.e., Eve has non-negligibly small probability to treat the pulses from different classes a little bit differently, even though the pulses have the same state. It is *insecure* if we simply use the asymptotic result in practice. Our remaining task is to verify a tight upper bound of Δ and the probability that the real value of Δ breaks the verified upper bound is exponentially close to 0.

The counting rate of any state ρ in class $Y_{\mu'}$ now can be slightly different from the counting rate of the same state ρ from another class, Y_μ, with non-negligible probability. We shall use the primed notation for the counting rate for any state in class $Y_{\mu'}$ and the original notation for the counting rate for any state in class Y_μ. Explicitly, Eqs. (9) and (11) are now converted to

$$\begin{aligned} e^{-\mu}s_0 + \mu e^{-\mu}s_1 + cs_c &= S_\mu, \\ cs'_c &\leq \frac{\mu^2 e^{-\mu}}{\mu'^2 e^{-\mu'}} \\ &\quad \times (S_{\mu'} - \mu' e^{-\mu'}s'_1 - e^{-\mu'}s'_0). \end{aligned} \tag{16}$$

Setting $s'_x = (1 - r_x)s_x$ for $x = 1$, c, and $s'_0 = (1 + r_0)s_0$, we can replace all primed parameters $\{s'_0, s'_1, s'_c\}$ by unprimed ones in the above equation; therefore, we obtain

$$\begin{aligned} \mu' e^{\mu}\left[(1 - r_c)\frac{\mu'}{\mu} - 1\right]\Delta &\leq \mu e^{\mu'}S_{\mu'}/S_\mu - \mu' e^{\mu} \\ &\quad + [(\mu' - \mu)s_0 + r_1 s_1 + r_0 s_0]/S_\mu. \end{aligned} \tag{17}$$

From this we can see, if μ and μ' are too close, Δ can be very large. The important question here is now whether there are reasonable values for μ', μ so that our method has significant advantage to the previous method [13]. The answer is yes.

Given $N_1 + N_2$ copies of state ρ, suppose the counting rate for N_1 randomly chosen states is s_ρ and the counting rate for the remaining states is s'_ρ, the probability that $s_\rho - s'_\rho > \delta_\rho$ is less than $\exp(-\frac{1}{4}\delta_\rho^2 N_0/s_\rho)$ and $N_0 = \mathrm{Min}(N_1, N_2)$. Now we consider the difference of counting rates for the same state from different classes, Y_μ and $Y_{\mu'}$. To make a faithful estimation for exponential certainty, we require $\delta_\rho^2 N_0/s_\rho = 100$. This causes a relative fluctuation $r_\rho = \frac{\delta_\rho}{s_\rho} \leq 10\sqrt{\frac{1}{s_\rho N_0}}$. To formulate the relative fluctuation r_1, r_c by s_c and s_1, we only need to check the number of pulses in ρ_c, $|1\rangle\langle 1|$ in each class in the protocol. That is, we can replace r_1, r_c in Eq. (16) by $10e^{\mu/2}\sqrt{\frac{1}{\mu s_1 N}}$, $10\sqrt{\frac{1}{cs_c N}}$, respectively, and N is the number of pulses in class Y_μ. Since we assume the case where the vacuum-counting rate is much less than the counting rate of state ρ_μ, we omit the effect of fluctuation in vacuum counting; i.e., we set $r_0 = 0$. With these inputs, Eq. (16) can now be solved numerically. The results are listed in Table I. From this table we

TABLE I. The verified upper bound of the fraction of tagged pulses in QKD. Δ_H is the result from Hwang's method. Δ_R is the true value of the fraction of multiphoton counts in case there is no Eve. Δ_H and Δ_R do not change with channel transmittance. Δ_{W1} is bound for pulses in class Y_μ, given that $\eta = 10^{-3}$. Δ_{W2} and Δ'_{W2} are bound values for the pulses in class Y_μ, $Y_{\mu'}$, respectively, given that $\eta = 10^{-4}$. We assume $s_0 = 10^{-6}$. The number of pulses is 10^{10} in class Y_μ, $Y_{\mu'}$ in calculating Δ_{W1} and 8×10^{10} in calculating Δ_{W2}, Δ'_{W2}. (Our results will only increase by 0.03 even if we only use 10^{10} pulses. Actually, a pretty good key rate can be obtained with only 10^{10} pulses [21].) 4×10^9 vacuum pulses are sufficient for class Y_0. The bound values will change by less than 0.01 if the value of s_0 is 1.5 times larger. The numbers inside the brackets are chosen values for μ'. For example, in the column of $\mu = 0.25$, data 30.9%(0.41) means, if we choose $\mu = 0.25$, $\mu' = 0.41$, we can verify $\Delta \leq 30.9\%$ for class Y_μ.

μ	0.2	0.25	0.3	0.35
Δ_H	44.5%	52.9%	60.4%	67.0%
Δ_R	18.3%	22.2%	25.9%	29.5%
Δ_{W1}	23.4%(0.34)	28.9%(0.38)	34.4%(0.43)	39.9%(0.45)
Δ_{W2}	25.6%(0.39)	30.9%(0.41)	36.2%(0.45)	41.5%(0.47)
μ'	0.39	0.41	0.45	0.47
Δ_H	71.8%	74.0%	78.0%	79.8%
Δ_R	32.3%	33.7%	36.2%	37.5%
Δ'_{W2}	40.1%	42.2%	45.8%	48.6

can see that good values of μ, μ' indeed exist and our verified upper bounds are sufficiently tight to make QKD over a very lossy channel. Note that so far this is the *only* nonasymptotic result among all existing works on decoy state. From Table I we can see that our nonasymptotic values are less than Hwang's asymptotic values already. Our verified values are rather close to the true values. Given the parameters in a typical real setup [19,20], we believe that our protocol works over a distance longer than 120 km with $\mu = 0.3$, $\mu' = 0.45$, and *a reasonable number* of total pulses.

In summary, following Hwang [13], we have proposed an efficient and feasible decoy-state method to do QKD over a very lossy channel. We have for the first time clearly demonstrated how it works efficiently with only three classes of different states. We have for the first time considered the effect of statistical fluctuation for the decoy-state method. Our method is further studied recently [21].

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Note added.—After this work was completed and presented, Ref. [22] was also presented and it was shortly mentioned there that they can also make it with only a few decoy states. We believe that prior to our presentation, none of decoy-state proposals can really work efficiently in practice [18].

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[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).
[2] N. Gisin, G. Ribordy, W. Tittle, and H. Zbinden, Rev. Mod. Phys. **74**, 145 (2002), references therein.
[3] C. H. Bennett and G. Brassard, in *Proceedings of the IEEE International Conference on Computers, Systems, and Signal Processing, Bangalore* (IEEE, New York, 1984), p. 175.
[4] W. K. Wootters and W. H. Zurek, Nature (London) **299**, 802 (1982).
[5] P. W. Shor and J. Preskill, Phys. Rev. Lett. **85**, 441 (2000).
[6] H.-K. Lo and H. F. Chau, Science **283**, 2050 (1999).
[7] D. Mayers, Journal of the Association for Computing Machinery **48**, 351 (2001). Its preliminary version appeared in *Advances in Cryptology-Proc. Crypto '96*, edited by N. Koblitz, of Lect. Notes Comput. Sci. Vol. 1109 (Springer-Verlag, Berlin, 1996), p. 343.
[8] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991); C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Phys. Rev. A **54**, 3824 (1996).
[9] H. Inamori, N. Lütkenhaus, and D. Mayers, quant-ph/0107017.
[10] D. Gottesman, H.-K. Lo, N. Lütkenhaus, and J. Preskill, Quantum Inf. Comput. **4**, 325 (2004).
[11] B. Huttner, N. Imoto, N. Gisin, and T. Mor, Phys. Rev. A **51**, 1863 (1995); G. Brassard, N. Lütkenhaus, T. Mor, and B. C. Sanders, Phys. Rev. Lett. **85**, 1330 (2000).
[12] N. Lütkenhaus and M. Jahma, New J. Phys. **4**, 44 (2002).
[13] W.-Y. Hwang, Phys. Rev. Lett. **91**, 057901 (2003).
[14] C. H. Bennett, Phys. Rev. Lett. **68**, 3121 (1992); M. Koashi, Phys. Rev. Lett. **93**, 120501 (2004).
[15] V. Scarani, A. Acin, G. Ribordy, and N. Gisin, Phys. Rev. Lett. **92**, 057901 (2004); A. Acin, N. Gisin, and V. Scarani, Phys. Rev. A **69**, 012309 (2004).
[16] H.-K. Lo, X.-F. Ma, and K. Chen, http://www.fields.utoronto.ca/programs/scientific/04-05/quantumIC/abstracts/lo.ppt; /lo.pdf: Decoy-State Quantum Key Distribution (QKD); and also: http://www.newton.cam.ac.uk/webseminars/pg+ws/2004/qisw01/0826/lo/.
[17] H.-K. Lo, *Proceedings of 2004 IEEE International Symposium on Inf. Theor., June 27–July 2, 2004, Chicago* (IEEE, New York, 2004), p. 17.
[18] X. B. Wang, quant-ph/0501143.
[19] C. Gobby, Z. L. Yuan, and A. J. Shields, Appl. Phys. Lett. **84**, 3762 (2004).
[20] H. Kosaka *et al.*, Electron. Lett. **39**, 1199 (2003).
[21] X. B. Wang, quant-ph/0411047 [Phys. Rev. A (to be published)]; X. Ma *et al.*, quant-ph/0503005.
[22] H.-K. Lo, X.-F. Ma, and K. Chen, this issue, Phys. Rev. Lett. **94**, 230504 (2005).