Scaling and Universality of Integer Quantum Hall Plateau-to-Plateau Transitions

Wanli Li,[1,2] G. A. Csáthy,[2] D. C. Tsui,[2] L. N. Pfeiffer,[3] and K. W. West[3]
[1]Department of Physics, Princeton University, Princeton, New Jersey 08544, USA
[2]Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA
[3]Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974, USA
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We have investigated the integer quantum Hall plateau-to-plateau transition in two-dimensional electrons confined to $Al_xGa_{1-x}As$-$Al_{0.33}Ga_{0.67}As$ heterostructures over a broad range of Al concentration x. For x between 0.65% and 1.6%, where the dominant contribution to disorder is from the short-range alloy potential fluctuations, we observe a perfect power-law scaling in the temperature range from 30 mK to 1 K with a critical exponent $\kappa = 0.42 \pm 0.01$.

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The plateau-to-plateau transition in the quantum Hall regime has been intensively studied [1–4] since the discovery of the integer quantum Hall effect (IQHE). In the IQHE, the Hall resistance R_{xy} has quantized values h/ie^2 over a wide range of the magnetic field B around integer Landau level filling factors i. The successive Hall plateaus correspond to separated energy regions of localized states, and in between them are extended states [5–7]. It was shown that between two plateaus there is only one such extended state at the critical energy E_c [1,8]. As the Fermi energy approaches this critical energy, the localization length is supposed to diverge following a power law $\xi \propto |E - E_c|^{-\nu}$ with a universal critical exponent ν [1–4]. The extended state can also be accessed by sweeping B, a case in which $\xi \propto |B - B_c|^{-\nu}$ [1–4]. In order to extract ν from experimentally measured quantities one has to invoke the finite size scaling theory [1–4,9] according to which the resistance tensor scales as $R_{uv} = R_{uv}(L/\xi)$ for a sample of finite size L. The quantum phase coherence length sets the effective sample size and from its temperature dependence of the $T^{-p/2}$ form [3,4,9] one obtains $R_{uv} = R_{uv}(|B - B_c|T^{-\nu})$, the scaling function of both the longitudinal resistance R_{xx} and the Hall resistance R_{xy}. Approaching zero temperature, the derivative of the Hall resistance R_{xy} taken at B_c diverges as a power law $(dR_{xy}/dB)|_{B=B_c} \propto T^{-\kappa}$, while the half width for the longitudinal resistance R_{xx} vanishes as $\Delta B \propto T^{\kappa}$, where the exponent κ is expressed as $\kappa = p/2\nu$.

The first experiment on electrons confined to the interface of InGaAs-InP heterostructures found $\kappa = 0.42$ [2]. Considering $p = 2$ [10–12], the exponent ν is found to be 2.4, a value independently obtained by subsequent theoretical calculations [3,4,13–17]. However, later studies in other experimental systems raised doubts about the universality of the critical exponent. In the silicon metal-oxide-semiconductor field-effect transistor (MOSFET) systems, κ was measured to range from 0.16 to 0.65 [18]; in GaAs-AlGaAs heterostructures, κ was found to vary from 0.28 to 0.81 [19,20] or to be totally absent [21]. These measurements show that κ is sample dependent and even varies for different transitions in a single sample. On the other hand, it has been long appreciated that the nature of disorder in the various systems mentioned above is fundamentally different [18,20,22,23]. While the disorder in Si-MOSFET and GaAs-AlGaAs systems is dominated by long-ranged ionized impurity potentials [24], there is a considerable contribution to the disorder from the short-range fluctuations of the alloy potential in the InGaAs-InP system [25], and the range of disorder seems to play an important role in the physics of the transition.

We propose a new approach to the fundamental problem of the plateau-to-plateau transition in IQHE, an approach that is focused on the nature of the scattering potential. More specifically, we have demonstrated experimentally that the random alloy potential in a two-dimensional sample is T independent and depends on the alloy concentration only. We are able to measure directly the alloy scattering potential and determine the strength of the alloy disorder relative to the disorder that arises from the long-range Coulomb force of charged impurities. Power-law scaling with the exponent $\kappa = 0.42$ is found only for various samples in which disorder is dominated by the short-range alloy potential fluctuations. The universality of the transition is thus firmly established in the short-range disordered regime.

The samples used in this work are based on the $GaAs$-$Al_{0.32}Ga_{0.68}As$ heterostructure, a two-dimensional electron system of high mobility ($n_e = 1.2 \times 10^{11}/\mathrm{cm}^2$, $\mu = 3.7 \times 10^6\ \mathrm{cm}^2/\mathrm{V\,s}$). By introducing a small amount of Al into the GaAs during the growth process we obtain $Al_xGa_{1-x}As$-$Al_{0.32}Ga_{0.68}As$ heterostructures [26]. A series of nine samples were grown with different Al concentration x by exactly the same molecular-beam epitaxy process. The Al content x is determined by controlling the growth rates of Ga and Al, which are calibrated by reflection high-energy electron diffraction technique oscillations. The relative error of x values is within 1%. The x value, the electronic density n_e, and mobility μ of each sample are summarized in Table I. In these samples, elec-

TABLE I. Sample properties. The Al concentration x, the electron density n_e and mobility μ, the ratio θ between the alloy and the background scattering rates at 0.3 K, and the scaling exponent κ of four transitions. There are two wafers with $x = 0.85\%$, and the three pieces A, B, and C were cut from the first one.

x		n_e	μ	θ	κ			
%		10^{11}/ cm^2	10^6 cm^2/V s		6-5	5-4	4-3	3-2
0		1.13	3.7	0	0.58	0.58	0.57	
0.21		1.32	2.05	0.8	0.57	0.56	0.58	
0.33		1.25	1.62	1.3	0.49	0.50	0.49	
0.85	A	1.16	0.89	3.3	0.43	0.42	0.42	0.41
	B				0.42	0.41	0.42	0.42
	C				0.42	0.42	0.42	0.41
0.85		1.18	0.91	3.2	0.41	0.42	0.42	0.42
1.4		1.14	0.56	5.6	0.43	0.43	0.42	0.42
1.9		1.26	0.46		0.49	0.49	0.50	0.51
2.6		1.22	0.34		0.58	0.60	0.59	0.58
4.1		0.83	0.20				0.58	0.57

trons accumulate on the $Al_xGa_{1-x}As$ side of the interface and experience alloy potential fluctuation around the Al atoms, which has been characterized in Ref. [26]. The amplitude of the alloy potential fluctuation was measured to be 1.13 eV, 3 orders of magnitude larger than the background potential fluctuation due to ionized impurities; however the range of the alloy potential fluctuations is only of the atomic size, 3 orders of magnitude smaller as compared to that of the ionized impurities [24,26]. The alloy potential can therefore be modeled as an uncorrelated δ-function-like disorder, which is an ideal short-range disorder. We also found that the alloy scattering rate is T independent and proportional to $x(1-x)$. The alloy scattering rate rises with increasing Al concentration. When x reaches 0.26%, the alloy scattering rate is about the same as the background ionized impurity scattering rate. We noticed that in samples with $x > 2\%$ the scattering rate has a large deviation from the linear dependence on $x(1-x)$. This deviation is believed to arise from the clustering of the Al atoms, which introduces correlations in the alloy scattering centers and thus renders the model of uncorrelated δ-function-like potential invalid [25]. For samples in which the linear dependence of the scattering rate on $x(1-x)$ still holds, we define θ as the ratio between the alloy scattering rate $1/\tau_a$ and the background scattering rate $1/\tau_b$. The parameter θ, listed in Table I, is a simple measure the dominance of the scattering due to alloy disorder.

For each sample the longitudinal resistance R_{xx} and the Hall resistance R_{xy} are measured simultaneously in a ^{3}He system from 0.3 to 1 K by using two lock-in amplifiers with a current excitation of 1 nA and frequency of 5.7 Hz. The sweeping rate of the magnetic field is kept sufficiently small to acquire at least five data points within 1 mT.

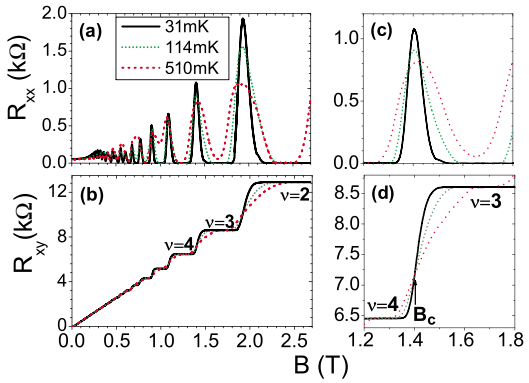

FIG. 1 (color online). (a), (b) The longitudinal resistance R_{xx} and Hall resistance R_{xy} at different temperatures for the sample with $x = 0.85\%$. In this plot, ν denotes the Landau level filling factors. (c), (d) The transition between the plateaus of $\nu = 4$ and $\nu = 3$. A critical magnetic field $B_c = 1.40$ T is observed.

Figs. 1(a) and 1(b) show the plots of R_{xx} and R_{xy} vs B at different temperatures for the sample with $x = 0.85\%$ and Figs. 1(c) and 1(d) show the transition between the plateaus around Landau level filling factors 4 and 3 (4-3 transition). According to the finite size scaling theory mentioned above [1–4], the critical exponent κ can be extracted from the power-law fit of $(dR_{xy}/dB)|_{B=B_c}$ against the temperature. Figure 2 shows the fitting of κ of the 4-3 transition for the samples with $x = 0$, $x = 0.85\%$, and $x = 4.1\%$. The R_{xy} vs B data was smoothed by averaging within 1 mT before the derivative was taken. We found that the exponents are all the same for the various transitions in the same sample, but vary from 0.42 to 0.59 for different samples. The fitting error is ± 0.01. The measured values of κ for different plateau-to-plateau transitions in each sample are shown in Table I. All the integer plateau-to-plateau transitions we studied are around Landau levels where spin splitting is already resolved at 1.2 K. Some high Landau level

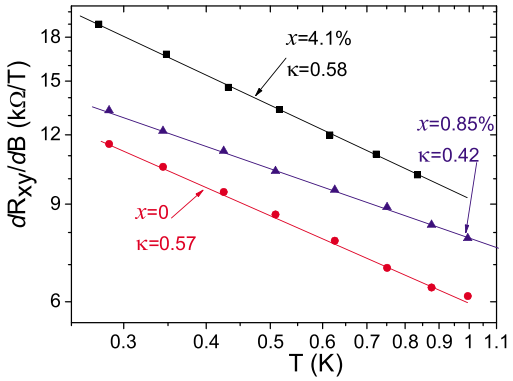

FIG. 2 (color online). $(dR_{xy}/dB)|_{B=B_c}$ vs T for the 4-3 transition. From down to up, $x = 0$, 0.85%, 4.1%, respectively. Data of different x has been shifted vertically in a log-log scale for a clear comparison. Scaling exponents κ are obtained from linear fits.

transitions (10-8, 12-10, 14-12) are observed spin unresolved at 1.2 K, but spin splitting occurs at about 0.5 K, therefore the spin-unresolved transitions are not studied. The transitions at lower Landau levels are also not studied in this paper because there are fractional quantum Hall states between those integer plateaus.

The dependence of the critical exponent κ on x is plotted in Fig. 3. Values of κ determine three regimes. In the first regime, when x is very small, κ is as large as 0.58 and decreases with increasing x. For the second regime x is between 0.65% and 1.6% and the alloy scattering rate is from 2.5 times to 6.5 times the background ionized impurity scattering rate. In this regime, κ is 0.42 for all samples. Finally, with x larger than 1.6%, the system is driven into the third regime and κ increases again with x. From the earlier characterization of scattering mechanisms we observe that, as shown by the large values of θ, in the second regime the disorder is dominated by the short-range alloy potential fluctuations. We have measured five pieces of samples from three different wafers grown in two different years in this regime. As is listed in Table I, all the results show consistently $\kappa = 0.42$ within the fitting error ± 0.01. Therefore we found that the exponent κ is sample and x independent only in the short-range disordered regime. Its value is the same as for the InGaAs-InP system [2] and it is consistent with theoretical calculations [3,4,13–17].

The universality observed in this second regime is further confirmed by our observation in a larger temperature range from 1 K down to 30 mK. Figure 4 shows the T dependence of ΔB and $(dR_{xy}/dB)|_{B=B_c}$ at the 4-3 transition of the sample with $x = 0.85\%$ in this T range. The data taken in the dilution refrigerator and the data taken from the ^{3}He system fall on top of each other where they overlap in temperature. Both the power law of ΔB vs T and that of $(dR_{xy}/dB)|_{B=B_c}$ vs T yield a critical exponent $\kappa = 0.42$. These power laws over nearly two decades of temperature confirm the scaling and define the exponent with a high precision.

The deviation of the exponent κ from the universal value 0.42 in the first and the third regimes shows that the nature of the transition is indeed affected by the nature of the disorder. The plateau-to-plateau transition is viewed as a localization-delocalization transition, while the physics of quantum localization [27] applies only within the range of the quantum phase coherence length, which is usually identified to be the inelastic scattering length l_{in} [9]. In the scaling theories, it is assured that the range of disorder is below the length scale of l_{in} by assuming the disorder to be an uncorrelated δ-function-like potential fluctuation [28]. However, for samples in the first regime where x is small, the disorder of the system is dominated by the potential of the ionized impurities. Being screened by the 2D electrons, the Coulomb potential fluctuation becomes slowly varying with a large correlation length of the order of μm [24]. With the disorder range comparable with or even larger than l_{in}, the quantum localization crosses over toward the classical percolation. In the second regime, where the disorder is dominated by the short-range potential fluctuations, the transport is quantum in nature and the universality of the plateau-to-plateau transition is restored. In the third regime, the likely clustering of Al atoms introduces correlations in the sample that may change the nature of the disorder destroying therefore the universal scaling.

In the theoretical calculations, the universal critical exponent $\nu = 7/3$ results from a network model of quantum percolation, where the quantum phase coherence is kept in the transport [13–17]. On the other hand, an exponent $\nu = 4/3$ was obtained with theories of classical percolation [16,29]. Using these values of ν and the $\kappa = p/2\nu$ relationship, we infer that the quantum-classical crossover

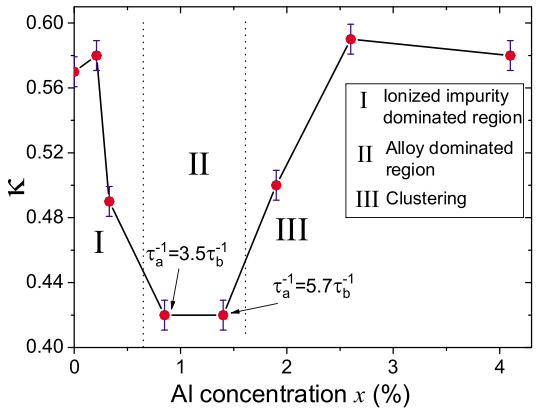

FIG. 3 (color online). Dependence of the exponent κ on the Al concentration x for the 4-3 transition. In the second regime, the alloy scattering rate τ_a^{-1} is from 2.5 times to 6.5 times the background long-range scattering rate τ_b^{-1}, and thus scattering is dominated by alloy disorder. In this regime the exponent κ is 0.42.

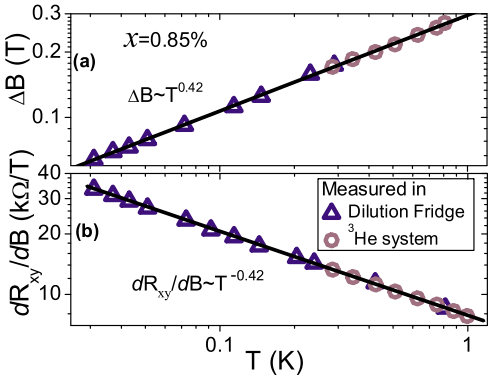

FIG. 4 (color online). Temperature scaling down to 30 mK of the 4-3 transition for the sample with $x = 0.85\%$. Data taken in the dilution fridge (up-triangles) and that from the ^{3}He system (circles) fall on the same straight line in the log-log plot. The slope of both curves in (a) and (b) give the critical exponent $\kappa = 0.42$ with a high precision.

effect increases the exponent κ from 0.42 up towards the classical value of 0.75. The κ values we obtained in the first and third regimes are still well below 0.75, showing that the system is still away from an ideal classical percolation regime.

There are other ways to explain the deviation of the exponent κ from the universal value in different samples by putting uncertainty on the temperature exponent p [30]. The exponent p was obtained in the Fermi liquid theory to be two and was confirmed by excitation current scaling experiments [11,12]. In samples with a high concentration of ionized impurities, it was proposed that the attractive Coulomb potential of the ions may be attributed to non-Born scattering and leads to a value of p that is larger than 2 [20,30,31]. However, in our experiments, the alloy potential as a neutral disorder does not give rise to inelastic scattering. At low temperatures, the inelastic scattering is mostly from the contribution of the electron-electron interaction [3,4,9], and the temperature exponent p should be all the same for the samples we studied. This suggests that the deviation of κ from 0.42 we observe is from a change of ν due to a fundamental crossover effect from quantum localization toward classical percolation.

Another approach to the universality of the plateau-to-plateau transition is from the hopping conductivity σ_{xx} [32] away from the critical magnetic field. In a recent experiment [33], information of the localization length is acquired by the T dependence of the hopping conductivity $\sigma_{xx} = \sigma_0 \exp[-(T_0/T)^{1/2}]$ assuming $\sigma_0 \propto 1/T$. However, the $1/T$ dependence of σ_0 was not fully confirmed by the experiments. The theory to obtain this T dependence of hopping conductivity demands the system to be short-range disordered as well, and we propose that the alloy systems would be useful to test this theory.

In conclusion, we have realized 2D electron systems where the dominant contribution of the disorder is from the short-range alloy potential fluctuations. In such systems, the disorder can be well modeled as an uncorrelated δ-function-like potential fluctuation and the physics of quantum localization prevails. We established the universality of the quantum Hall plateau-to-plateau transition in this regime and found a universal critical exponent $\kappa = 0.42 \pm 0.01$. With the nature of disorder being changed from short ranged to long ranged, a crossover effect from quantum localization toward classical percolation is observed accompanied by a substantial increase of the exponent κ.

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