# Role of Bremsstrahlung Radiation in Limiting the Energy of Runaway Electrons in Tokamaks

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Bremsstrahlung radiation of runaway electrons is found to be an energy limit for runaway electrons in tokamaks. The minimum and maximum energy of runaway electron beams is shown to be limited by collisions and bremsstrahlung radiation, respectively. It is also found that a massive injection of a high-$Z$ gas such as xenon can terminate a disruption-generated runaway current before the runaway electrons hit the walls.

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Runaway electron (RE) generation is a phenomenon with a wide range of applications in different areas such as astronomy, accelerators, and fusion devices [1,2]. During the disruptions in fusion devices these runaway electrons can reach energies in excess of 100 MeV and they can damage the wall of the vacuum vessel when they hit it in large numbers. In small- and medium-size tokamaks the energy of the runaway electrons is limited by different mechanisms, such as synchrotron radiation, that are mainly determined by the size of the device. However, these size-related mechanisms may not be adequate during disruptions in next-step fusion devices such as the international thermonuclear experimental reactor (ITER) to keep the number of runaway electrons and their energy below values that are harmless to the vacuum vessel and the structures that surround the plasma. Reducing the energy of runaway electrons and the mitigation of their effects is a key issue for ITER [3–7] during fast plasma shutdowns and disruptions.

The physics behind runaway electron generation is the excess of the driving force from the electric field over the collisional drag force from plasma particles which results in the free acceleration of electrons. On the other hand, there are some limits on the energy the runaway electrons can reach, particularly in tokamaks, which are set by, amongst others, synchrotron radiation [8,9], drift orbits [1], acceleration time [1], and magnetic filed ripples [10]. These phenomena can limit the energy of runaway electrons in ITER to some value between 55 to 300 MeV. Diffusion of runaway electrons due to magnetic fluctuations [9,11,12] is one mechanism that had been used to interrupt the runaway electron generation or suppress it by removing the runaway electrons out of the plasma [13–15]. The combined effect of pitch-angle scattering and synchrotron radiation has been considered to be a physics phenomena that damps the runaway current in JET and JT-60U [6,13]. Runaway electrons do also radiate bremsstrahlung in the vicinity of heavy ions which can be detected in fusion reactors using soft and/or hard x-ray detectors [6,7]. However, to our knowledge, bremsstrahlung radiation has not been taken into account as an energy limit for runaway electrons in tokamaks. In this Letter we use bremsstrahlung to calculate an energy limit for runaway electrons in tokamaks. Based on those calculations, we propose a new method to cool down disruption-generated runaway electrons. These results can be applied to study the physics behind high pressure gas injection experiments in DIII-D [4] as well.

The energy loss rate of high-energy electrons passing through a gas can be expressed in terms of the stopping power $dW_e/dx$, with $W_e = (\gamma - 1)mc^2$ the kinetic energy of the runaway electron, $m$ the electron rest mass, $\gamma = 1/\sqrt{(1 - v^2/c^2)}$, $v$ the electron velocity, and $x$ the distance traveled by the electron. A calculation of the average energy transferred to randomly distributed background electrons and ions leads to the Bethe formula [16] for the collision stopping power of relativistic electrons:

$$F_C = \left(\frac{dW_e}{dx}\right)_C = 2N_m \kappa Z \Phi, \tag{1}$$

with $N_m$ the atomic density of the medium in atoms per cubic meter, $Z$ the atomic number of the medium, $\kappa = 2\pi r_e^2 mc^2$, $r_e = e^2/4\pi\varepsilon_0 mc^2$ the classical electron radius, and

$$\Phi(\gamma, I) = \frac{\gamma^2}{\gamma^2 - 1}\left[\ln\left(\frac{mc^2\sqrt{\gamma^2 - 1}\sqrt{\gamma - 1}}{\sqrt{2}I}\right) - \left(\frac{2}{\gamma} - \frac{1}{\gamma^2}\right)\frac{\ln 2}{2} + \frac{1}{2\gamma^2} + \frac{(\gamma - 1)^2}{16\gamma^2}\right], \tag{2}$$

with $I$ the average ionization potential [17]. Equation (1) gives the energy loss per distance of an electron when it travels through matter.

Energetic electrons not only lose energy due to collisions but they also lose energy in the form of bremsstrahlung when they are strongly deflected by the nuclei of atoms. The radiation stopping power is [18],

$$F_R = \left(\frac{dW_e}{dx}\right)_R \cong 2N_m \kappa Z(Z + 1)\frac{\alpha}{\pi}\frac{W_e}{mc^2}[\ln(2\gamma) - 1/3], \tag{3}$$

 

where $\alpha$ is fine structure constant. The radiation loss is proportional to $Z^2$, while collision loss is proportional to $Z$. Furthermore, the radiation loss is proportional to $W_e$, while the collision loss for relativistic electrons is almost independent of $W_e$. The total stopping power is obtained by adding the collision and radiative stopping powers: $F_T = F_C + F_R$.

The electric field, $E$, in tokamaks is the driving force for runaway electrons: $F_E = eE$ ($e$ is the electron charge). When $F_E$ exceeds $F_T$, runaway electrons are generated. In general, the equation $F_T = F_E$ has two roots when $F_E$ exceeds a critical value given by: $F_E^{\rm crit} = 2N_m Z\kappa\Phi_{\rm min} = eE_c$ with $\Phi_{\rm min} = \Phi(\gamma_{\rm min}, I)$, $\gamma_{\rm min} = 3.8$, and $\Phi_{\rm min}$ is given in Table I for a number of gases. At the low-energy root the stopping power is dominated by $F_C$ whereas the high-energy root is dominated by $F_R$.

The low-energy root, $W_{c1}$, is determined by collisions and can be written as [2]:

$$W_{c1} \approx mc^2 \frac{E_c}{E}. \tag{4}$$

The high-energy root, $W_{c2}$, is dominated by bremsstrahlung radiation and straightforwardly can be obtained as:

$$W_{c2} \approx mc^2 \frac{A}{2\mathcal{W}_L(A)}, \qquad A = 2\frac{\pi\Phi_{\rm min}}{\alpha Z}\left(\frac{E}{E_c} - 1\right), \tag{5}$$

where $\mathcal{W}_L(A)$ is the Lambert's $W$ function [19] and the critical electric field $E_c$ is given by:

$$E_c = 2N_m Z\kappa\Phi_{\rm min} e^{-1}. \tag{6}$$

Equation (5) is the new energy limit for runaway electrons and it is plotted for different gases in Fig. 1. As can be seen from this figure, $W_{c2}$ decreases with increasing $Z$. As expected, only for high-$Z$ gases, this limit is effective for large $E/E_c$. For low $E/E_c$, all gases can limit the energy gain of runaway electrons.

In the high pressure gas injection experiments [4,20], it was found that the energy of the runaway electrons was limited almost independently of the used gas species. In such a massive injection, $E/E_c \sim 1$ and $W_{c2}$ decreases greatly. This means that even already existing runaways, which can generate a runaway electron avalanche, are cooled down and cannot trigger the avalanche process. In fact, in Ref. [20], it was found experimentally that runaway electron suppression occurs at a lower gas injection pressures and/or smaller amounts of high-$Z$ gases than was calculated with a radiative cooling model. Taking the bremsstrahlung radiation into account decreases the difference between the calculations and experiments. In Fig. 2 the lower and upper critical energies $W_{c1,2}$ for a given $E$ are illustrated for xenon gas. The data were calculated using the ESTAR code [21]. Electrons with energies between $W_{c1}$ and $W_{c2}$ gain energy from the electric field and are accelerated to $W_{c2}$. Electrons with energies less than $W_{c1}$ are slowed down to thermal velocities by collisions. Electrons with energies greater than $W_{c2}$ will cool down by strong radiation.

TABLE I. $\Phi_{\rm min}$ for different noble gases and hydrogen. Values of $I$ are obtained from Ref. [17].

| Gas | H | He | Ne | Ar | Kr | Xe |
|---|---|---|---|---|---|---|
| $\Phi_{\rm min}$ | 12.4230 | 11.5871 | 10.3117 | 9.9717 | 9.2978 | 8.9601 |

In fact, for energies larger than a critical value, $W_{\rm cr}$, the radiation losses are dominant, and for the rest collisional losses dominate (see Fig. 2). The width of the runaway energy range depends on $E/E_c$ and it is clear that it can be narrowed or diminished by increasing $E_c$. In fact, $E_c$ depends on the number of particles and on its atomic number. In Fig. 3 we show $E_c$ and $W_{\rm cr}$ for five different noble gases and hydrogen: $E_c$ is an increasing function of $Z$ while $W_{\rm cr}$ is a decreasing one. A gas with low $W_{\rm cr}$ forces the runaway electrons to cool down to $W_{\rm cr}$ mostly by radiation. Around $W_{\rm cr}$ the effect of collisions and radiation are both important. Larger electric fields are required to generate runaway electrons in a gas with higher $E_c$.

These results lead also to a new method for stopping a disruption-generated runaway electron current. Returning to the stopping powers, the total time it takes an electron to come to rest can be approximately calculated through

$$t \approx \int_{W_0}^{0} \frac{-dW_e}{v(F_T + eE)}, \tag{7}$$

where $v$ is the electron velocity. [In the mildly relativistic regime, which we are considering in this Letter, Eq. (7) is a good approximation. However, Eq. (7) is strictly correct in

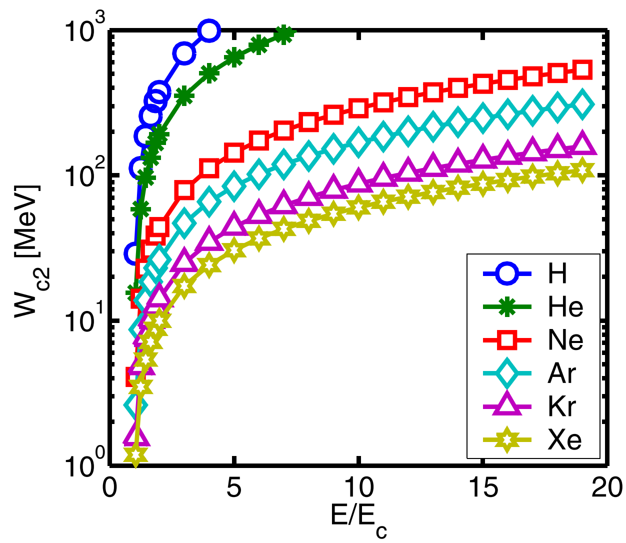


FIG. 1 (color online). Bremsstrahlung limit for runaway electrons in the presence of hydrogen and different noble gases vs $E/E_c$. All gases affect the energy of runaway electrons in low $E/E_c$. High-$Z$ noble gases limit the energy of runaways even for high $E/E_c$.

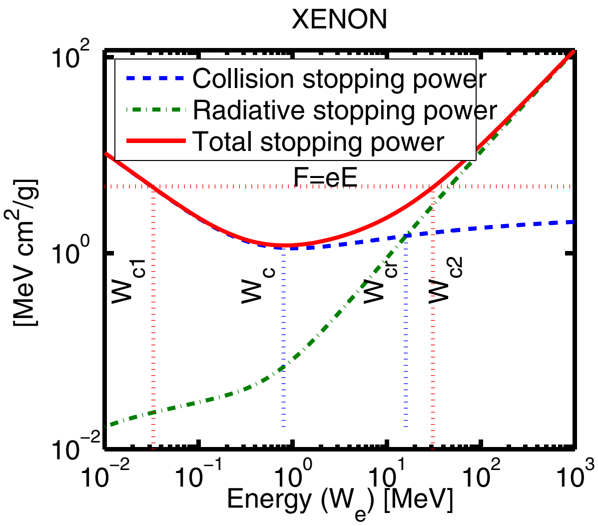


FIG. 2 (color online). Stopping powers for electrons in xenon gas. The electric driving force, $F = eE$, on the runaway electrons crosses the curves in two points. One in the collision dominated region, $W_{c1}$, and the other in the radiative dominated region, $W_{c2}$. Electrons with energies $W_{c1} < W_e < W_{c2}$ can accelerate freely up to $W_{c2}$.

the nonrelativistic and highly relativistic limits.] In Fig. 4 we show the stopping time for electrons in different noble gases for $N_m = 10^{20}$ m$^{-3}$ and $E = 0$ because during the runaway electron plateau after a disruption the plasma is highly conductive and $E \sim 0$. According to this figure we can inject a value about $N_m = 10^{20}$ m$^{-3}$ or more of xenon during the runaway electron plateau in large tokamaks to stop the runaway electron current in a short time of less than 30 ms. Injecting such amounts of xenon in such a short time is possible by using high pressure gas injection or it may even be possible by using a conventional gas puff system. The benefit of this method is that the runaway electrons will be slowed down mainly by radiation to around $W_{cr} \sim 15$ MeV, and then during the collision processes they return to thermal velocities before hitting the wall. As it is seen in Fig. 4, runaway electrons with energy around 100 MeV are stopped in a hydrogen plasma with $n_e \sim 1 \times 10^{20}$ m$^{-3}$ in about 4 sec, which is comparable with the observation of runaway current decay in the Joint European Torus (JET) [6]. In Ref. [20] a radiative cooling model was used to show that at low temperatures the resistive electric field is almost independent of the effective charge or the injected amount of impurities. This indicates that there is no strong electric field to compensate the effect of the stopping powers and the assumption of $E \sim 0$ is valid. Note that in the case of a massive xenon injection, high-energy runaway electrons slow down in ~30 ms but the runaway current can persist for a much longer time: because runaway electrons move with relativistic velocities, runaway currents are independent of the electron energy. For more precise calculations on runaway current termination, the inductive electric field due to current decay must be included as well.

Synchrotron radiation is a very important effect which has been considered as an energy limit for runaway electrons in magnetized plasmas, in particular, in tokamaks [8]. In Ref. [8] it was shown that runaway electrons can be slowed down by the combined effects of pitch-angle scattering and radiation reaction. The flow of a test particle in a two-dimensional momentum space is a method to study this effect along with other effects [22]. We have used the same method as in [22] but we included bremsstrahlung radiation to study its effect along with the synchrotron radiation on runaway electron currents generated by low pressure argon gas puffing [5] and by killer pellet (neon ice pellet) injection [13] in the JT-60U tokamak. The runaway current generated by argon gas puffing lasted for less than 300 ms while the one generated by the killer pellet injec-

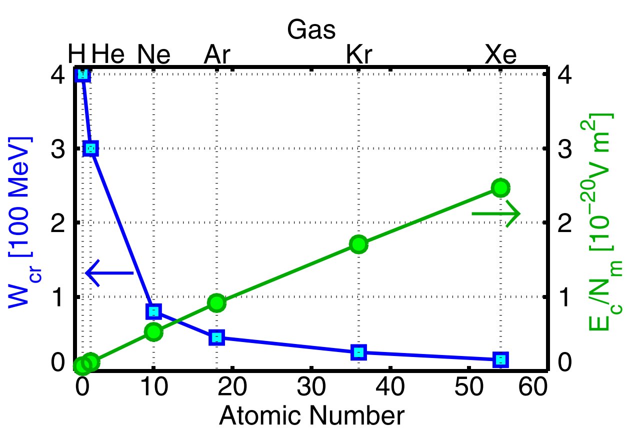


FIG. 3 (color online). Critical energy, $W_{cr}$, over which the radiative stopping power of electrons is dominant over the collision stopping power and the critical electric field, $E_c$, below which the runaway electrons population is not enhanced.

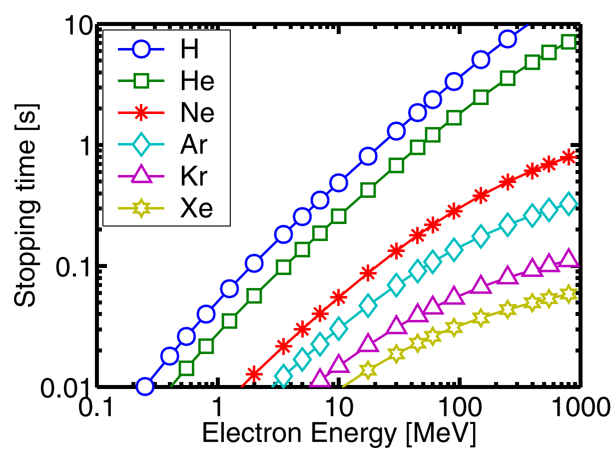


FIG. 4 (color online). The time it takes to cool down energetic electrons to 10 keV for hydrogen and different noble gases with a number density of $N_m = 10^{20}$ m$^{-3}$. Such an amount of xenon atoms can cool down runaway electrons with energy of 100 MeV in ~30 ms.

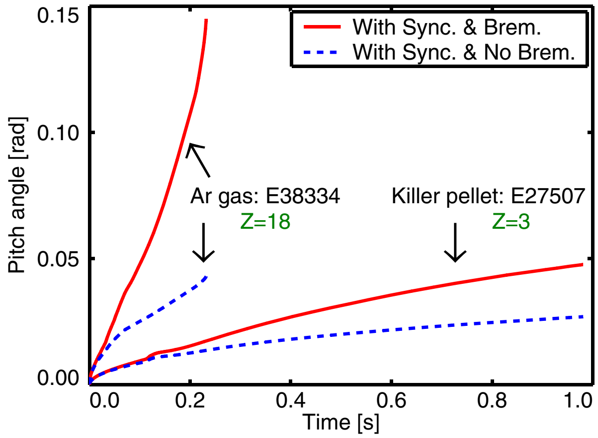


FIG. 5 (color online). The evolution of the pitch angle of a 200 MeV electron in presence of argon and neon.

tion lasted for more than 1 s. We did the 2D momentum space calculations for both discharges with and without bremsstrahlung radiation. The result of these calculations for the pitch angle of an electron with energy of 200 MeV is shown in Fig. 5. The effective charge ($Z_{\mathrm{eff}}$) when a killer pellet is injected was taken to be 3 since the number of neon atoms in a pellet does not affect the plasma impurity content much. From this figure one can find that injecting high-$Z$ impurities enhances the effect of synchrotron radiation by increasing the pitch angle (compare the argon gas and killer pellet curves). When bremsstrahlung is taken into account, the pitch angle, which is important for synchrotron radiation, becomes larger and hence the synchrotron radiation is larger [22]. So ignoring the scattering of REs from high-$Z$ impurities in the plasma, which generates bremsstrahlung, leads to an underestimation of the synchrotron radiation. It is difficult to separate the contribution of bremsstrahlung and synchrotron radiation since those contributions strongly depend on the machine size, evolution of the plasma parameters, and the energy of the runaway electrons. In the experiments shown in Fig. 5 for the neon case ~9% of the energy of a 200 MeV electron is radiated by bremsstrahlung while for the argon case this percentage is ~45%. For a 100 MeV electron, 28% of the energy is radiated by bremsstrahlung in the neon case while this percentage is 78% for argon case. Therefore, bremsstrahlung and synchrotron radiation are both effective in controlling the energy of runaway electrons in tokamaks.

In summary, by using the collision and radiative stopping powers of different noble gases we have shown that bremsstrahlung radiation can be an effective energy limit for runaway electrons in Tokamaks. It was shown that runaway electrons cannot gain high energies in the presence of high-$Z$ noble gases, in particular, xenon, and the runaway electrons are also cooled down to thermal velocity due to a combination of collisions and bremsstrahlung. This means that for ITER a fast injection of heavy noble gas during a fast plasma shutdown or disruption the damaging effects due to very high-energy runaway electrons can be avoided.

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