Fischer Replies: The Comment [1] on my Letter [2] criticizes the statement of a possible protection of the negative coupling constant Bose gas under rotation against collapse if a Chern-Simons term is present, as well as the solutions describing the order parameter distribution.

The predominantly relevant Letters in the original literature are [3,4]. The first establishes the Jackiw-Pi solitons, the second investigates these objects in a constant external magnetic field, corresponding to the external rotation field Ω in my Letter [2]. In the introductory paragraph after Eq. (1) of [4], we find the phrase "corresponds in the second-quantized problem to a two-body δ-function attraction," the physical situation described in [2]. The Jackiw-Pi solitons for $\Omega = 0$ are derived in [3]; they are solutions of the self-dual Bogomol'nyi equation minimizing the energy for an attractive pseudopotential interaction at a particular value of the coupling constant, $g = -\frac{1}{m|\Theta|}$, where Θ is the Chern-Simons constant.

I agree with the author of the Comment that the unfortunate claim contained on the third page of [2], asserting the boosted Jackiw-Pi solitons for nonzero rotation field $\Omega \neq 0$ to be solutions of the Bogomol'nyi Eq. (11), is obviously not correct: Their energy per particle is simply not equal to the lower bound satisfied by solutions of the Bogomol'nyi equation, $E_{\rm self}/N = -\Omega$. In Ref. [4], the energy of the boosted soliton solutions is calculated (see Eq. (32) in [4]). This energy is given by the expression $E_\Omega = \Omega[\frac{\Omega m r_0^2}{2} \frac{\pi/n}{\sin(\pi/n)} - 4\pi\Theta n]$, where r_0 is soliton size and n the winding number [4]. Ginzburg-Landau-Chern-Simons theory hence allows for an exact soliton solution at negative coupling constant $g = -\frac{1}{m|\Theta|}$. However, this solution represents an excited state, and not a state corresponding to a solution of the Bogomol'nyi equation.

On the other hand, the primary statement of the Comment [1], that the system collapses for any negative semidefinite total energy, hinges upon the fact that the action under consideration obeys scaling invariance and that the energy is unbounded from below. The Ginzburg-Landau-expanded potential $\mathcal{U}(|\psi|^2) = \frac{1}{2} g|\psi|^4 + \frac{1}{6}\gamma|\psi|^6 + \cdots$, Eq. (9) in [2], does, for $\gamma \neq 0$, not yield a scale-invariant action. The term $\frac{1}{6}\gamma|\psi|^6$ in $\mathcal{U}(|\psi|^2)$, for $\gamma > 0$, corresponding to the fact that the system possesses sixth-order stability, protects the scalar order parameter Bose gas against decay. Thus, the $\frac{1}{6}\gamma|\psi|^6$ term is crucial for the anyonic statistics in a fractional quantum Hall state to play an effective dynamical role which is due to the fact that part of the kinetic energy generates a $|\psi|^4$ term if the Chern-Simons constant is nonzero.

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[1] Pijush K. Ghosh, preceding Comment, Phys. Rev. Lett. **94**, 208903 (2005).
[2] U. R. Fischer, Phys. Rev. Lett. **93**, 160403 (2004).
[3] R. Jackiw and S.-Y. Pi, Phys. Rev. Lett. **64**, 2969 (1990).
[4] R. Jackiw and S.-Y. Pi, Phys. Rev. Lett. **67**, 415 (1991).

