Quantum Electrodynamics in Strong Electric Fields: The Ground-State Lamb Shift in Hydrogenlike Uranium

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X-ray spectra following radiative recombination of free electrons with bare uranium ions (U^{92+}) were measured at the electron cooler of the ESR storage ring. The most intense lines observed in the spectra can be attributed to the characteristic Lyman ground-state transitions and to the recombination of free electrons into the K shell of the ions. Our experiment was carried out by utilizing the deceleration technique which leads to a considerable reduction of the uncertainties associated with Doppler corrections. This, in combination with the 0° observation geometry, allowed us to determine the ground-state Lamb shift in hydrogenlike uranium (U^{91+}) from the observed x-ray lines with an accuracy of 1%. The present result is about 3 times more precise than the most accurate value available up to now and provides the most stringent test of bound-state quantum electrodynamics for one-electron systems in the strong-field regime.

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Quantum electrodynamics (QED), the basis of all present field theories, is still the best confirmed theory in physics. Despite the enormous success of QED in predicting the properties of electrons in weak fields [1–3], a precise test in the strong-field limit where novel phenomena might show up is still pending. Thus, a primary goal of atomic-structure studies at high Z is to explore the behavior of electrons in the strongest electromagnetic fields accessible to experimental investigation. Precision measurements of electron binding energies are best suited to deduce characteristic QED phenomena in intense fields. Therefore, the comparison of predicted with experimentally determined level energies of strongly bound electrons provides a critical test of QED in strong fields [4–6]. In this respect one-electron high-Z ions are of particular interest since they represent the most fundamental atomic systems where one has a unique possibility to study the behavior of a single strongly bound electron. This allows for an unambiguous test of bound-state QED in the nonperturbative regime of extremely strong Coulomb fields in the absence of many-electron effects. Besides, in combination with accurate measurements for high-Z Li-like systems, precise results for the 1s binding energy in H-like ions may help to disentangle between nuclear, one- and multielectron contributions to the 2s binding energy in Li-like species where very accurate experimental results have been obtained already [7]. For the case of hyperfine structure studies at high-Z (test of bound-state QED in extreme magnetic fields), such a combined investigation for the ground states in the H- and the Li-like ions has already been proposed theoretically [8]. Until recently the largest sources of theoretical uncertainties in predicting the binding energies for high-Z one-electron systems were unevaluated higher-order QED contributions [9,10]. During the last few years significant progress took place, resulting in QED calculations for high-Z hydrogenlike ions which do now comprise all second order (in α) corrections [11,12]. However, one has to note that there are still discrepancies between the results obtained by different groups [11–13].

The ESR storage ring with its brilliant beams of cooled heavy ions has proven to provide unique conditions for this kind of precision investigations. In a series of experiments carried out at the gas-jet target [14] and at the electron cooler [15], accurate values for the ground-state Lamb shift in heavy hydrogenlike ions were obtained. For such systems, the most direct experimental approach for the investigation of the effects of quantum electrodynamics in strong Coulomb fields is a precise determination of x-ray energies for transitions into the ground state of the ion. In particular, the Lyman-α transitions are used as they appear most intense and well resolved in the x-ray spectra. In these experiments the Lamb-shift value is deduced from the measured transition energy by comparison with the Dirac energy eigenvalue for the 1s ground state of a pointlike nucleus and the additional assumption that the binding energies of the excited states involved are known to high accuracy. Although the ESR provides brilliant, monochromatic beams, the main challenge encountered is still caused by the uncertainties introduced from the Doppler shift corrections. In order to deduce the transition energy for the rest frame of the ion (emitter frame), the value obtained in the laboratory system must be corrected for the relativistic Doppler shift given by $E = E_{\mathrm{lab}}\gamma(1 - \beta\cos\theta_{\mathrm{lab}})$. Here, E and E_{lab} are the x-ray energies in the emitter and in the laboratory frame, respectively, $\theta_{\rm lab}$ denotes the laboratory observation angle, and γ is the relativistic Lorentz factor. This correction is usually large, because the ions are accelerated up to very high velocities ($\beta \approx 0.7$), needed for an efficient production of the bare charge state via stripping. Besides statistics, the uncertainties introduced by errors in $\theta_{\rm lab}$ and β limit the final accuracy of the x-ray energy in the emitter frame. A significant reduction of the error introduced by an uncertainty in the laboratory observation angle can be achieved by exploiting the 0° geometry as it was done in one of the first $1s$-Lamb-shift experiments conducted at the ESR [15]. Here, in general, both the error due to the uncertainty in the observation angle $\Delta\theta$ as well as the Doppler broadening are negligible. However, in this case the error due to $\Delta\beta$ is largest. Nevertheless, the latter can also be considerably reduced by exploiting a deceleration of the ion beam as it was applied in the most recent Lamb-shift measurement reported for H-like uranium [14]. We have performed an experiment at the ESR electron cooler, utilizing for the first time a combination of these two techniques; this allowed us to obtain a value for the ground-state Lamb shift in hydrogenlike uranium with an accuracy of 4.6 eV which is about 3 times more precise than the most accurate value available up to now.

At 360 MeV/u, close to 10^8 bare uranium ions were injected into the ESR and subsequently decelerated to an energy of 43.59 MeV/u. Directly after the injection from the heavy-ion synchrotron (SIS) (before the deceleration) the ions were first cooled at the high energy, then electron cooling was switched off, the coasting beam was bunched, and the deceleration mode was applied. At the low energy used for data taking, the electron cooling was switched on again guaranteeing for a well defined constant beam velocity, generally of the order of $\Delta\beta/\beta \approx 10^{-4}$. The cooler current and voltage applied after deceleration were 100 mA and 23 kV, respectively. For the decelerated ions, the accumulated ion current in the ESR was about 550–600 μA. In our experiment, projectile photon emission is entirely produced via radiative recombination transitions of the free cooler electrons and subsequent cascades. Inside the cooler, these electrons are copropagating with the ions at exactly the same mean velocity (relative velocity $\langle v_{\rm REL}\rangle = 0$), and their relative momentum distribution is characterized by the corresponding longitudinal and transversal temperatures of $T_{\parallel} \approx 10^{-3}$ eV and $T_{\perp} \approx 0.1$ eV, respectively. The x rays emitted out of the cooler region were detected by three independent strips of a segmented germanium detector. The active area and the thickness of each strip were 390 mm^2 and 12 mm, respectively. The energy resolution was about 700 eV at an x-ray energy of 150 keV for all strips. The detector was mounted 4.1 m downstream of the midpoint of the 2.5 m long straight cooling section and could be moved vertically by means of a stepping motor. During the measurement it was placed close to the ion beam, at a distance of 1 ± 0.1 cm, allowing for an almost 0° observation angle for all strips of the segmented detector. The x rays were recorded in coincidence with down-charged uranium ions, as produced by the capture of one electron in the cooler. The down-charged ions were registered in a gas-filled multiwire proportional counter (MWPC) which was installed in a pocket behind the first dipole magnet downstream from the electron cooler.

In Fig. 1 we display a calibrated x-ray spectrum as observed for initially bare uranium ions at an energy of 43.59 MeV/u (laboratory frame). The spectrum is almost background free, partially because of the coincidence technique applied and also due to the deceleration mode which results in a much lower electron current ($\approx$100 mA) and a low cooler voltage ($\approx$23 kV). Compared to high beam energies, the latter reduced substantially the bremsstrahlung intensity caused by the cooler electrons. The most intense lines observed in the spectrum are the characteristic Lyman-α ground-state transitions (Ly-α_2: $2p_{1/2} \rightarrow 1s$; **M1**: $2s_{1/2} \rightarrow 1s$; Ly-α_1: $2p_{3/2} \rightarrow 1s$). Because of the intrinsic resolution of the detectors used, the **M1** line blend of the Ly-α_2 decay cannot be resolved experimentally since the $2s_{1/2}$–$2p_{1/2}$ line spacing amounts to 70 eV (classical Lamb shift). Because of this ambiguity we did not consider the Ly-α_2 transition in the determination of the $1s$ Lamb-shift value. At the Lyman series limit, a prominent x-ray line (K-RR) is visible which is produced by direct recombination transitions into the $1s$ ground state of H-like uranium. Note, at the "cooling conditions," relevant for our current x-ray study, the K-RR line centroid in the emitter frame corresponds exactly to the $1s$ ground-state binding energy of H-like uranium, a unique feature of this experimental scenario. The distinctive tails in the low-energy side of the Lyman transitions which are observed in the x-ray spectrum (see Fig. 1) can be explained by the population characteristics of the radiative recombination process which is known to populate at low relative veloci-

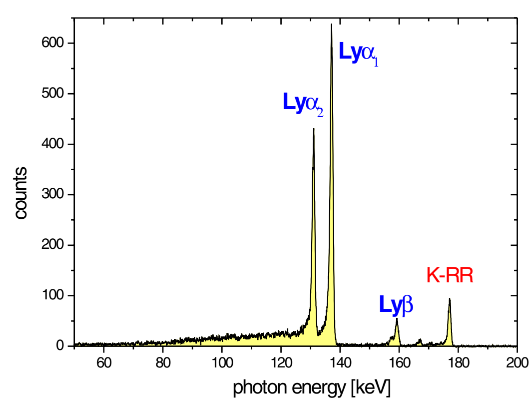

FIG. 1 (color online). X-ray spectrum (laboratory frame) following the radiative recombination of electrons with bare uranium ions.

ties predominantly high-n, l states [16]. The cascades following recombination into highly-excited levels may result in a delayed Lyman emission, which then takes place within the 3 m long distance between the end of the electron cooler and the Ge(i) detector. As a consequence, such events appear considerably Doppler shifted towards lower energies [15]. One should note that the tails of the Lyman-α transition lines, caused by cascade feeding of the L-shell levels, are not present in the case of the K-RR line. Moreover, due to the low β value of 0.29565 and the experimental time resolution of about 20 ns, the photon events which occurred inside the cooler section can be distinguished from the events where the emission took place just in front of the x-ray detector. The events which are associated with the delayed Lyman emission have different coincidence time as compared to the ones stemming from the prompt emission inside the cooler. As a consequence, application of a proper time condition for the x-ray spectrum analysis excludes the cascade contributions leading to the low-energy tails [17] (see Fig. 2).

The intense Ly-α_1 transitions as well as the K-RR line can be exploited to obtain an accurate value for the ground-state Lamb shift [14,15]. In order to accurately deduce the x-ray line centroid energies we took advantage of the fact that for solid state detectors small energy differences between two closely spaced lines can be determined with high accuracy [18]. For this purpose the particular ion beam energy of 43.59 MeV/u was chosen. Via the Doppler effect this beam energy allows us to park the Ly-α_1 and the K-RR line close to the 130.523 and 177.213 keV γ lines of ^{169}Yb which were used for calibration. The γ energy values are those as given in [19] and corrected by new results for the energy-wavelength conversion factor and for the silicon lattice spacing [20]. During the experiment the calibration source was regularly placed in front of the detector in order to gain control over possible drifts. The data acquired were divided into individual groups and were analyzed separately. In all cases the x-ray line centroid energies (Ly-α_1 and K-RR) were determined relative to the calibration lines by a least square fit of a function comprising a Gaussian with a shelf on the

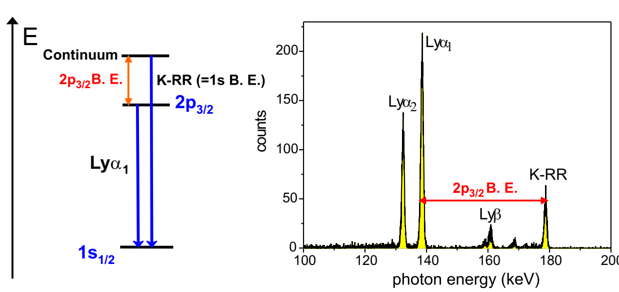

FIG. 2 (color online). Left side: schematic presentation of the Ly-α_1 ($2p_{3/2} \rightarrow 1s_{1/2}$) and K-RR transitions in a hydrogenlike ion. Right side: x-ray spectrum accumulated with a condition on the coincidence time (see above).

low-energy side [18]. In the following step, the results deduced from each of the individual data sets were compared and checked for consistency. As an example, in Table I we present the values obtained for the different data sets (for the strip number 1) together with the error-weighted mean value.

Afterwards, combining the results from the different data sets, the final numbers (for each of the strips) were obtained. They are given in the third and fourth columns of Table II.

In order to obtain a value for the 1s Lamb shift, the energies of the Ly-α_1 and K-RR lines were transformed into the emitter frame. According to the transformation formula (see above), the uncertainties for the observation angle $\theta_{\rm lab}$ and β contribute to the overall error of the x-ray centroid energies for the emitter frame. The observation angles for strip 1, strip 2, and strip 3 were measured to be 0.35°, 0.53°, and 0.71°, with an uncertainty of $\Delta\theta = 0.02°$. This translates into a relative uncertainty $\Delta E/E = 10^{-6}$ and can therefore be neglected in the following. The value for the absolute beam velocity β is determined by the acceleration voltage of the electron cooler. It is given by the relation $(\gamma - 1)mc^2 = eU_e$, where e and mc^2 are the charge and the rest mass of the electron, respectively. U_e is an effective electron acceleration voltage represented by the following formula [21] $U_e = U \times 1.0011 - 375 I_c[A]$. The first term is the voltmeter reading corrected for a calibration of the cooler voltage and the second term represents the space charge correction. The latter was determined by a measurement of the Schottky revolution frequency of the circulating beam as function of the cooler current I_c. In our experiment, $U = 23924$ V and $I_c = 100$ mA, which gives an effective cooler voltage U_e of 23913 V. From this, a β value of $\beta = 0.29565$ follows with an uncertainty of $5.8 \times 10^{-6} \Delta U_e$, where ΔU_e refers to the accuracy achieved in the calibration of the cooler voltage. The high-voltage generator has been precisely calibrated for the voltage range corresponding to the low beam energies with an accuracy of $\Delta U = 5$ V (at $U = 23.913$ kV) [21], which results in $\Delta\beta = 2.9 \times 10^{-5}$. This implies an uncertainty for the Doppler correction of $\Delta E/E = 3.16 \times 10^{-5}$. From the above values of β and $\theta_{\rm lab}$, the Doppler correction factors for the three strips can be deduced to be 0.737 309(23) for the first, 0.737 317(23) for the second,

TABLE I. Laboratory x-ray energies derived from the different data sets for strip number 1; Ly-$\alpha_{1,\rm lab}$: the Ly-α_1 centroid energy, K-$RR_{\rm lab}$: the K-RR centroid energy, mean: the error-weighted mean of the three values. (All results are in eV.)

Data set	Ly-$\alpha_{1,\rm lab}$	K-$RR_{\rm lab}$
1	138 577.0 ± 10.6	178 800.3 ± 30.9
2	138 587.7 ± 9.7	178 743.5 ± 21.8
3	138 562.2 ± 9.8	178 791.5 ± 19.4
Mean	138 575.7 ± 5.8	178 775.7 ± 13.1

TABLE II. Experimental results obtained. For details, compare text.

Strip	$Ly\alpha_{1,lab}$	$K\text{-}RR_{lab}$	$Ly\alpha_{1,EM} + 2p_{3/2}$B.E.	$K\text{-}RR_{EM}$
1	138 575.7 ± 5.8	178 775.7 ± 13.1	131 814.1 ± 4.3	131 812.96 ± 9.7
2	138 584.7 ± 5.0	178 787.8 ± 10.3	131 821.9 ± 3.7	131 823.26 ± 7.6
3	138 582.6 ± 8.4	178 819.6 ± 16.3	131 821.7 ± 6.2	131 848.5 ± 12.1

and 0.737 327(23) for the third strip. From these numbers it is evident that for the x-ray transitions considered, Doppler broadening amounts to about 1 eV and is therefore negligible. In the case of the *K-RR* lines, the Doppler transformation into the emitter frame yields directly the $1s$ binding energy. For the Ly-α_1 line, however, we assume that the $2p_{3/2}$ binding energy of -29640.99 eV [22] is known exactly from theory which also includes a Lamb-shift correction of 8.8 eV. The $2p_{3/2}$ binding energy must be added to the experimental Ly-α_1 transition energy in order to compare with the Dirac eigenvalue for the $1s$ state of $-132\,279.96$ eV. In total, we yield six independent values for the $1s$ binding energy as it is given in Table II, three from the Ly-α_1 transitions ($Ly\alpha_{1,EM} + 2p_{3/2}$B.E.) and three from the *K-RR* lines ($K\text{-}RR_{lab}$). Note, the uncertainties in the table refer only to the counting statistic achieved. Finally, taking a weighted average of these results we obtain a value of $460.2 \pm 2.3 \pm 3.5$ eV for the ground-state Lamb shift in H-like uranium. The uncertainty of 2.3 eV is entirely statistical whereas the one of 3.5 eV stems from the imprecision of the beam velocity determination (see above). The error resulting from an uncertainty in the observation angle introduces a negligible contribution of about 0.1 eV. In addition, we estimate an uncertainty of 2 eV to account for possible systematic errors introduced by the line-shape analysis. Adding quadratically these various contributions results in an uncertainty of 4.6 eV for the experimental $1s$ Lamb-shift value. Here, we like to emphasize that in this measurement, contributions from systematic errors to the total uncertainty are considerably reduced as compared to the previous studies of this kind [14,15]. Our result is consistent with the values from these former experiments and is almost 3 times more precise than the most accurate result reported up to now [14].

In Table. III we compare our experimental result for the $1s$ Lamb shift with the newest theoretical value [11,12]. In order to emphasize the achieved experimental precision, several individual contributions to the total theoretical Lamb shift including the recently calculated 2nd order QED corrections are listed separately as well. The comparison shows that our result is sensitive to the QED contributions of the first order (in α) at the 2% level and thus represents the most precise test of the bound-state QED in high-Z one-electron systems.

TABLE III. The ground-state Lamb shift in eV for H-like uranium.

Finite nuclear size	198.81
1st order QED	266.45
2nd order QED	−1.26(33)
Total theory [11,12]	464.26 ± 0.5
This work	460.2 ± 4.6

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