

Controlled Observation of a Nonequilibrium Ising-Bloch Transition in a Nonlinear Optical Cavity

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We report the controlled observation of the nonequilibrium Ising-Bloch transition in a broad area nonlinear optical cavity (a quasi-1D single longitudinal-mode photorefractive oscillator in a degenerate four-wave mixing configuration). Our experimental technique allows for the controlled injection of the domain walls. We use cavity detuning as control parameter and find that both Ising and Bloch walls can exist for the same detuning values within a certain interval of detunings; i.e., the Ising-Bloch transition is hysteretic in our case. A complex Ginzburg-Landau model is used for supporting the observations.

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Spatially extended bistable systems exhibit a large variety of localized structures as bistability allows that different states occupy different spatial regions. An interesting subclass is that of systems with broken phase invariance [1], which can display defects in the form of interfaces, so-called fronts, across which the system passes from one of the bistable phases to the other one in adjacent spatial domains. Here we consider one-dimensional systems.

In equilibrium systems, characterized by a well-defined free energy, two cases can be distinguished depending on whether the two states that the front connects have equal or different free energy: When equivalent (e.g., the two possible orientations of magnetization in a ferromagnet) fronts are static, while when both states have different free energy, fronts move so that the lower energy state finally invades the whole system. Front dynamics is richer in nonequilibrium systems whose dynamics does not derive from a free energy. As in equilibrium systems, a front connecting two nonequivalent states is a transient state. Contrarily, the behavior is different from equilibrium systems when the front connects two equivalent states (these fronts are known as domain walls, DWs): Motion in this case is still possible through a parity breaking bifurcation occurring at the front core [2]. Before the bifurcation the (resting) structure is “odd” with respect to the front and is known as an Ising wall; after the bifurcation that symmetry is lost and the structure, known as a Bloch wall, moves. This nonequilibrium Ising-Bloch transition (NIBT) [3] [borrowing its name from the (equilibrium) Ising-Bloch transition of ferromagnets [4]] is generic in self-oscillatory systems parametrically forced at twice their natural oscillation frequency [2,5].

There are few experimental observations of this phenomenon. As far as we know, it has been reported only in liquid crystals [6] either subjected to rotating magnetic fields, [7,8] or to an alternate electrical voltage [9]. This last experiment constitutes a particularly clear observation of the NIBT free of 2D effects, which complicate front dynamics through curvature effects.

Nonlinear optical cavities provide several examples of systems with broken phase invariance for which the NIBT

has been predicted but not yet observed (intracavity type II second-harmonic generation [3], degenerate optical parametric oscillation [10–12], and cavity degenerate four-wave mixing [13]). In an optical system, the DW connects two stable states of equal light intensity but opposite (i.e., separated by π) phase. In the Ising wall the phase jumps sharply by π , and the light intensity is null at the wall core, while in the Bloch wall the phase angle rotates continuously through π , and the light intensity is minimal but not null at the wall core. As two senses of rotation are possible, Bloch walls are chiral (have the symmetry of a corkscrew), which has important dynamical consequences as walls with opposite chirality move in opposite directions [2]. In this Letter we report the first experimental observation of the NIBT in a nonlinear optical system. Moreover, the NIBT we describe below presents a distinctive feature that, to the best of our knowledge, has not been described before: It is a hysteretic NIBT; i.e., there is a domain of coexistence of both Ising and Bloch walls.

Our system is a single longitudinal-mode photorefractive oscillator in a degenerate four-wave mixing configuration [14–16], exactly the same system used in [16]. Two counterpropagating coherent pump beams of equal frequency illuminate a BaTiO₃ crystal which produces scattered light, part of which seeds the oscillation into the (plane mirrors) Fabry-Perot resonator. Oscillation is dynamically ruled by gain, losses, cavity detuning (the mismatch between the frequency of the pump fields and that of the longitudinal cavity mode in which the system oscillates), and diffraction. Each of these parameters can be modified in our experiment up to a certain extent. In particular, the detuning Δ can be finely tuned by our adjusting of the stabilized cavity length [16] and plays the role of control parameter in our experiment. Two characteristics of the cavity are particularly relevant. First, we are using a near self-imaging resonator [17] that allows us to have a very large Fresnel number, i.e., allows the oscillation of a very large number of transverse modes. Second, we intentionally make our system quasi-1D in the transverse dimension by placing slits in the Fourier planes. The width of the slits is adjusted to the size of the diffrac-

tion spot in these planes. In this way, beams with an inclination such that their transverse wave vector does not lie in the plane defined by the center line of the slit are not compatible with the diffraction constraints of the cavity. The Fourier filtering allows the use of finite width slits and still gets rid of most 2D effects, such as front curvature. All this amounts to saying that our nonlinear cavity is equivalent to a large aspect ratio 1D nonlinear system. Figure 1 displays a simplified scheme of the photorefractive oscillator and the reader is referred to [16] for full details.

In [15] the dynamics of spontaneously formed phase domains was experimentally studied in a setup very similar to the one we use here but, importantly, two dimensional in the transverse plane. Larionova *et al.* [15] characterize the closed wall separating different phase domains and find Bloch-type and Ising-type segments along the same wall. They did not observe, however, any NIBT because of the 2D character of their system.

In [16] pattern formation in the same system we are using here was discussed. The basic bifurcation diagram found in [16] can be summarized as follows: For large enough negative detuning Δ , the system forms a periodic structure (stripe pattern) whose spatial frequency varies as $k^2 \sim -\Delta$ (this law allows the sharp determination of the cavity resonance); for small negative Δ , aperiodic patterns are formed; and for positive Δ , homogeneous states are found. In this last region, DWs (isolated or not) form spontaneously, as already described in [16]. In this Letter we address the dynamic behavior of these DWs as Δ is varied.

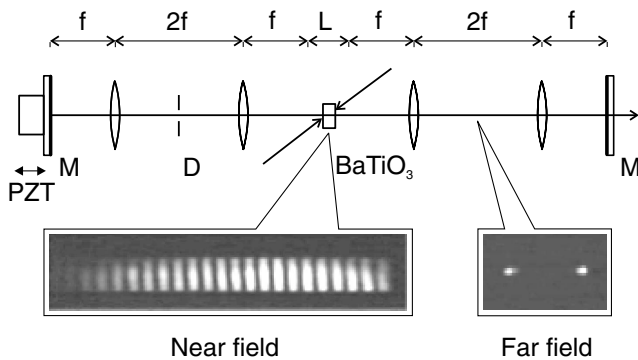


FIG. 1. Scheme of the experimental setup: M, cavity mirrors; L, effective cavity length; D, rectangular diaphragm that makes the system quasi-1D; PZT, piezotransducer for control of the cavity detuning; BaTiO₃, photorefractive nonlinear crystal ($5 \times 5 \text{ mm}^3$). The tilted arrows indicate the two counterpropagating pump beams. Four lenses of focal length f are arranged in two telescopic systems for forming a (near) self-imaging resonator. See [16] for a detailed description of the experimental setup. Photographs: Output field intensity in the near and far fields for a striped pattern showing the quasi-1D nature of the system.

A crucial difference with respect to [16] and from any previous experiment is that in the present experiment DWs are created in a controlled way. This is achieved by injecting, for a short time, a tilted laser beam of the same frequency as the pumps into the photorefractive oscillator. This tilted beam has a phase profile $\phi(x) = \phi_0 + (2\pi/\lambda \sin\alpha)x$ (ϕ_0 is a constant, λ the light wavelength, α the tilt angle with respect to the resonator axis, and x the transverse coordinate). As the degenerate four-wave mixing process is phase sensitive [18], only two phase values (say 0 and π , modulo 2π) are amplified. Consequently, the portions of the transverse plane whose phase lies in $]-\pi/2, +\pi/2[$ (in $]+ \pi/2, +3\pi/2[$) are attracted towards the phase value 0 (π). The points at which $\phi = \pm\pi/2$ (modulo 2π) are dark as those phase values are not amplified and, more importantly, because these points are separating adjacent domains with opposite phases. These points (the domain walls) are thus topological defects [19]. Once DWs are envisaged, the injection is removed. In this way a single DW (or more, for larger tilt angles in the writing beam) can be written at the desired location along the transverse dimension.

The intensity distribution on the x - y plane, perpendicular to the resonator axis z , is not homogeneous due to several reasons, the Gaussian profile of the pump beam among them. On the other hand, the intracavity slits make the system quasi-1D but, obviously, not strictly 1D. Then in order to analyze quantitatively the intensity and phase properties of the field, either a particular value of y is chosen or some averaging in the y direction is made. Our strategy has consisted in isolating the central region in the y direction where the output intensity is approximately constant (for fixed x) and then making an average of the field inside this region. We chose this option because this procedure gets rid of local imperfections, which could affect the wall characterization if a particular y value were chosen. Notice that in this way the outer borders in the y direction are discarded, a convenient procedure in order to avoid border effects in the DW characterization. The in-

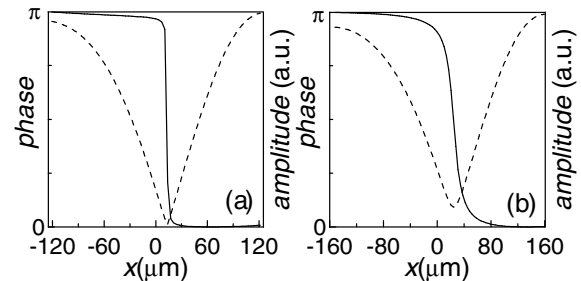


FIG. 2. Amplitude (dashed line, arbitrary units) and phase (full line) of an Ising (a) and a Bloch (b) domain wall as a function of the transverse coordinate, as reconstructed with the interferometric technique in [15]. The profiles correspond to the cases represented in the fourth row snapshots in Figs. 3(a) and 3(b), respectively.

terferometric reconstruction technique giving the complex field $A(x, y)$ is described in full detail in [15], and it is from these data that we obtain the average field $\langle A(x) \rangle_y$.

Depending on the cavity detuning value, two different types of DWs are found: For small (large) values of Δ , static Ising-type (moving Bloch-type) DWs are seen. The Ising or Bloch character of the DWs is clearly appreciated in Fig. 2: The Ising wall exhibits a discontinuous phase jump and the field intensity is nearly zero at the wall center [Fig. 2(a)], while in the Bloch wall the phase variation is smooth and the field intensity minimum is clearly different from zero [Fig. 2(b)]. But the most striking difference between Ising and Bloch walls is in their different dynamic behavior. In Fig. 3 interferometric snapshots of Ising [Fig. 3(a)] and Bloch [Figs. 3(b) and 3(c)] DWs are shown. The difference between Figs. 3(b) and 3(c) lies in the sign of the velocity, which is different due to the different chirality of the Bloch walls in the two figures. It can be appreciated that the wall velocity is not constant; i.e., there is some acceleration. This is due to the spatial inhomogeneity of the field along the x direction as the intensity gradient introduces spurious effects on the wall dynamics. Then, when evaluating the wall velocity (see below), we discard the part of the trajectory where the acceleration is more obvious.

Let us see now where (in terms of the cavity detuning Δ , the control parameter) and how the NIBT occurs. We proceeded as follows: Starting with one of the Ising walls, Δ is increased in small steps [20]. We observe that the Ising wall remains at rest until some critical detuning value is reached, call it Δ_{IB} , where the wall spontaneously starts to move [Figs. 3(b) and 3(c)], thus signaling a NIBT. As the illuminated region is finite, after some time the Bloch wall disappears through the illuminated border. Then, in order to follow the dynamics for $\Delta > \Delta_{IB}$, new DWs are injected for each increasing value of Δ . We check in this way that for $\Delta > \Delta_{IB}$ walls are always of the Bloch type. Next we proceeded to a reverse scan of the detuning: Starting with large positive Δ , we inject a DW and see that it is of Bloch type. Then we decrease Δ and repeat the operation until a value of Δ is reached, say Δ_{BI} , below which only Ising

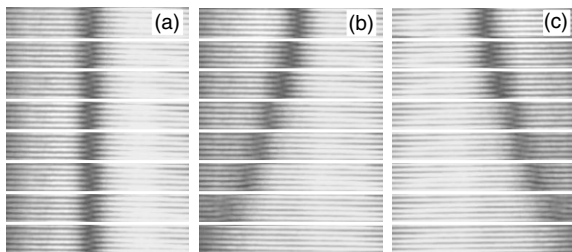


FIG. 3. Interferometric snapshots of Ising (a) and Bloch (b) and (c), domain walls. In (a) [(b) and (c)] the detuning is 0 MHz (25 MHz). The interferograms show sharp phase jumps in (a), and smooth ones in (b) and (c). Time runs from top to bottom in steps of 5 s, and the horizontal (transverse) dimension is 1.6 mm.

walls are observed. The interesting feature is that $\Delta_{BI} < \Delta_{IB}$; i.e., there is a domain of detuning values ($\Delta_{BI} < \Delta < \Delta_{IB}$) where both Ising and Bloch walls exist. In fact we have checked repeatedly that within this domain of hysteresis both Bloch and Ising walls are observed alternatively after subsequent wall injections for fixed Δ (in this detuning range the final state obviously depends on the initial condition, which seemingly we are not able to control). We repeated the procedure until we convinced ourselves of the reproducibility of the observation. Let us remark that the hysteresis cannot be attributed to mechanical hysteresis in the piezoelectric mirror that controls Δ , as subsequent observations of the two types of walls *for fixed* Δ rule out this possibility. Figure 4 summarizes these findings. The upper and lower arrows indicate the values of Δ where the NIBTs are observed (which we have called Δ_{BI} and Δ_{IB}), and the squares (crosses) denote increasing (decreasing) detuning scan. These results firmly establish the existence of the hysteresis cycle.

Although one could think that the observed hysteresis could be due to a subcritical character of the NIBT in our system, we argue that it is due to the existence of a hysteresis cycle in the homogeneous states of the system. In the first case, the homogeneous states connected by the DW, differing only in their phase, would have the same intensity irrespective of the wall type (Ising or Bloch). Contrarily, in the second case the homogeneous states should be different for Ising and Bloch walls. The latter is our case: The two equivalent homogeneous states (of opposite phases) connected by the Ising wall have larger intensities than those connected by the Bloch wall. This implies that there are two homogeneous states in the system that are bistable in a limited range of cavity detun-

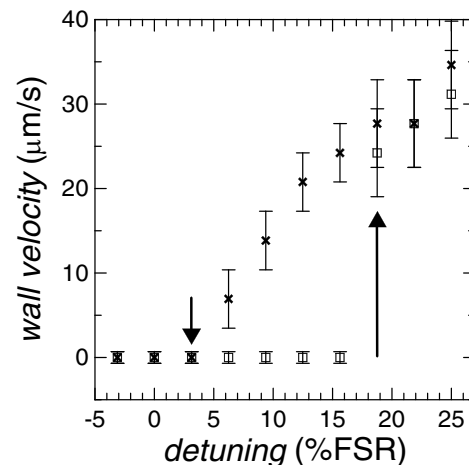


FIG. 4. DW velocity versus cavity detuning in units of the cavity free-spectral range (which is 120 MHz). Squares (crosses) indicate increasing (decreasing) detuning scan, the segments indicate the error bars, and the arrows mark the two nonequilibrium Ising-Bloch transitions. The hysteresis cycle is clearly apparent; see text for details.

ing values. We have measured the homogeneous state intensity by scanning Δ and have found evidence of the existence of a bistability loop. Very likely, the NIBT occurs somewhere in the middle of the unstable branch of the homogeneous state (the one connecting the higher and the smaller intensity stable states). This would explain why we never observe Bloch walls moving at small velocities.

Finally, in order to provide some theoretical support we consider the following phenomenological, adimensional model:

$$\begin{aligned} \partial_t E = & (1 - i\Delta)E + \gamma E^* + (1 + i\alpha)\partial_x^2 E \\ & - (1 + i\beta_3)|E|^2 E - i\beta_5|E|^4 E, \end{aligned} \quad (1)$$

where E is proportional to the complex intracavity field amplitude, γ accounts for parametric gain (the typical phase symmetry breaking term of degenerate four-wave mixing [21]), Δ is the detuning, α accounts for diffraction, and β_3 and β_5 account for nonlinear dispersion. When $\beta_5 = 0$, Eq. (1) is the complex Ginzburg-Landau equation investigated in [2]. This minimum correction to the model of [2], the term $-i\beta_5|E|^4 E$, mimics the refractive index saturation existing in photorefractive media. We have found that Eq. (1) displays an intensity-bistable homogeneous state in coincidence with a hysteretic NIBT (similar to Fig. 4) for a wide range of parameters (as an example, for $\gamma = 2$, $\alpha = 1$, $\beta_3 = -2.7$, and $\beta_5 = 1$, the hysteretic NIBT is observed; details will be given elsewhere). Of course, we do not claim that this constitutes a theoretical interpretation of our experimental results as Eq. (1) has not been derived from first principles. In any case, the fact that the modified Ginzburg-Landau Eq. (1) shows a hysteretic NIBT indicates that the phenomenon we are reporting could be found in other systems.

In conclusion, we have reported the first observation of the nonequilibrium Ising-Bloch transition in an optical system, a quasi-1D single longitudinal-mode photorefractive oscillator in a degenerate four-wave mixing configuration, using the cavity detuning as the control parameter. The NIBT we report is special in the sense that it is hysteretic; i.e., in terms of the control parameter, there is a domain of coexistence of both Ising and Bloch walls. The origin of this hysteretic cycle lies in the bistability exhibited by the homogeneous state of the system.

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