Two-Photon-Exchange Correction to Parity-Violating Elastic Electron-Proton Scattering

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Higher-order QED effects play an important role in precision measurements of nucleon elastic form factors in electron scattering. Here we introduce a two-photon-exchange QED correction to the parity-violating polarization asymmetry of elastic electron-proton scattering. We calculate this correction in the parton model using the formalism of generalized parton distributions, and demonstrate that it can reach several percent in certain kinematics, becoming comparable in size with existing experimental measurements of strange-quark effects in the proton neutral weak current.

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Introduction.—Recently, the two-photon-exchange mechanism in elastic electron-proton scattering has attracted a lot of attention. The reason is that this mechanism arguably accounts for the difference between the high-Q^2 values of the electric to magnetic proton form factor ratio G_{Ep}/G_{Mp} [1,2] as measured in unpolarized (Rosenbluth) and polarized electron scattering. Calculations of Ref. [3] using a formalism of generalized parton distributions (GPDs) [4] confirm the possibility, and decisive experimental tests are under development [5]. See also Ref. [6] for a review of the problem.

The Rosenbluth-polarization controversy is resolved (putatively) by including nonsoft photon exchange into the box and cross-box diagrams of electron-proton scattering. Since the additional exchanged photon is not soft, it alters the spin structure of the electron-proton elastic scattering amplitude and contributes to polarization observables [7]. These effects exceed spin-dependent corrections due to bremsstrahlung [8], because bremsstrahlung photons can be constrained to be soft by experimental cuts. The magnitude of the additional correction at fixed Q^2 depends on the electron scattering angle, reaching a few percent. This result motivated us to take a closer look at other electron scattering measurements that require high precision.

In this Letter, we discuss the implications of two-photon exchange for parity-violating electron scattering. The possibility of a radiative correction of this type has been mentioned in the literature (see, e.g., Ref. [9]), but neither a general nor a model-based analysis has been done so far. In our study, we derive expressions for parity-violating observables in terms of generalized form factors, and then present numerical results using the formalism of GPDs [3]. We conclude that the two-photon-exchange mechanism introduces a systematic correction to parity-violating electron-proton scattering asymmetry at a level of one or more percent in certain kinematics, which is comparable in size with existing experimental constraints of the strange-quark contribution to this asymmetry.

Currents and observables.—At tree level, parity violation in electron scattering is caused by interference of the diagrams with the single photon and single Z^0-boson exchange shown in Figs. 1(a) and 1(b). Leading-order radiative corrections include, but are not limited to, the box diagrams of Figs. 1(c) and 1(d), where the blobs between photon and Z-boson coupling to the nucleon denote excitations of all possible intermediate states by the electromagnetic and weak neutral currents. Therefore a calculation of such corrections depends on nucleon structure and requires modeling of the nucleon radiative weak amplitude and Compton amplitude under the loop integral. The mechanism of Fig. 1(d), the γZ box, contributes to parity-violating electron-nucleon interaction through interference with one-photon exchange [Fig. 1(a)]. It was calculated by Marciano and Sirlin [10] for the case of atomic parity violation corresponding to the limit of zero momentum transfer.

Let us consider one more possibility, namely, the radiative correction due to interference between the diagrams with Z-boson exchange [Fig. 1(b)] and two-photon exchange [Fig. 1(c)]. This mechanism of $2\gamma \times Z$ interference contributes to the observed asymmetry at the same order $\mathcal{O}(\alpha)$ as the γZ box discussed in Ref. [10]; it comes in

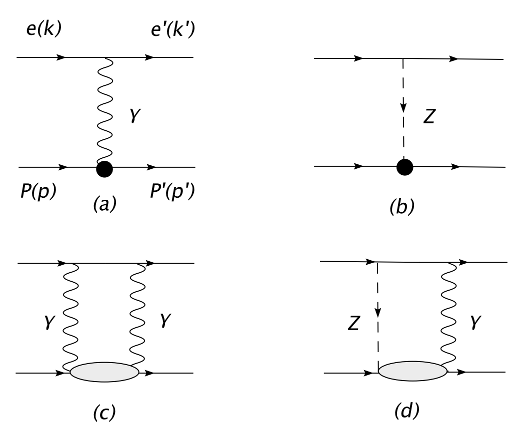

FIG. 1. (a),(b) Diagrams of the Born approximation, (c) two-photon exchange, and (d) the γZ box for elastic e-p scattering in a standard model of electroweak interactions. Corresponding cross-box diagrams are implied.

addition to the parity-conserving correction to the unpolarized cross section from two-photon exchange. For purely leptonic processes, the contribution of the box diagrams is calculable exactly as a part of the leading-order electroweak radiative correction [11]. However, additional knowledge of hadronic structure is needed to calculate a similar correction for semileptonic weak processes. A partial solution of this problem was introduced by Mo and Tsai in the classic paper [12] on radiative corrections to electron-proton scattering, in which the photon in diagram Fig. 1(c) was treated in an infrared approximation, thereby reducing the hadronic-structure dependence to Born-approximation form factors. We should note that if the infrared approximation is used, then the combined contribution of the boxes [Figs. 1(c) and 1(d)] would factor out and exactly cancel in the polarization asymmetries. To obtain the correction to polarization asymmetries, one needs to go beyond the infrared approximation for two-photon exchange, and this is the goal of the present Letter.

Let us first present general formulas for the electromagnetic and weak currents and the parity-violating asymmetry. The Born matrix element of Fig. 1(a) is described in a standard way in terms of proton electromagnetic form factors $F^{\gamma}_{1,2p}$. The exchange of the Z boson [Fig. 1(b)] is parametrized at tree level as [13]

$$\mathcal{M}^{Z} = -\frac{G_F}{\sqrt{2}} j^{\mu}_{Z} J^{Z}_{\mu}, \tag{1}$$

$$j^{\mu}_{Z} = \bar{u}(k')\gamma^{\mu}(g^{e}_{V} - g^{e}_{A}\gamma^{5})u(k), \tag{2}$$

$$J^{Z}_{\mu} = \bar{u}(p')\left[\gamma_{\mu}F^{Z}_{1} + i\sigma_{\mu\nu}q^{\nu}\frac{F^{Z}_{2}}{2M} + \gamma_{\mu}\gamma_{5}G^{Z}_{A}\right]u(p), \tag{3}$$

where G_F is Fermi constant, $F^{Z}_{1,2p}$ are the form factors of the proton neutral weak current, M is the proton mass, and $q = k - k'$. In the standard model of electroweak interactions, the weak form factors $F^{Z}_{1,2p}$ are related to the proton and neutron electromagnetic form factors by the expression

$$F^{Z}_{1,2p}(Q^2) = (1 - 4\sin^2\theta_W)F^{\gamma}_{1,2p} - F^{\gamma}_{1,2n} - F^{s}_{1,2}, \tag{4}$$

where θ_W is the weak mixing (Weinberg) angle and the quantities $F^{s}_{1,2}$ denote contributions to the neutral weak current from strange (and heavier) quarks. Other combinations of form factors are also used, namely, $G_E = F_1 - \tau F_2$ and $G_M = F_1 + F_2$, where $\tau = Q^2/(4M^2)$.

The axial form factor is

$$G^{Z}_{A}(Q^2) = -\tau_3 G_A(Q^2) + \Delta s, \tag{5}$$

where $\tau_3 = +1(-1)$ for protons (neutrons) and Δs stands for the strange-quark contribution. The isovector axial form factor is normalized at $Q^2 = 0$ to the neutron β-decay constant as $G_A(0) = -g_A/g_V = +1.2670 \pm 0.0035$. Note that all the above expressions are valid at tree level, when higher-order radiative corrections are neglected.

Interference between photon and Z-boson exchange produces an experimentally observable parity-violating asymmetry $A_{\rm PV}$ in the scattering of longitudinally polarized electrons on an unpolarized target [14]. It can be expressed in terms of the above quantities as

$$A^{\rm Born}_{\rm PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}}\frac{A^{\rm Born}_{E} + A^{\rm Born}_{M} + A^{\rm Born}_{A}}{[\epsilon(G^{\gamma}_{Ep})^2 + \tau(G^{\gamma}_{Mp})^2]}, \tag{6}$$

where

$$A^{\rm Born}_{E} = -2g^{e}_{A}\epsilon G^{Z}_{Ep}G^{\gamma}_{Ep}, \qquad A^{\rm Born}_{M} = -2g^{e}_{A}\tau G^{Z}_{Mp}G^{\gamma}_{Mp}$$
$$A^{\rm Born}_{A} = 2g^{e}_{V}\sqrt{\tau(1+\tau)(1-\epsilon^2)}G^{Z}_{A}G^{\gamma}_{Mp}$$
$$\epsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta_e}{2}\right)^{-1}, \tag{7}$$

and where $g^{e}_{V} = -(1 - 4\sin^2\theta_W)/2$, $g^{e}_{A} = -1/2$ are the electron coupling constants from Eq. (1).

Let us now consider the case when electron-proton scattering is caused by the exchange of more than one photon. Neglecting the electron mass, the electron-nucleon scattering amplitude can be presented in the form described by three complex scalar invariants; see also [1,3]:

$$\mathcal{M}^{\gamma} = \frac{e^2}{Q^2}\Big[\bar{u}(k')\gamma^{\mu}u(k)\times\bar{u}(p')\left(\gamma_{\mu}F'_1 + i\sigma_{\mu\nu}q^{\nu}\frac{F'_2}{2M}\right)u(p) + \bar{u}(k')\gamma^{\mu}\gamma^{5}u(k)\times\bar{u}(p')\gamma_{\mu}\gamma_{5}G'_{A}u(p)\Big]. \tag{8}$$

The invariants, or generalized form factors, may be separated into parts coming from one-photon exchange [Fig. 1(a)] and parts from two- or more-photon exchange [Fig. 1(c)],

$$G'_{M}(\epsilon, Q^2) = G^{\gamma}_{M}(Q^2) + \delta G'_{M}(\epsilon, Q^2)$$
$$G'_{E}(\epsilon, Q^2) = G^{\gamma}_{E}(Q^2) + \delta G'_{E}(\epsilon, Q^2) \tag{9}$$
$$G'_{A}(\epsilon, Q^2) = \delta G'_{A}(\epsilon, Q^2),$$

where $G^{\gamma}_{M}(Q^2)$ and $G^{\gamma}_{E}(Q^2)$ are the usual magnetic and electric form factors that parametrize matrix elements of nucleon electromagnetic current, and the corrections of order $\mathcal{O}(\alpha)$ from multiphoton exchange are represented by the quantities $\delta G'_{M}$, $\delta G'_{E}$, and G'_{A}.

Equipped with the corrected expression for the proton electromagnetic amplitudes, it is straightforward to update the formula for the parity-violating asymmetry $A_{\rm PV}$ which properly includes the two-photon box contributions. The asymmetry takes the form

$$A_{\rm PV} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}}\frac{A_E + A_M + A_A + A'_M + A'_A}{\epsilon|G'_{Ep}|^2 + \tau|G'_{Mp}|^2 + 2\sqrt{\tau(1+\tau)(1-\epsilon^2)}G^{\gamma}_{Mp}\,{\rm Re}(G'_{Ap})}, \tag{10}$$

where the formulas for $A_{E,M,A}$ are obtained from the corresponding formulas for $A^{\rm Born}_{E,M,A}$ of Eq. (7) by replacing electromagnetic form factors $G^{\gamma}_{E,M}$ with real parts of their generalized versions $\mathrm{Re}(G'_{E,M})$. Two additional terms, A'_M and A'_A, arise from interference between the axial-like terms in the two-photon-exchange amplitude and Z-boson exchange,

$$\begin{aligned} A'_A &= 2g^e_V(1+\tau)G^Z_A\,\mathrm{Re}(G'_{Ap}) \\ A'_M &= -2g^e_A\sqrt{\tau(1+\tau)(1-\epsilon^2)}G^Z_M\,\mathrm{Re}(G'_{Ap}). \end{aligned} \quad (11)$$

One can see from Eq. (11) that the parity-violating asymmetry acquires new terms from the modified spin structure of electron-proton scattering amplitude coming from two-photon exchange. Because of these terms, the updated expressions for A_{PV} cannot be obtained by just updating the nucleon electromagnetic form factors, but also require including this new $\mathcal{O}(\alpha)$ correction to the observed asymmetry.

Thus the two-photon-exchange correction to A_{PV} is a combined effect from the correction to the unpolarized cross section in the denominator of Eq. (10) and the correction to the polarization-dependent numerator. To the leading order in (electromagnetic) α, the former is due to interference between one- and two-photon-exchange diagrams [Figs. 1(a) and 1(c)], whereas the latter arises due to interference of the Z boson and two-photon exchange [Figs. 1(b) and 1(c)].

GPD calculation of two-photon-exchange correction to A_{PV}.—We evaluate the contribution of two-photon exchange to the parity-violating asymmetry using a partonic formalism. The calculation of the generalized form factors given in Eq. (9) is described in detail in Refs. [3,7], and here we will only briefly remind the reader of the main steps.

For the two-photon-exchange mechanism of Fig. 1(c), a "handbag" approximation is used, in which the two photons are coupled directly to a pointlike quark. The photon phase space in the four-dimensional loop integral is separated into "soft" and "hard" regions using the gauge-invariant procedure of [15]. The soft part is treated separately in accordance with a low-energy theorem, while the hard part calculation is performed using GPDs to describe the emission and reabsorption of a quark by a nucleon. The same parametrization is used for the GPDs as in Ref. [3], with constraints on x (quark momentum fraction) and Q^2 dependence coming from available data on elastic nucleon form factors and parton distribution functions of inclusive deep-inelastic scattering. The calculation is not extended to small values of ϵ because the model [3] in its present form has limited applicability in this region, which corresponds to small values of Mandelstam variable $-u < M^2$. The parton model approach used here also restricts four-momentum transfers to the region $Q^2 > M^2$.

The results of the calculation are shown in Figs. 2 and 3 for the ratio

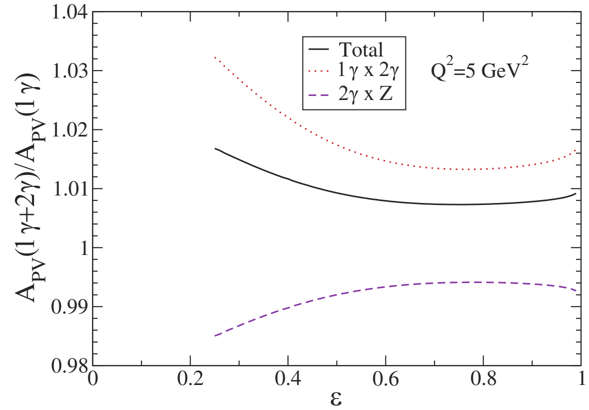

FIG. 2 (color online). Two-photon-exchange correction to parity-violating asymmetry as a function of ϵ at $Q^2 = 5\ \mathrm{GeV}^2$. Also shown are separate effects from the parity-conserving $1\gamma \times 2\gamma$ interference and parity-violating $2\gamma \times Z$ interference.

$$R = \frac{A_{\mathrm{PV}}(1\gamma + 2\gamma)}{A_{\mathrm{PV}}(1\gamma)}, \quad (12)$$

where $A_{\mathrm{PV}}(1\gamma)$ is the Born asymmetry of Eq. (6) and $A_{\mathrm{PV}}(1\gamma + 2\gamma)$ is the calculation with two-photon-exchange correction included. It is instructive to compare contributions to the above ratio from the polarization-dependent numerator and the polarization-independent cross section in the denominator of Eq. (10), because they come from interference of different pairs of diagrams. The two-photon-exchange correction reduces the magnitude of both the denominator and the numerator of Eq. (10) by several percent, leading to partial cancellation of the effect for the asymmetry and increasing the magnitude of A_{PV} compared to its Born value.

The correction is both Q^2 and ϵ dependent. It is increasing toward backward electron scattering angles (small ϵ), reaching above 1%. Quantitatively, a 1% increase in the magnitude of A_{PV} due to two-photon exchange results in about the same percentage decrease in the magnitude of the

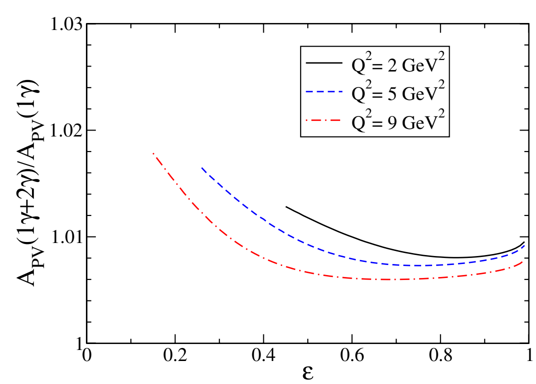

FIG. 3 (color online). Q^2 evolution of two-photon-exchange correction to parity-violating asymmetry for several values of $Q^2 = 2\ \mathrm{GeV}^2$ (solid curve), 5 GeV^2 (dashed curve), and 9 GeV^2 (dash-dotted curve).

extracted G_{Mp}^{Z}, so that the extracted strange G_M^s would become more positive by about 1% of the value of G_{Mp}^{Z}. It would have an even larger impact were one extracting Δs at backward angles, changing the extracted Δs by about 10% the value of $G_A(Q^2)$ [Eq. (5)]. The magnitude of the correction in the model is therefore comparable in size with experimentally measured contributions (albeit done at lower $Q^2 < 1$ GeV^2) to the scattering asymmetry from strange quarks in the proton neutral weak current [16]; see also Ref. [17] for a review of the current experimental status.

Our result also calls for an update of the model corrections to the proton neutral weak current from the γZ box and an extension of model calculations to the region of lower momentum transfer $Q^2 < 1$ GeV^2.

Let us discuss the possibilities of setting experimental constraints on two-photon exchange and γZ corrections to $A_{\rm PV}$. It is known that the difference between the cross sections for electron and positron electromagnetic scattering on a nuclear target provides a direct measure of the two-photon-exchange contribution to the cross section [18]. However, the situation is more complicated for weak currents, for which the vector and axial-vector terms change relative sign under charge conjugation. As a result, the parity-violating asymmetry at tree level can be presented in the form

$$A_{\rm PV}^{\pm} \propto g_V^e \alpha_A \pm g_A^e \alpha_V, \tag{13}$$

where the upper (lower) sign corresponds to scattering positrons (electrons) and the quantities $\alpha_V(\alpha_A)$ are due to the vector (axial-vector) hadronic neutral weak current interfering with the electromagnetic current. With two-photon exchange and γZ contributions included, each term in the above expression (13) receives a correction $\delta\alpha_{A,V}$ that has opposite signs for electrons and positrons, i.e., $\alpha_{A,V} \to \alpha_{A,V} \pm \delta\alpha_{A,V}$. As a result, a comparison of electron vs positron scattering on a nucleon target is not sufficient for separating out the contributions with an extra photon exchange between the lepton and the hadron. However, there is an exception for specific quantum numbers of the target. For spin-zero targets (e.g., ^{4}He) and elastic scattering, the hadronic weak current has only a vector component and thus only the term $\propto g_A^e$ remains in Eq. (13). Therefore for such a target, the sum of asymmetries from positrons and electrons, $A_{\rm PV}^{+} + A_{\rm PV}^{-}$, is proportional to the effects with an extra photon exchange shown in Figs. 1(c) and 1(d). Such an experiment, however, is difficult to implement at this time without high-current polarized positron beams.

As far as the nucleon target is concerned, the two-photon-exchange mechanism can be constrained experimentally from parity-conserving observables both from electron vs positron scattering comparisons and from dedicated polarization measurements [7]. These data can further be used to provide necessary input for theoretical models needed to evaluate additional contributions from the γZ box.

In summary, we have demonstrated that the two-photon-exchange corrections to the parity-violation electron scattering asymmetries can reach a few percent; i.e., they are comparable in size to existing experimental measurements [16,17] on the contribution of strange quarks to these asymmetries.

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