Pixel Entanglement: Experimental Realization of Optically Entangled $d = 3$ and $d = 6$ Qudits

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We demonstrate a simple experimental method for creating entangled qudits. Using transverse-momentum and position entanglement of photons emitted in spontaneous parametric down-conversion, we show entanglement between discrete regions of space, i.e., pixels. We map each photon onto as many as six pixels, where each pixel represents one level of our qudit state. The method is easily generalizable to create even higher dimensional, entangled states. Thus, the realization of quantum information processing in arbitrarily high dimensions is possible, allowing for greatly increased information capacity.

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Qubits, quantum mechanical two-level states, are the analog of classical bits. Just as most classical information processing relies on bits, qubits form the foundation of nearly all protocols in quantum information. Extending beyond two-level superposition states to d-dimensional qudit states has distinct advantages. By allowing each particle to carry d possible states instead of the usual two, the information flux increases. Recently, several quantum cryptographic protocols for qutrits and higher dimensional states were shown to increase security against eavesdropping attacks [1–3]. Furthermore, since entangled states [4] play a key role in many applications of quantum information including *quantum key distribution* [5], the secure sharing of information, *dense coding* [6], the encoding two classical bits onto a single photon of a biphoton pair, and *teleporation* [7], the remote reconstruction of a quantum state, methods for creating entangled qudits are of particular interest.

Traditionally, spontaneous parametric down-conversion (SPDC) has offered a straightforward method for creating entangled photon pairs. To date, much of the work with optically entangled pairs concentrates on the polarization entanglement of photons created in SPDC [8]. In general, though, type-II down-converted photon pairs are entangled in continuous variables as well, such as in transverse momentum and position [9–14], which has proved useful in quantum imaging [15–17]. It was recently shown that position-momentum entangled photons created in SPDC violate separability criteria by 2 orders of magnitude [18].

In this work, we perform a proof-of-principle experiment that creates entangled qudits from the entangled transverse-momentum and position spaces of SPDC. The positions and transverse momenta of the entangled pairs are imaged into discrete regions of space forming our qudits. Thus, the system displays *pixel entanglement.* Alternative methods have been demonstrated for creating entangled qudits, but they suffer from complications when trying to generalize them to higher dimensions. For instance, angular momentum properties of light have been shown to create higher dimensional states [19,20], but the diffraction efficiencies of computer-generated holograms limit the number of levels possible [21]. In addition, states with 11 levels have been realized using time binning [22]. This technique's reliance on time entanglement makes it challenging to implement in quantum information protocols. Further methods involving multiport beam splitters have been proposed, but to the best of our knowledge they have not been experimentally realized [23]. Also multipair polarization-entangled photons were demonstrated to obey spin-1 statistics [24], but these states are produced with less efficiency than the biphotons considered in our method. Additionally, our method is experimentally simple to realize and to generalize to high dimensions.

We used type-II phase-matching conditions for degenerate, nearly collinear SPDC. Because of momentum conservation, the two down-converted photons, A and B (see Fig. 1), are anticorrelated in transverse momentum $\mathbf{q}_{A,B}$. In the limit of an infinite plane-wave pump field, the emitted photons are in a maximally entangled EPR output state, with delta function correlations in transverse momentum [16]. Thus, although each photon is emitted with a range of transverse momenta, measuring the momentum of one

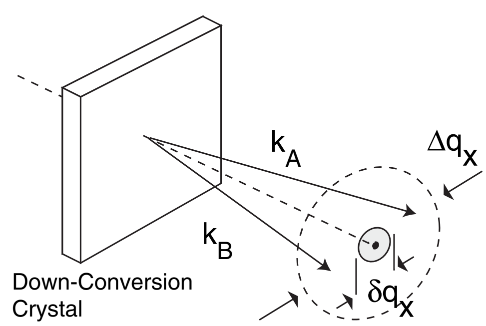

FIG. 1. Visualization of pixels in the down-conversion process for the case of transverse-momentum entanglement. The photons are emitted with anticorrelated transverse momenta $\mathbf{k}_{A_\perp}, \mathbf{k}_{B_\perp}$ within a range Δq_x. The sum of the two momenta are uncertain over a range δq_x which is determined by the angular spread of the pump beam.

uniquely determines the momentum of the other. In practice, the pump beam is not an infinite plane wave, but rather a Gaussian beam with a finite waist. In this case, the correlations are no longer perfect. Instead, based on the approach in [11], the two-photon term of the output state is a superposition of Fock states $|1_{\mathbf{q}_A}\rangle|1_{\mathbf{q}_B}\rangle$ given by

$$|\psi\rangle = N\int d^2\mathbf{q}_A \int d^2\mathbf{q}_B \tilde{E}(\mathbf{q_A}+\mathbf{q_B}) \times \frac{e^{i\Delta k_z L}-1}{i\Delta k_z}|1_{\mathbf{q}_A}\rangle|1_{\mathbf{q}_B}\rangle,$$

with normalization constant N, longitudinal wave-vector mismatch $\Delta k_z = k_{p_z} - k_{A_z} - k_{B_z}$, nonlinear crystal length L, and angular spectrum of the pump field $\tilde{\mathbf{E}}(\mathbf{q})$. The measurement of one photon's momentum determines the other's within a small region of uncertainty. This uncertainty or correlation area defines a limit to the system's resolution. The correlation length in momentum space $\delta q_i \equiv \langle(\mathbf{q}_A + \mathbf{q}_B)_i^2\rangle^{1/2}$ thus gives an estimate to the momentum pixel size in transverse dimension $i = x, y$. In the focal plane of a lens, this length maps into real space through the relation $\delta q_i^{(x)} = \delta q_i(\lambda f/2\pi)$. Entangled photons are strongly correlated in position as well. Thus, we can correspondingly define a correlation length in position as $\delta x_i \equiv \langle(\mathbf{x}_A - \mathbf{x}_B)_i^2\rangle^{1/2}$, where the position representation of the state is simply a Fourier transform of the state in the momentum basis. We can discretize this two-photon state by considering each pixel, i.e., a discrete region of space, as a distinct quantum level. These regions, however, must be separated by a distance larger than the parameters δx, $\delta q^{(x)}$ to ensure nearly perfectly correlated or anticorrelated fields when measuring photon coincidences in position or transverse momentum. Under these assumptions, we can describe the two-photon state as an entangled d-level qudit state.

We first considered the case of three entangled pixels for which $d = 3$. We performed an experiment that images either the position or the transverse momentum of the entangled photons emitted from the face of a 2-mm-thick β-barium borate (BBO) crystal onto a triangular optical fiber array of multimode fibers with core and cladding diameters of 62.5 and 125 μm, respectively (see Fig. 2). The coincidences of detection events involving the fibers in arms A and B were measured. The crystal was pumped by a 30 mW, cw beam at 390 nm with an rms intensity width of 0.17 mm. The orthogonally polarized entangled pair, emitted at 780 nm, was separated using a polarizing beam splitter. To measure the position of an emitted photon, we use only lens L_1, which images the exit face of the BBO crystal onto the cleaved ends of the fibers. Lens L_1 used in combination with lens $L_{2,3}$ performed a measurement of the transverse momentum of the photon. Here, the Fourier transform properties of lenses were employed to map transverse momentum into positions in the back focal plane. A 10-nm-wide spectral filter placed before the imaging objectives limited the detection to photons of near degenerate frequency. Since the transverse momenta of the entangled pair are anticorrelated, a Dove prism was used to rotate the beam by 180°. The rms intensity width of the field is 0.41 mm in the position detection plane and 0.48 mm in the transverse-momentum detection plane. Thus, the fiber array fills the entire field in both planes, ensuring that we sample the photons equivalently for each measurement. The correlation lengths in real space, after accounting for system magnifications, were calculated to be $\delta x = 50$ μm and $\delta q_x^{(x)} = 15$ μm, both smaller than the fiber separation. Thus to reasonable approximation our state is an entangled qudit state.

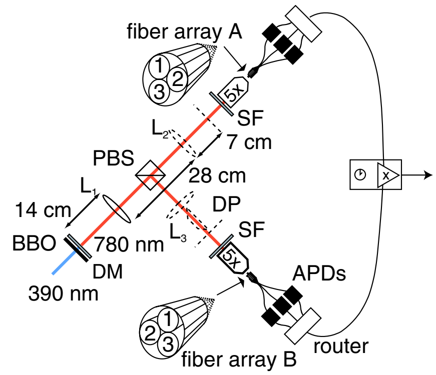

FIG. 2 (color online). Experimental configuration used for measuring photon spatial and momentum correlations. Dichroic mirrors (DM) filter out the pump light. Lenses L_1, L_2, L_3 have focal lengths of 100, 50, and 50 mm, respectively. The Dove prism (DP) inverts the image in momentum measurements. The distances indicated are the same in both arms.

We observed coincidences between the photons in each of the arms for each possible lens configuration. The results are shown in Fig. 3. We normalized all the graphs to the $3_A 3_B$ peak in Fig. 3(a), corresponding to 475 coincidences/second, also taking into account the fiber transmission coefficients. When both arms were configured to measure in the same basis, three sharp coincidence peaks occurred. The absence of any prominent off-diagonal peaks demonstrates that the two-photon state can be adequately described as a qutrit state. In the position basis, these peaks correspond to correlated pixels, and in the transverse-momentum basis, they correspond to anticorrelated pixels. We note near perfect correlations exist when imaging in both the position and the momentum spaces. The conditional uncertainty product is estimated to be $\Delta x_{A|B}\Delta q_{A|B} \lesssim 0.13$ where we assume the distance between two neighboring pixels (separated by 125 μm) represents the $1/e^2$ radius of the conditional distributions, justified by the high visibility observed. This uncertainty product is well below the classical limit of 0.5, demonstrating that the photon pairs are entangled in both position and momentum, a well-established property of SPDC [14,18].

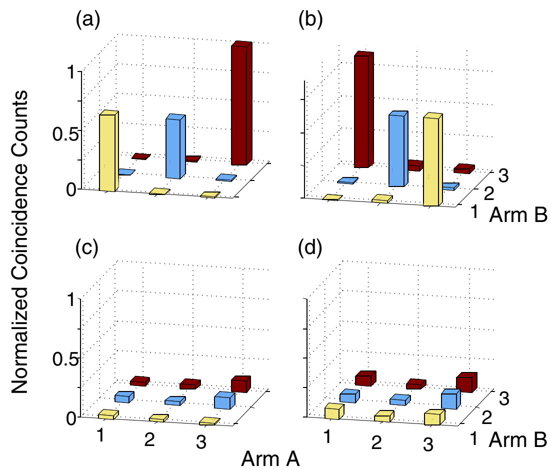

FIG. 3 (color online). Normalized coincidence counts between photons in the two arms, when both arms measure (a) photon position and (b) photon transverse momentum, as well as when the arms measure (c),(d) conjugate variables.

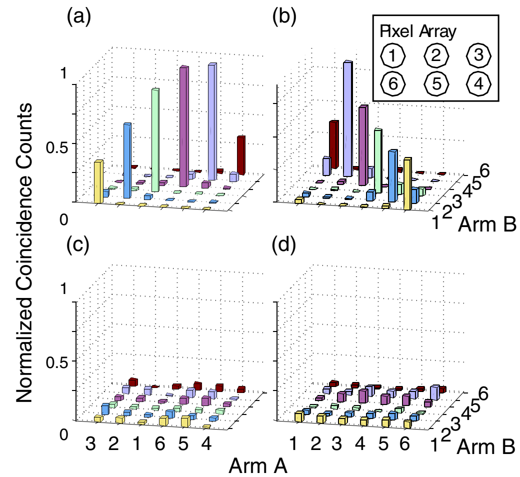

FIG. 4 (color online). Normalized coincidence counts between two six-pixel arrays when both arms measure (a) photon position and (b) transverse photon momentum, as well as when the arms measure (c),(d) conjugate variables. The inset shows the pixel array configuration.

When the two arms are set to measure conjugate variables, i.e., one measuring position and one measuring momentum, we expect a uniform distribution of coincidences among all nine possible pixel combinations. As seen in Figs. 3(c) and 3(d), there is a nearly flat distribution of coincidences, indicating that the pixels are uncorrelated. This property is important for quantum cryptography, since a measurement in conjugate bases yields nine possible results, decreasing the odds of successfully eavesdropping in comparison to a qubit system.

We also investigated the scalability of the process by considering the case $d = 6$. We scanned a single fiber in the detector planes through six positions, creating a pixel array and measured coincidences as before. The results are depicted in Fig. 4, normalized again to the highest peak. They show the behavior required for quantum information applications, namely, highly correlated and anticorrelated pixels when measuring the photons' position and transverse momentum and an even distribution of correlations when measuring conjugate variables. Although, as a scanning system, this system does not realize the increased information flux advantage of qudit systems, it shows that this technique scales and can produce six-level qudit states.

The question of how many realizable states are possible remains. Intuitively, the number of states should be the square of the ratio of the total field width to the correlation length, within an appropriate pixel-spacing factor to mitigate unwanted cross-pixel correlations. Alternatively, the Schmidt number is a measure of the effective dimension of the entangled state and can set an upper bound to the number of pixels. Recently, Law and Eberly estimated the Schmidt number for parametric down-conversion to be $K \geq (\Delta q/2\delta q)^2$ where $(\Delta q)^2 = \langle q^2 \rangle$ is the rms spread of the transverse wave vector of the down-converted photon and δq is defined as before [25]. This agrees with the intuitive result in the transverse-momentum basis. In position space, we can define a similar bound $N = (\Delta x/2\delta x)^2$. For our system we find $N = 16$, which is significantly less than the calculated Schmidt number $K \gtrsim 360$ since, in general, the Schmidt modes are not well localized in space. Hence, all K levels are not realizable as localized pixels in both the position and the transverse-momentum bases. Even so, N is highly scalable since Δx depends only on the pump waist and δx depends only on the crystal length and phase-matching conditions.

Each of these N states, in position space, for example, is conjugate to the entire transverse-momentum space and vice versa. Hence these can be considered qudits, but not in the usual sense. This property appears to be advantageous in quantum key distribution. In this case, one wants to effectively detect the presence of an eavesdropper while maximizing the number of highly correlated pixels. We consider an eavesdropper Eve intercepting a signal sent from Alice to Bob using a quantum cryptographic protocol. Eve's presence is typically revealed when Alice and Bob measure in the same basis and Eve measures in the conjugate basis. After Eve's measurement, the photons of Alice and Bob will be uncorrelated and found anywhere in the detection plane. Thus, if a photon is detected, it will be randomly measured in one of the N states, increasing the bit error rate (BER) and resulting in a nonviolation of the EPR bound. Additionally, because of the detection "dead" area, Eve will cause a drop in the overall key generation rate. For practical purposes, sufficient dead area is required to ensure high visibility correlations and low BER due to the finite correlation widths of the source. In practice, photons in this area would still be collected for security purposes, but would not contribute to key generation. Hence, these dead areas actually provide an additional signature for the presence of eavesdropping. At present, we envision this system to be useful for cryptographic applications in free-space applications because the quality of entanglement strongly depends on the preservation of the wave front, which cannot be done using current fiber technologies.

In summary, we have experimentally demonstrated a simple method for producing entangled qudits using the two-photon output of type-II SPDC. We created qutrits using a three-pixel detector array and investigated six-level qudits. The method is easily extended to higher dimensional qudits. The number of levels possible is restricted ultimately only by the Schmidt number which can be engineered to be large [25]. Very high dimensional entangled states are therefore easily conceivable using the position and transverse-momentum entanglement of SPDC, though for applications in quantum cryptography it may be beneficial to use only a small fraction of these states. This method presents a simple, yet powerful, tool for investigating entangled qudit states for quantum information protocols.

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[1] H. Bechmann-Pasquinucci and W. Tittel, Phys. Rev. A **61**, 062308 (2000).
[2] C. Macchiavello and D. Bruss, J. Mod. Opt. **50**, 1025 (2003).
[3] T. Durt, N. J. Cerf, N. Gisin, and M. Żukowski, Phys. Rev. A **67**, 012311 (2003).
[4] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[5] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
[6] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
[7] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[8] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, Rev. Mod. Phys. **74**, 145 (2002), and references within.
[9] D. N. Klyshko, Sov. Phys. JETP **56**, 753 (1982).
[10] A. Joobeur, B. E. A. Saleh, T. S. Larchuk, and M. C. Teich, Phys. Rev. A **53**, 4360 (1996).
[11] C. H. Monken, P. H. Souto Ribeiro, and S. Pádua, Phys. Rev. A **57**, 3123 (1998).
[12] C. K. Hong and L. Mandel, Phys. Rev. A **31**, 2409 (1985).
[13] E. Brambilla, A. Gatti, M. Bache, and L. A. Lugiato, Phys. Rev. A **69**, 023802 (2004).
[14] A. Gatti, E. Brambilla, and L. A. Lugiato, Phys. Rev. Lett. **90**, 133603 (2003).
[15] D. V. Strekalov, A. V. Sergienko, D. N. Klyshko, and Y. H. Shih, Phys. Rev. Lett. **74**, 3600 (1995).
[16] T. B. Pittman, Y. H. Shih, D. V. Strekalov, and A. V. Sergienko, Phys. Rev. A **52**, R3429 (1995).
[17] R. S. Bennink, S. J. Bentley, R. W. Boyd, and J. C. Howell, Phys. Rev. Lett. **92**, 033601 (2004).
[18] J. C. Howell, R. S. Bennink, S. J. Bentley, and R. W. Boyd, Phys. Rev. Lett. **92**, 210403 (2004).
[19] A. Mair, A. Vaziri, G. Weihs, and A. Zeilinger, Nature (London) **412**, 313 (2001).
[20] A. Vaziri, J.-W. Pan, T. Jennewein, G. Weihs, and A. Zeilinger, Phys. Rev. Lett. **91**, 227902 (2003).
[21] X.-F. Ren, G.-P. Guo, B. Yu, J. Li, and G.-C. Guo, J. Opt. B **6**, 243 (2004).
[22] H. de Riedmatten, I. Marcikic, H. Zbinden, and N. Gisin, Quantum Inf. Comput. **2**, 425 (2002).
[23] M. Żukowski, A. Zeilinger, and M. A. Horne, Phys. Rev. A **55**, 2564 (1997).
[24] J. C. Howell, A. Lamas-Linares, and D. Bouwmeester, Phys. Rev. Lett. **88**, 030401 (2002).
[25] C. K. Law and J. H. Eberly, Phys. Rev. Lett. **92**, 127903 (2004).