Atom Interferometric Detection of the Pairing Order Parameter in a Fermi Gas

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We propose two interferometric schemes to experimentally detect in real space the onset of pair condensation in a two-spin-component Fermi gas. Two atomic wave packets are coherently extracted from the gas at different positions and are mixed by a matter-wave beam splitter: we show that the spatial long-range order of the atomic pairs in the gas reflects in the atom counting statistics in the beam splitter output channels. The same long-range order is also shown to create a matter-wave grating in the overlapping region of the two extracted wave packets, grating that can be revealed by a light-scattering experiment.

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The experimental possibility of controlling at will the scattering length a between two spin components of fermionic atoms *via* a Feshbach resonance has opened the way to a comprehensive study of the pairing transition in a degenerate Fermi gas [1–3]. The weakly interacting limits are well understood theoretically: the phase transition is the Bose-Einstein condensation (BEC) of diatomic molecules ($a = 0^+$) or the BCS transition due to pairing in momentum space ($a = 0^-$). But one can now investigate experimentally the theoretically challenging crossover region, including the unitary limit $|a| = \infty$ [4].

While the techniques used for atomic BECs have allowed detection and characterization of a molecular BEC [2], a debate is still in progress about experimental signatures of pair condensation for $a < 0$. Most of the proposed signatures [5] do not directly address pair condensation. A recent idea is to detect pair condensation in momentum space by analysis of the noise in the atomic density pattern after ballistic expansion [6]. No proposal allows us to directly reveal the real-space long-range order. First experimental evidences of a condensation of fermionic pairs in the crossover regime have been recently presented [3], based on a fast ramping of the magnetic field to convert pairs on the $a < 0$ side into bound molecules on the $a > 0$ side, and on the observation of the Bose condensed fraction of the resulting gas of dimers. This method is expected to work only when the fermionic pairs are small enough, that is, in the vicinity of the Feshbach resonance, $k_F|a| > 1$ where k_F is the Fermi momentum.

In this Letter, we propose a general way of proving the condensation of pairs, by measuring the pairing order parameter directly in real space, not restricting to the small pair regime $k_F|a| > 1$. This proposal is the fermionic analog of the atom interferometric measurement of the first-order coherence function $G^{(1)}$ of a Bose gas [7]. More subtle schemes than the observation of fringes on the mean atomic density, however, have to be introduced as there is no long-range first-order coherence for fermions. Their experimental implementation would constitute a remarkable transposition of quantum optics techniques to a fermionic matter field.

In fermionic systems [4,8], the onset of pair condensation is defined by a nonzero long-distance limit $x_{AB} \equiv |\mathbf{x}_A - \mathbf{x}_B| \to +\infty$ of the pair coherence function

$$G^{(1)}_{\rm pair}(\mathbf{x}_A, \mathbf{x}_B) = \langle \hat{\Psi}^\dagger_\uparrow(\mathbf{x}_A)\hat{\Psi}^\dagger_\downarrow(\mathbf{x}_A)\hat{\Psi}_\downarrow(\mathbf{x}_B)\hat{\Psi}_\uparrow(\mathbf{x}_B)\rangle, \quad (1)$$

this function then factorizing in the product of the order parameter in $\mathbf{x}_B$ and of its complex conjugate in $\mathbf{x}_A$. We propose two methods to measure $G^{(1)}_{\rm pair}$ directly in real space, both relying on the coherent extraction of two atomic wave packets in $\mathbf{x}_{A,B}$ and their subsequent beating. The first method is based on a two-atom interferometric technique inspired by two-photon techniques [9]: it relies on atom counting in the two output channels of a matter-wave beam splitter, a procedure mastered experimentally for bosons [10]. The second method is based on the coherence properties of light elastically scattered off the matter-wave interference pattern of the two overlapping wave packets. Contrary to momentum space ones, our real-space techniques do not require measuring narrow lines scaling as the inverse of the system size, which, experimentally, may be a crucial advantage in the large system size limit.

Consider a gas of spin-$1/2$ fermionic atoms at thermal equilibrium in a trap. At the time $t = 0$, the trap potential is suddenly switched off and the atom-atom interactions brought to a negligible strength, so that the subsequent propagation is the one of a free atomic field [11]. Simultaneously, a suitable pulse of duration Δt of spin-independent optical potential is applied (Fig. 1) to the atoms situated in regions of size ℓ_u around the points $\mathbf{x}_A$ and $\mathbf{x}_B$ to impart to them a momentum kick of $\hbar(\mathbf{k}_0 \pm \mathbf{k}_1)$ by means of Bragg processes and to produce wave packets which are a coherent copy of the field in the trap, but for a shift in momentum space [10,12]. $\mathbf{k}_0$ is taken orthogonal to $\mathbf{x}_A - \mathbf{x}_B$, while $\mathbf{k}_1$ is parallel to it. The magnitude $\hbar k_1$ of the counter-propagating momentum kicks is taken to be larger than the momentum width Δp of the gas, which is on the order of the Fermi wave vector k_F in the resonance

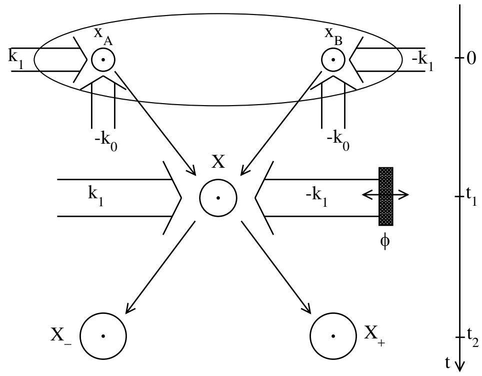

FIG. 1. First proposed setup: atoms are extracted by a Bragg process from the gas at points $\mathbf{x}_{A,B}$, using pairs of laser beams of wave vectors $-\mathbf{k}_0$, $\mathbf{k}_1$ and $-\mathbf{k}_0$, $-\mathbf{k}_1$ respectively; at their overlap position $\mathbf{X}$, the two atomic wave packets are coherently mixed by a laser standing wave acting as a 50-50 beam splitter with adjustable phase shift ϕ; the number of atoms in each wave packet is measured at the final positions $\mathbf{X}_{\pm}$.

region ($|a| = +\infty$) and in the weakly interacting BCS regime ($a < 0$), or on the order of $\hbar/a$ for a molecular condensate ($a > 0$). The extraction region size ℓ_u is taken to be much smaller than the distance x_{AB} between the extraction points. This latter is taken as macroscopic, that is, much larger than any other length scale of the problem, e.g., the Fermi distance $1/k_F$ and the Cooper-pair size $\ell_{\rm BCS}$.

In a Heisenberg picture the field operator at the end of the optical pulse can be related to the initial one by [13]

$$\hat{\Psi}_\sigma(\mathbf{x}, \Delta t) = u(\mathbf{x} - \mathbf{x}_A) e^{i(\mathbf{k}_0+\mathbf{k}_1)\cdot(\mathbf{x}-\mathbf{x}_A)} \hat{\Psi}_\sigma(\mathbf{x}) + u(\mathbf{x} - \mathbf{x}_B) e^{i(\mathbf{k}_0-\mathbf{k}_1)\cdot(\mathbf{x}-\mathbf{x}_B)} \hat{\Psi}_\sigma(\mathbf{x}) + \hat{\Psi}_\sigma^{\rm other}(\mathbf{x}). \tag{2}$$

The atoms which are left in their original momentum state as well as the ones having received a different momentum kick during the extraction process are included in the field $\hat{\Psi}_\sigma^{\rm other}$: as they spatially separate during the evolution, they will be omitted in the discussion [15]. This out-coupling scheme produces two atomic wave packets traveling with momentum $\hbar(\mathbf{k}_0 \pm \mathbf{k}_1)$ starting from respectively $\mathbf{x}_{A,B}$. At a time $t_1 = m x_{AB}/2\hbar|\mathbf{k}_1|$, they superimpose around $\mathbf{X}$. As mentioned in the introduction, the mean density profile in the overlap region does not show fringes so that more elaborate manipulations have to be performed in order to measure the pair coherence function $G^{(1)}_{\rm pair}(\mathbf{x}_A, \mathbf{x}_B)$.

Atom-number correlations.—At $t = t_1$, the two overlapping wave packets of momentum $\hbar(\mathbf{k}_0 \pm \mathbf{k}_1)$ can be coherently mixed by a spin-insensitive 50-50 matter-wave beam splitter, with reflection and transmission amplitudes of momentum-independent phase difference ϕ. Such a beam splitter may be realized [10,12] by applying a pulse of sinusoidal optical potential $U(\mathbf{x}, t) = 4\hbar\Omega(t) \times \sin^2(\mathbf{k}_1 \cdot \mathbf{x} + \phi/2)$ [14]. At a time t_2 after the splitting procedure, the two emerging wave packets of momentum $\hbar(\mathbf{k}_0 \pm \mathbf{k}_1)$ will be again spatially separated and centered at $\mathbf{X}_\pm = \mathbf{X} + \hbar(\mathbf{k}_0 \pm \mathbf{k}_1)(t_2 - t_1)/m$. The total field can be written as $\hat{\Psi}_\sigma(\mathbf{x}, t_2) = \hat{\Psi}_\sigma^+(\mathbf{x}, t_2) + \hat{\Psi}_\sigma^-(\mathbf{x}, t_2) + \hat{\Psi}^{\rm other}(\mathbf{x}, t_2)$ where the contribution of each packet is

$$\hat{\Psi}_\sigma^\pm(\boldsymbol{\xi} + \mathbf{X}_\pm, t_2) = \frac{e^{i(\mathbf{k}_0 \pm \mathbf{k}_1)\boldsymbol{\xi}} e^{i\theta}}{\sqrt{2}} \int d\boldsymbol{\xi}' \mathcal{R}(\boldsymbol{\xi}, \boldsymbol{\xi}'; t_2) u(\boldsymbol{\xi}') \times [\hat{\Psi}_\sigma(\mathbf{x}_{A,B} + \boldsymbol{\xi}') + i e^{\pm i\phi} \hat{\Psi}_\sigma(\mathbf{x}_{B,A} + \boldsymbol{\xi}')]. \tag{3}$$

$\mathcal{R}(\boldsymbol{\xi}, \boldsymbol{\xi}'; t)$ is the free-particle propagator and θ is an irrelevant propagation phase which depends on the details of the beam splitting procedure. The unitarity of $\mathcal{R}$ ensures that the results to come do not depend on t_2.

The operator $\hat{N}_\sigma^\pm$ giving the number of atoms with spin σ in the wave packet $\pm$ is obtained by integration of $\hat{\Psi}_\sigma^\dagger(\mathbf{x}, t_2)\hat{\Psi}_\sigma(\mathbf{x}, t_2)$ over the spatial extension of the packet $\pm$ at time t_2. The operator giving the atom-number difference between the two wave packets is then $\hat{D}_\sigma = \hat{N}_\sigma^+ - \hat{N}_\sigma^-$. Its expectation value $\langle \hat{D}_\sigma \rangle$ involves the first-order coherence function $\langle \hat{\Psi}_\sigma^\dagger(\mathbf{x}_A + \boldsymbol{\xi})\hat{\Psi}_\sigma(\mathbf{x}_B + \boldsymbol{\xi})\rangle$ of the initially trapped atoms, and therefore vanishes for a macroscopic distance $x_{AB} \gg \hbar/\Delta p$: we shall now take $\langle \hat{N}_\sigma^+ \rangle = \langle \hat{N}_\sigma^- \rangle = \bar{N}_\sigma$. Information on the pair coherence function $G^{(1)}_{\rm pair}$ is obtained from the correlation between the two spin components:

$$C_{\uparrow\downarrow} = \langle \hat{D}_\uparrow \hat{D}_\downarrow \rangle = \int d\boldsymbol{\xi} d\boldsymbol{\xi}' |u(\boldsymbol{\xi})|^2 |u(\boldsymbol{\xi}')|^2 [-e^{2i\phi} \langle \hat{\Psi}_\uparrow^\dagger(\mathbf{x}_A + \boldsymbol{\xi}) \hat{\Psi}_\downarrow^\dagger(\mathbf{x}_A + \boldsymbol{\xi}') \hat{\Psi}_\downarrow(\mathbf{x}_B + \boldsymbol{\xi}') \hat{\Psi}_\uparrow(\mathbf{x}_B + \boldsymbol{\xi}) \rangle + \langle \hat{\Psi}_\uparrow^\dagger(\mathbf{x}_A + \boldsymbol{\xi}) \hat{\Psi}_\downarrow^\dagger(\mathbf{x}_B + \boldsymbol{\xi}') \hat{\Psi}_\downarrow(\mathbf{x}_A + \boldsymbol{\xi}') \hat{\Psi}_\uparrow(\mathbf{x}_B + \boldsymbol{\xi}) \rangle + \text{H.c.}]. \tag{4}$$

$C_{\uparrow\downarrow}$ is obtained by counting the atoms [10,16] in each output channel, $N_\sigma^\pm$, then by forming the product $(N_\uparrow^+ - N_\uparrow^-)(N_\downarrow^+ - N_\downarrow^-)$ and finally averaging over many realizations of the whole experiment. From an experimental measurement of the ϕ dependence of $C_{\uparrow\downarrow}$, one can therefore determine whether the system has long-range order or not.

An explicit calculation of $C_{\uparrow\downarrow}$ as a function of the energy gap Δ can be performed with the zero temperature BCS theory [8] in the local density approximation. In the large x_{AB} limit, one sees from the Wick theorem that only the ϕ dependent part of Eq. (4) has a nonzero value: $C_{\uparrow\downarrow} = C_{\uparrow\downarrow}^{(0)} \cos(2\phi)$. For a wide extraction region $\ell_u \gg \ell_{\rm BCS}$, the anomalous correlation function $\langle \hat{\Psi}_\downarrow \hat{\Psi}_\uparrow \rangle$ has a width $|\boldsymbol{\xi} - \boldsymbol{\xi}'| \sim \ell_{\rm BCS}$ much smaller than the one of u so that one may set $u(\boldsymbol{\xi}') \simeq u(\boldsymbol{\xi})$. For equal mean densities in $\mathbf{x}_{A,B}$, the anomalous correlation functions around $\mathbf{x}_{A,B}$ are the same and one is left with

$$\int d\boldsymbol{\xi}' |\langle \hat{\Psi}_\downarrow(\mathbf{x}_B + \boldsymbol{\xi}') \hat{\Psi}_\uparrow(\mathbf{x}_B + \boldsymbol{\xi}) \rangle|^2 = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\Delta^2}{4E_k^2} \tag{5}$$

where $E_k = [\Delta^2 + (\hbar^2 k^2/(2m) - \mu)^2]^{1/2}$ is the BCS spectrum and μ is the chemical potential. Taking for the extraction function $u(\boldsymbol{\xi}) = u_0 e^{-\xi^2/2\ell_u^2}$, with $|u_0| \leq 1$, we get in the BCS regime $\mu \simeq E_F$ and $\Delta \ll \mu$:

$$C_{\uparrow\downarrow}^{(0)} = -\frac{3\pi}{8\sqrt{2}}|u_0|^2 \frac{\Delta}{E_F}\bar{N}_\sigma, \quad (6)$$

while for a molecular condensate $C_{\uparrow\downarrow}^{(0)} = -|u_0|^2 \bar{N}_\sigma/\sqrt{2}$. We estimate the statistical noise in the measurement of $C_{\uparrow\downarrow}$ by the standard deviation of $\hat{D}_\sigma$ in the BCS state: to leading order in u_0, $\langle \hat{D}_\sigma^2 \rangle \simeq 2\bar{N}_\sigma$, which shows that the shot noise in the initial extraction process is the dominant source of noise. The number of realizations over which to average therefore scales as $(\bar{N}_\sigma/C_{\uparrow\downarrow}^{(0)})^2$, ~ 1 in the BEC limit and $\sim (E_F/\Delta)^2$ in the BCS limit.

Light scattering off the matter-wave grating.—Information on the pairing coherence function $G_{\rm pair}^{(1)}$ of the trapped gas can also be obtained by means of light scattering off the matter-wave interference pattern formed by the overlapping wave packets at $t = t_1$ around point $\mathbf{X}$, which is taken in what follows as the origin of the coordinates. As already mentioned, the mean density does not show fringes. On the other hand, fringes appear in the opposite spin density-density correlation function $\mathcal{G}_{\uparrow\downarrow}^{(2)}(\mathbf{x}, \mathbf{x}') = \langle \hat{\Psi}_\uparrow^\dagger(\mathbf{x}, t_1) \times \hat{\Psi}_\downarrow^\dagger(\mathbf{x}', t_1)\hat{\Psi}_\downarrow(\mathbf{x}', t_1)\hat{\Psi}_\uparrow(\mathbf{x}, t_1)\rangle$. As a guideline, one performs an explicit calculation for BCS theory: one finds that, as soon as a condensate of pairs is present, fringes appear as a function of the center of mass coordinates $(\mathbf{x} + \mathbf{x}')/2$ of a pair, with a sinusoidal oscillation of wave vector $4\mathbf{k}_1$. Their amplitude is proportional to the product of the in-trap anomalous averages in $\mathbf{x}_A$ and $\mathbf{x}_B$ and extends up to relative distances $|\mathbf{x} - \mathbf{x}'|$ of the order of the Cooper-pair size $\ell_{\rm BCS}$. This matter-wave grating is not easily detected in position space since its spatial period is smaller than the mean interatomic distance. We therefore switch to Fourier space:

$$\tilde{\mathcal{G}}_{\uparrow\downarrow}^{(2)}(\mathbf{q}, \mathbf{q}') \equiv \int d\mathbf{x} d\mathbf{x}' e^{-i(\mathbf{q}\cdot\mathbf{x} + \mathbf{q}'\cdot\mathbf{x}')} \mathcal{G}_{\uparrow\downarrow}^{(2)}(\mathbf{x}, \mathbf{x}'). \quad (7)$$

Since fringes show up on the center of mass coordinate with a wave vector $4\mathbf{k}_1$ we limit ourselves to the region $\mathbf{q}' = \mathbf{q} \simeq -2\mathbf{k}_1$. Taking into account the free expansion during t_1, one then obtains

$$\tilde{\mathcal{G}}_{\uparrow\downarrow}^{(2)}(\mathbf{q}, \mathbf{q}) = e^{i\hbar\Delta q^2 t_1/m} \int d\boldsymbol{\xi} d\boldsymbol{\xi}' e^{-i(\mathbf{q}+2\mathbf{k}_1)\cdot(\boldsymbol{\xi}+\boldsymbol{\xi}')} u^*(\boldsymbol{\xi}) u^*(\boldsymbol{\xi}') u(\boldsymbol{\xi} - \mathbf{b}) u(\boldsymbol{\xi}' - \mathbf{b})$$
$$\times \langle \hat{\Psi}_\uparrow^\dagger(\mathbf{x}_A + \boldsymbol{\xi}) \hat{\Psi}_\downarrow^\dagger(\mathbf{x}_A + \boldsymbol{\xi}') \hat{\Psi}_\downarrow(\mathbf{x}_B + \boldsymbol{\xi}' - \mathbf{b}) \hat{\Psi}_\uparrow(\mathbf{x}_B + \boldsymbol{\xi} - \mathbf{b}) \rangle \quad (8)$$

where $\mathbf{b} = \hbar(\mathbf{q} + 2\mathbf{k}_1)t_1/m$ and $\Delta q = |\mathbf{q} + 2\mathbf{k}_1|$. For $\mathbf{q} = -2\mathbf{k}_1$ one recovers the factor in front of $e^{2i\phi}$ in Eq. (4).

This Fourier component of $\mathcal{G}_{\uparrow\downarrow}^{(2)}$ is detectable by means of the elastic light scattering from the atomic cloud. The incoming laser intensity has to be weak enough to avoid saturation of the atomic transition. Optical pumping processes have to be negligible during the whole measurement time: the mean number of scattered photons per atom has to be much less than one not to wash out the information on the internal atomic state. The imaging sequence is assumed to take place in a short time so that the positions of the atoms can be safely considered as fixed. For each realization of the whole experiment, a different distribution of the atomic positions is obtained, and consequently a different angular pattern for the elastic scattering. Information on the density-density correlation function will be obtained by taking the average over many different realizations.

We consider here the simple case when the laser is close to resonance with the transition from a $F_g = 1/2$ ground state to a $F_e = 1/2$ excited state. In this case, $\sigma_\pm$ polarized light interacts only with atoms in the $\downarrow, \uparrow$ spin state, respectively. The geometry adapted to get information on the condensation of pairs is shown in Fig. 2: a pair of mutually coherent laser beams with a common intensity $I_{\rm inc}$ is sent on the atomic cloud with opposite circular polarizations $\sigma_\pm$ and opposite wave vectors $\pm\mathbf{k}_1$. We look at the mutual coherence of two back-scattered beams in opposite directions $\pm\mathbf{k}_{\rm sc}$ [17], with opposite circular polarizations. Within the Born approximation (valid if the cloud is optically dilute and optically thin), the amplitudes in Fourier space of the scattered light on the circular polarizations $\sigma_\pm$ are related to the ones of the incoming field by $E_\pm^{\rm sc}(\pm\mathbf{k}_{\rm sc}) = A\hat{\rho}_{\downarrow,\uparrow}(\pm\mathbf{q}, t_1)E_\pm^{\rm inc}(\pm\mathbf{k}_1)$, where A is a factor depending on the dipole moment of the transition and on the atom-laser detuning, and $\hat{\rho}_\sigma(\mathbf{q}, t_1)$ is the Fourier component at the transferred wave vector $\mathbf{q} = \mathbf{k}_{\rm sc} - \mathbf{k}_1$ of the density operator $\hat{\Psi}_\sigma^\dagger(\mathbf{x}, t_1)\hat{\Psi}_\sigma(\mathbf{x}, t_1)$ at time t_1. This mutual coherence is quantified by the correlation function

$$I_{-+}/I_{\rm inc} = \langle [E_-^{\rm sc}(-\mathbf{k}_{\rm sc})]^\dagger E_+^{\rm sc}(\mathbf{k}_{\rm sc})\rangle/I_{\rm inc}$$
$$= |A|^2 \langle [\hat{\rho}_\uparrow(-\mathbf{q}, t_1)]^\dagger \hat{\rho}_\downarrow(\mathbf{q}, t_1)\rangle = |A|^2 \tilde{\mathcal{G}}_{\uparrow\downarrow}^{(2)}(\mathbf{q}, \mathbf{q}). \quad (9)$$

By using (8), one indeed sees that the correlation function I_{-+} can reveal the pair long-range order. In the $T = 0$ BCS theory, it has a simple expression for $\mathbf{q} \simeq -2\mathbf{k}_1$:

$$I_{-+} = -\frac{|A|^2}{2} C_{\uparrow\downarrow}^{(0)} e^{-\ell_I^2 \Delta q^2/2} I_{\rm inc}. \quad (10)$$

As a function of Δq, it shows a narrow peak of height proportional to the signal $C_{\uparrow\downarrow}$ of the first proposed setup and width $1/\ell_I$ such that $\ell_I^2 = \ell_u^2 + (\hbar t_1/m\ell_u)^2$. Experimentally, this can be determined by beating the two scattered beams: as a function of the mixing phase $\phi_{\rm mix}$, the resulting intensity presents oscillations of amplitude $2|I_{-+}|$ on a background $\simeq 2|A|^2 \bar{N}_\sigma I_{\rm inc}$.

We now discuss the feasibility of the proposed schemes. In present experiments $\sim 10^5$ ^{6}Li atoms can be trapped with

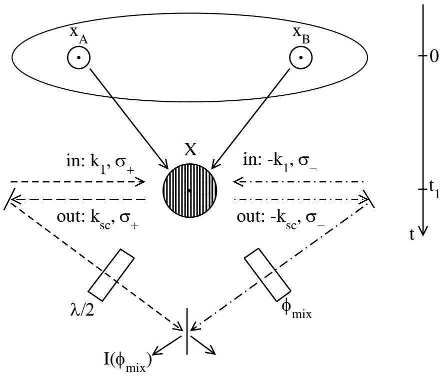

FIG. 2. Second proposed setup: atoms are extracted from the cloud at $\mathbf{x}_{A,B}$ and create a matter-wave grating when the wave packets overlap at $\mathbf{X}$; one detects this grating by shining a pair of counter-propagating, $\sigma_\pm$ polarized laser beams on the overlap region and by beating the two resulting back-scattered light beams on a beam splitter. As function of the mixing phase ϕ_{mix}, the beating intensity $I(\phi_{\text{mix}})$ averaged over many realizations presents fringes revealing the pair long-range order. Dashed (dot dashed) lines: σ_+ (σ_-) polarization.

oscillation frequencies 20 Hz axially and 250 Hz radially, leading to cloud radii of 20 and 250 μm and to $k_F/k_1 = 0.3 \ll 1$. Using Bragg beams with a waist of 14 μm leads to $\ell_u = 10\ \mu\text{m} \ll 250\ \mu\text{m}$ and $\bar{N}_\sigma \leq 1500$. Taking $k_F|a| = 0.6$ leads, in the BCS theory, to $\Delta/E_F = 0.1$ and $\ell_{\text{BCS}} = 7.8\ \mu\text{m} < \ell_u$. For these parameters, the first scheme is therefore realistic and requires a few hundreds of realizations. The second one is made difficult by the low photon number (<0.05 per realization) scattered in the cone $\Delta q \ell_I < 1$, which would require averaging over a large number of realizations; this can be overcome by using 10^7 atoms as done at MIT [2] or by spatially compressing the cloud in $\mathbf{X}$ radially with respect to $\mathbf{k}_1$ to increase the cone aperture: the second scheme then becomes as efficient as the first one.

In conclusion, we have proposed two ways of detecting a long-range pairing order in a degenerate Fermi gas by measuring the coherence function of the pairs directly in real space via matter-wave interferometric techniques. The proposed scheme is an application of quantum optics techniques to Fermi fields, a line of research expected to open new possibilities in the experimental manipulation and characterization of fermionic systems.

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[14] Scattering into higher, nonresonant, momentum states is suppressed for an optical potential pulse duration $\tau \gg \omega_R^{-1}$. The transition amplitudes from one momentum component to the other are constant over the initial momentum spread of the gas if $\tau^{-1} \gg \hbar k_1 \Delta p/m$. The two conditions are compatible as $\Delta p \ll \hbar k_1$.
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