

## Comment on “Temperature in Nonequilibrium Systems with Conserved Energy”

In a recent Letter [1], Bertin, Dauchot, and Droz (BDD) presented a model with stochastic energy conserving dynamics. BDD’s central claim is that their model exhibits nonequilibrium behavior. The results BDD present as evidence for this claim are (ostensible) deviations from the microcanonical and canonical distributions, and (ostensible) violation of fluctuation-dissipation (FD) relations. In this Comment we show that these results are insufficient for proving nonequilibrium behavior, and demonstrate that in fact the same results are obtained assuming equilibrium behavior for the same system and dynamics defined in [1].

BDD define a system together with its dynamics, and claim the resulting distributions, Eqs. (5) and (9) in [1] [henceforth (B5) and (B9)], differ from the microcanonical and canonical equilibrium distributions. Were the model’s dynamics to stem from some Hamiltonian description, the corresponding generalized coordinates would define the system’s phase space, and one could compare the distributions in that space to the corresponding equilibrium distributions. BDD do not define a Hamiltonian which yields the system’s dynamics, hence one cannot know what phase space is. Thus, one cannot infer whether the calculated distributions imply any nonequilibrium behavior.

To make this point more explicit, we note that there is a description for BDD’s model, within which (B5) and (B9) are obtained as the equilibrium distributions. For BDD’s dynamics (B3) to be understood as dynamics of an equilibrium system, all final states consistent with energy conservation should be chosen at every time step with equal probability [2]. The density of states (DOS) for BDD’s model to sample phase space in such a uniform fashion is  $d(E) \sim E^{\eta-1}$  [ $d(E_i) \sim E_i^{\eta_i-1}$  for the inhomogeneous system defined by (B12)]. This DOS may be thought of as arising from some “hidden coordinates”  $\{s_i\}$ , such that  $|x_i| = s_i^{1/2\eta}$ , where the  $s_i$ ’s have a flat DOS, and thus define phase space. Since the energy of each particle is given by  $E_i = x_i^2/2$ , the DOS as a function of  $x$  is  $d(x) \sim |x|^{2\eta-1}$ , and the state defined by  $\{x_i\}$  is a collection of  $d(\{x_i\}) \sim \prod_{i=1}^N |x_i|^{2\eta-1}$  actual states defined by  $\{s_i\}$ . Therefore, the probabilities for the occurrence of the state  $\{x_i\}$  in (B5) and (B9) are the equilibrium microcanonical and canonical distributions of the actual states  $\{s_i\}$  multiplied by the DOS.

In calculating FD relations, it is necessary to know the measure of phase space. The FD measurement that is performed in [1] *tacitly assumes* a flat DOS for  $x_i$ , which results in their obtaining  $T_{\text{FD}} = 2\epsilon$ , rather than  $T_{\text{FD}} = T_{\text{th}}$ , where  $T_{\text{th}}$  is the actual equilibrium temperature defined in (B8), and  $\epsilon$  is the average per particle energy. Since no DOS may be inferred from the model, it is impossible to

construct a correct FD measurement: one cannot prescribe the DOS (flat or otherwise) independent of the dynamics. If the DOS is not flat, using (B3) [or (B12)] does not modify the DOS as a function of  $E$  and hence effectively changes the DOS as a function of  $x$ . Introducing an external field should not affect the structure of the system as reflected in its DOS  $d(x)$ , but merely change the relation between  $x$  and  $E$  (see [3] for an example of modeling FD relations in stochastic models). When assuming the DOS  $d(x) \sim x^{2\eta-1}$ , proposed above, one obtains  $T_{\text{FD}} = T_{\text{th}}$ , irrespective of  $\eta$ .

Extending this to inhomogeneous systems, BDD’s  $T_{\text{FD}}$  characterizes the average energy of each type of particle according to its DOS (or value of  $\eta$ ), and is thus spatially nonuniform, while  $T_{\text{th}}$  is the actual temperature, which is uniform throughout the system. A proper FD measurement would yield a uniform  $T_{\text{FD}}$ , coinciding with  $T_{\text{th}}$ . This is not the reason for the different effective temperatures observed in granular mixtures [4], where species differ in their mechanical properties which determine energy dissipation, rather than in their densities of states. Moreover, FD measurements in granular gases are on the position and not on the momentum (the equivalent of  $x$  in BDD’s model), and measure FD relations, which yield the actual temperature in the equilibrium limit, and not the average energy as is the case in BDD’s procedure. Even though  $T_{\text{FD}}$  does not strictly measure the average energy in granular gases, its value is similar to that of the average energy because the energy distribution differs only slightly from the Boltzmann distribution [3,5].

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