Comment on "Dynamical Role of Anyonic Excitation Statistics in Rapidly Rotating Bose Gases"

It has been claimed in a recent Letter that attractive Bose gases under rapid rotation can be stabilized in a fractional quantum Hall state due to the anyonic statistics of their quasiparticle excitations [1]. Further, it is claimed that the solutions of the self-dual equation satisfying the Bogomol'nyi bound with a nonzero "external" field can be obtained exactly from that of the Jackiw-Pi solutions through time-dependent coordinate transformations [1]. We argue in this Comment, with the help of known results [2,3], that the anyonic excitation statistics does not play any dynamical role in stabilizing the attractive Bose gases. We also point out that the assertion of obtaining exact solutions of the self-dual Eq. (11) in Ref. [1] with $\Omega \neq 0$ is invalid.

The dynamical stability of the system described by the Hamiltonian H in Ref. [1] with quartic self-interaction has been studied previously [2]. Results obtained in subsequent papers [3,4] are also equally valid for H due to an underlying universality. The mean-square radius $I = \frac{m}{2} \times \int d^2 r r^2 \psi^* \psi$ has the same dynamical behavior for the system with or without the Chern-Simons gauge field, if it is evolved with the same set of initial conditions and a fixed $H > 0$ for both the cases [2,3]. On the other hand, the field ψ blows up (i.e., I collapses) at a finite time independent of initial conditions if $H \leq 0$ [2,3]. This result is again valid for both in the presence and in the absence of the Chern-Simons gauge field.

Analyzing Eq. (8) of Ref. [1] with an attractive quartic self-interaction (i.e., $\mathcal{U} = g|\psi|^4$ with $g < 0$), at the self-dual point $g_{\text{eff}} = 0$ and $V(x) = 0$, we find that H is negative for $\Theta\Omega > 0$. The fractional quantum Hall state with the filling factor $\nu < 1$ is obtained in Ref. [1] for both Θ and Ω positive, implying $H < 0$. Hence, the system is not dynamically stable and the field ψ blows up at a finite time. In fact, there are no finite energy solutions of the self-dual Eq. (11) for $g < 0$ and $\Theta\Omega > 0$, since the energy functional is not bounded from below.

Our second point concerns the claim of obtaining exact solutions of the self-dual equation (11) with $\Omega \neq 0$. Any solution of a self-dual equation saturating the Bogomol'nyi bound is necessarily a solution of the second order equations of motion that are obtained by varying the action. However, the converse is not true. There are solutions of the second order equations of motion with higher energy which are not solutions of the self-dual equation. The mapping that relates S_Θ with $\Omega \neq 0$ to that S_Θ with $\Omega = 0$ is only at the level of action. Thus, an exact solution of the second order equations of motion of S_Θ at $g_{\text{eff}} = 0$, can definitely be obtained from Jackiw-Pi solitons of S_Θ with $\Omega = 0$, which however is not a solution of Eq. (11). In fact, unlike these exact solutions, the density $\rho = \psi^* \psi$ of the self-dual system is not determined by the Liouville equation for $\Omega \neq 0$ [5].

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