# Dynamics of Periodic Pulse Collisions in a Strongly Dissipative-Dispersive System

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We report on the dynamics of the periodic collision process occurring between different pulse bound states in a stretched-pulse erbium-doped fiber laser. The acquisition of a large number of second-order correlation traces allows us to reconstruct the dynamics of a single collision event. The measurements clearly demonstrate that, unlike true solitons in the case of integrable systems, the pulses do not fully overlap in the course of a collision. Instead, the collision proceeds through the exchange of bonds between the individual pulses constituting the bound states.

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Since the early proposal by Hasegawa *et al.* [1] for the nonlinear propagation of self-maintaining pulses in optical fibers and its report by Mollenauer *et al.* [2], solitons have become major workhorses in many subfields of modern optics. As light beams and/or light pulses, many behaviors of nonlinear light propagation are well described with these ubiquitous entities [3]. For example, the generation of light pulses in passively mode-locked solid state lasers using the nonlinear refractive index of Kerr media (through either self-focusing or nonlinear rotation of the polarization ellipse) has been successfully described in terms of sech waveforms typical of solitons [4]. However, for some laser systems where the pulses suffer large losses (and gain) and encounters significant amounts of dispersion (positive and negative) upon propagation along the laser cavity, the nonlinear Schrödinger equation fails to describe several behaviors. Rather, the pulse formation and interaction inside these systems, under the strong influence of dissipative and dispersive mechanisms, may be understood within the formalism of a complex Ginzburg-Landau equation [5] and described by means of other waveforms known as dispersion-managed solitons [6] and self-similar pulses [7].

New phenomena, yet unknown of integrable systems, have been observed in passively mode-locked solid-state lasers where several pulses are emitted per cavity round trip due, in part, to the saturation of the nonlinear gain mechanism. For example, the coherent binding of closely spaced pulses in distinct multiplets has been reported by several groups using various laser materials and configurations [8,9]. Recently two groups have independently reported the observation of periodic collisions occurring between several of these bound states traveling with slightly different group velocities [10,11]. We then suggested that the different group velocities were due to the stimulated Raman scattering that is shifting differently the carrier frequencies of the colliding pulse states. However, details of the collision process remain unknown because of the finite resolution of the electronics with which the phenomenon was observed. Thus, whether the distinct light pulses fully overlap (i.e., pass through each other) or simply interact through their wings in the course of a collision remains an open question.

In this Letter, we report time-resolved measurements revealing the dynamics of the collisions taking place between a single-pulse and a two-pulse bound state in a stretched-pulse erbium-doped fiber ring laser. The measurements are made using an autocorrelator which makes it possible to monitor the interacting femtosecond pulses typically located a few picoseconds apart. The acquisition of a significant number of autocorrelation traces over a large number of collisions allows one to reconstruct the history of the collision process over a time span of a few tens of picoseconds. The observations are then compared to an expression introduced to describe the intensity autocorrelation of a nonstationary signal; the agreement between theory and experiment is convincing, allowing one to elucidate the nature of the collision process. As for solitons in integrable systems, the collisions proceed elastically (in the sense that the pulse number is conserved and that no energy is shed away by the interacting pulses). However, unlike integrable systems, the pulses do not fully overlap in the course of a collision; instead, the exchange of a bond takes place between the colliding pulses. This is due to the interaction between the trailing and leading edges of the stretched pulses which follows from the strongly dissipative-dispersive nature of the system.

Previously interaction forces among solitons in optical fibers have been described [12] and accordingly observed [13]. As well, some theoretical studies have described collisions occurring between interacting pulses in fiber lasers [14] and in the similar system of dispersion-managed optical communication lines [15]. However, there appears to be no report stating clear observational evidence of the dynamics underlying such a process. This is to be compared to the spatial domain where a wide variety of collision scenarios have already been reported [16]. The reason is that the ultrashort time scale typical of temporal solitons ($\sim 10^{-12}$ s) commands more intricate technical means and a procedure for the data analysis that is not as straightforward as for the observation of spatial phenomena. In addition, the collision process reported herein appears to have no analogue in the realm of spatial solitons, despite the richness of phenomena the latter encompasses.

Passively mode-locked fiber lasers have long been known for their inherent ability to emit several pulses per cavity round trip [8,17]. This is due to the high gain of doped fibers, deleterious pulse shaping mechanisms (e.g., higher-order dispersion) which often lead to pulse breakup and the pulse limiting response of the mode-locking mechanism at high power. Recently, it has been shown that the formation and annihilation of these multiple pulses is following a cascade of first-order phase transitions [18]. In this regime, for instance, the multiple pulses may arrange themselves in distinct phase-locked states of bound pulses following each phase transition; the bound states are, in fact, multiplets of a fundamental, single-pulse state with quantized energy [19–21]. Even more puzzling is the unique dynamics which characterizes the periodic collisions that may occur in a particular phase which involves two or more of these bound states, as revealed in the present study.

The stretched-pulse fiber ring laser cavity used for the experiment and its operation have been detailed elsewhere [10]. The average cavity dispersion is near zero, albeit slightly positive with $D = 0.004\ \mathrm{ps}^2$. Typical operation of this laser yielded about 80 fs chirp-compensated pulses at a pulse repetition rate of 28.7 MHz in the single-pulse regime. The energy per pulse was then about 300 pJ and the time-bandwidth product was close to 0.6. Increasing the pump power above a certain level resulted in the collision process shown in Fig. 1 which involves a two-pulse bound state with a pulse spacing of 2.1 ps (not resolved here because of the finite bandwidth of the photodiode) traveling faster than the single pulse. The two-pulse bound state and the single pulse keep colliding indefinitely (i.e., at least for a day) every 1.8 s.

In order to gain insight into the collision process, we performed second-order optical correlation measurements since no electronics could resolve features of the collision process over the subpicosecond scale. A rotating-arm delay autocorrelator with a resolution better than 0.1 ps and a quasilinear scan range of several tens of picoseconds was used. We performed the acquisition of a very large number of autocorrelation traces with the use of a digital oscilloscope. In fact, we did record $10^6$ traces, of which about 1000 displayed features we could attribute to an ongoing collision process. This is easily understood when one compares the 30 ps time span that was recorded from each autocorrelator scan while one complete cavity round trip was about $10^3$ times longer ($\approx 35$ ns). Bearing in mind that there was no synchronization between the trigger of the oscilloscope and the collision process, it was thus expected that about one trace over a thousand would yield information about the collision process. Then we had to sort these traces according to the time $\tau_C$ elapsed between the trigger of the oscilloscope (which was set on the central peak of the trace) and the collision itself (i.e., the instant at which the three pulses are equidistant). The result of this ordering process is shown in Fig. 2. We emphasize that Fig. 2 does not represent, as such, a real-time view of the collision process; the underlying dynamics has to be unfolded, as is discussed later.

The surface plot shown in Fig. 2 features about 1000 autocorrelation traces of the nonstationary laser signal in

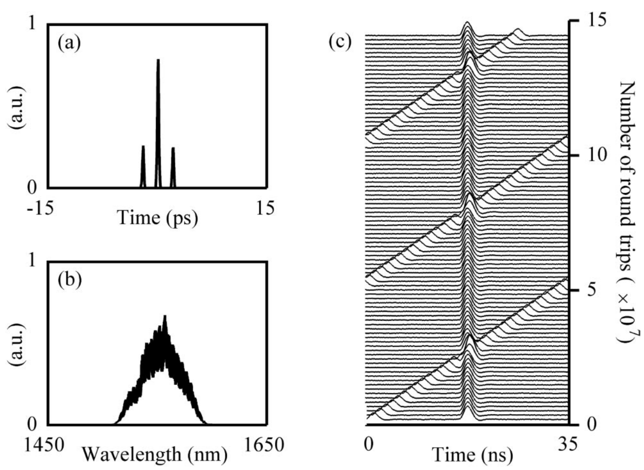


FIG. 1. A single pulse colliding periodically with a two-pulse bound state: (a) the observed autocorrelation trace, (b) the optical spectrum, and (c) the signal measured with a fast photodiode as a function of the cavity round trip number (note that one cavity round trip is about 35 ns long and that the oscilloscope trigger was set on the central peak which represents the unresolved two-pulse bound state).

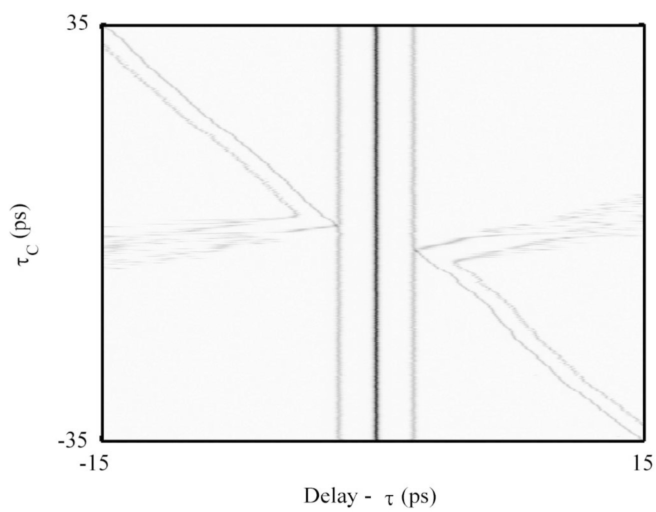


FIG. 2. Autocorrelation traces sorted according to the time elapsed between the oscilloscope trigger and the pulse collision. The oscilloscope trigger was set on the central peak (that is, for a delay $\tau = 0$).

the course of the collision process. This is why all the traces that are displayed are lacking the usual time-reversal symmetry which characterizes second-order optical correlation measurements taken from stationary signals. The correlation traces in Fig. 2 are sorted in chronological order; that is to say, the collision occurs later than the oscilloscope trigger in the first half traces ($\tau_C < 0$) while the reverse holds for the second half ($\tau_C > 0$). As well, the collision nearly coincides with the oscilloscope trigger in the case of the middle traces (i.e., $\tau_C \approx 0$). The central vertical line in Fig. 2 represents the sum of the autocorrelations of each pulse with itself while the parallel lines on each of its sides results from the cross correlation between the two pulses in the two-pulse bound state. The other lines appearing in the surface plot follow from the cross correlation of the single pulse with the two-pulse bound state.

The preceding remarks can all be summed up in the following expression for the second-order intensity correlation of the nonstationary signal with itself (details regarding its derivation will be published elsewhere):

$$\Gamma(\tau, \tau_C) = \int_{-\infty}^{+\infty} I(t, \tau + \tau_C) I(t - \tau, \tau + \tau_C) dt, \qquad (1)$$

where the explicit dependence of the pulse train distribution $I(t, \tau + \tau_C)$ upon the time delay $\tau$ of the autocorrelator has been included as a separate independent variable and where the time $\tau_C$ which adds up to the time delay effectively displaces the time origin of the collision process. This explicit dependence translates the fact that the signal distribution has at least one of its components whose position relative to the others is delayed with respect to $\tau$. The nonstationary component may be described according to a delay function $\Delta(\tau + \tau_C)$, i.e., $I_{\mathrm{ns}}(t, \tau + \tau_C) = I_{\mathrm{ns}}(t - \Delta(\tau + \tau_C))$ whose precise form determines the position of the peaks in the correlation traces $\Gamma(\tau, \tau_C)$ arising from the cross correlation of the nonstationary component with the stationary ones. For instance, in Fig. 2, the near 45° sharper lines display a spacing smaller than the central ones while the wide-angled (nearly horizontal) more diffuse lines are farther apart from one another. The latter is due to the concurrent delay of the retarding pulse (with respect to the two-pulse bound state) relative to the scanning autocorrelator while the former is explained by a countercurrent delay between both of them. The same holds for the width of the peaks.

Assuming the collision dynamics between the two-pulse bound state and the single pulse to behave as the one illustrated in the inset of Fig. 3, one can now compute the resulting autocorrelation traces following Eq. (1). Very few experimental parameters are needed in order for the theory to reproduce the measurement conditions. Indeed, the relative speed between both pulse states of the nonstationary laser signal (i.e., $\approx 20$ ns/s) with respect to the scanning speed of the autocorrelator (i.e., 30 ns/s) fixes the details of the measurement portrayed in Fig. 2 [i.e., $\Delta(\tau + \tau_C) = \kappa(\tau + \tau_C)$, with $\kappa \approx 2/3$]. The similarity between the measured traces (Fig. 2) and the calculated ones (see Fig. 3) is striking. However, since our calculation did not account for the fluctuations in the rate at which the collision repeated (the latter being directly linked to the pulse spacing in the two-pulse bound state [10]), the near-horizontal lines in the surface plot illustrated in Fig. 3 appears straight and sharper by comparison with the actual measurement (Fig. 2). Nonetheless, we conclude that the actual pulse dynamics during a collision is following closely the dynamics of an elastic collision as depicted in Fig. 3. If the colliding pulses were to cross each other (see inset of Fig. 4), the experiment discussed above would have resulted in the traces shown in Fig. 4 instead of the ones in Fig. 3; for instance, the lines that result from the

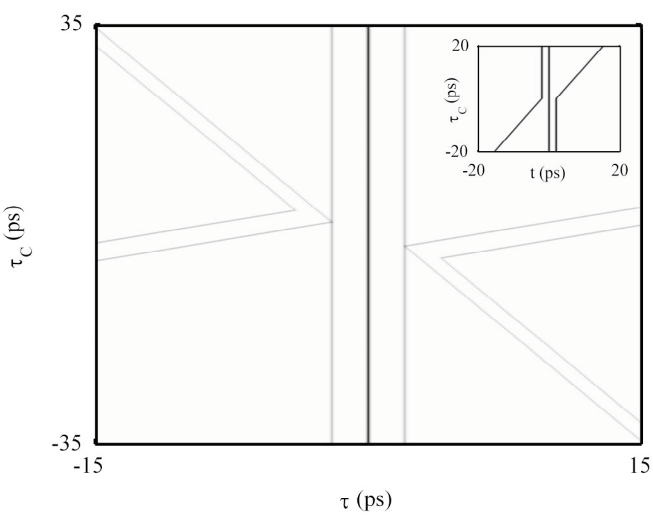


FIG. 3. Reconstruction of the measurement illustrated in Fig. 2 assuming the elastic collision dynamics illustrated in the inset (as seen in the reference frame moving with the central pulse).

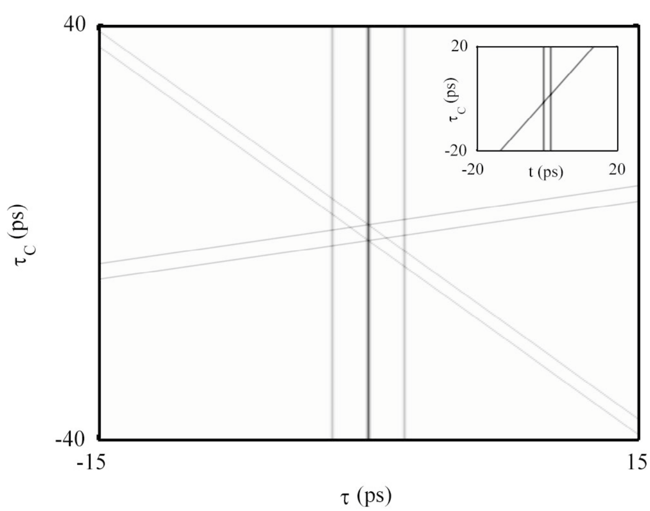


FIG. 4. Simulation of a measurement similar to the one illustrated in Fig. 2 assuming the pulses to fully overlap during the collision as shown in the inset (to be compared with Fig. 3).

cross correlation of the single pulse with the two-pulse bound state would not have been broken and shifted in the central region, in contradiction with the measurements shown in Fig. 2.

At first sight, it might appear counterintuitive that the envelopes of the colliding pulses do not fully overlap in the collision process (that is, that the pulses do not cross each other), but rather interact (attract or repel) with each other through their tails [21]. In fact, the opposite would encounter several difficulties. First and foremost, the inherent incapacity of the laser cavity to support pulses with twice the energy is fundamental to the breakup in multiple pulses. Indeed, one should think of the excessive nonlinear effects such a pulse would undergo as deleterious mechanisms which inhibit the complete overlap of the two colliding pulses. In addition, the pulse width is stretched by a factor of 10–20 twice per cavity round trip due to the dispersion management performed in the laser cavity. This is certainly having an effect on closely spaced pulses since their envelopes will extend farther from each pulse in comparison to locations where the pulse duration is minimum.

The results reported herein are believed to have no precedent in the literature covering the subject of nonlinear optical wave propagation. The performed time-resolved measurement of the collision process between ultrashort pulses of light has revealed the unique dynamics which characterizes dissipative-dispersive systems as the laser described above. Soliton interactions, in the realm of integrable systems, have long been known to have a great influence on their stability. However, the latter does not appear to hold anymore in the case of dissipative systems as the endlessly repeating pulse collisions seem to show in the study reported here.

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[1] A. Hasegawa and F. Tappert, Appl. Phys. Lett. **23**, 142 (1973).
[2] L. F. Mollenauer, R. H. Stolen, and J. P. Gordon, Phys. Rev. Lett. **45**, 1095 (1980).
[3] N. N. Akhmediev and A. Ankiewicz, *Solitons: Nonlinear Pulses and Beams* (Chapman & Hall, London, 1997).
[4] H. A. Haus, IEEE J. Sel. Top. Quantum Electron. **6**, 1173 (2000), and references therein.
[5] N. N. Akhmediev, A. Ankiewicz, and J. M. Soto-Crespo, J. Opt. Soc. Am. B **15**, 515 (1998).
[6] K. Tamura *et al.*, Opt. Lett. **18**, 1080 (1993).
[7] F. Ö. Ilday *et al.*, Phys. Rev. Lett. **92**, 213902 (2004); M. E. Fermann *et al.*, Phys. Rev. Lett. **84**, 6010 (2000).
[8] M. J. Guy, D. U. Noske, and J. R. Taylor, Opt. Lett. **18**, 1447 (1993).
[9] C. Spielmann *et al.*, IEEE J. Quantum Electron. **30**, 1100 (1994); C.-Y. Wang *et al.*, Opt. Commun. **137**, 89 (1997); B. Chassagne *et al.*, Opt. Commun. **150**, 355 (1998).
[10] M. Olivier *et al.*, Opt. Lett. **29**, 1461 (2004).
[11] Ph. Grelu and N. Akhmediev, Opt. Express **12**, 3184 (2004).
[12] J. P. Gordon, Opt. Lett. **8**, 596 (1983).
[13] F. M. Mitschke and L. F. Mollenauer, Opt. Lett. **12**, 355 (1987).
[14] Y. Kodama, M. Romagnoli, and S. Wabnitz, Electron. Lett. **28**, 1981 (1992).
[15] Y. Chen and H. A. Haus, Opt. Lett. **24**, 217 (1999).
[16] G. I. Stegeman and M. Segev, Science **286**, 1518 (1999).
[17] D. J. Richardson *et al.*, Electron. Lett. **27**, 1451 (1991).
[18] B. Vodonos *et al.*, Phys. Rev. Lett. **93**, 153901 (2004); A. Gordon and B. Fischer, Phys. Rev. Lett. **89**, 103901 (2002).
[19] D. Y. Tang *et al.*, Phys. Rev. A **64**, 033814 (2001).
[20] Ph. Grelu *et al.*, Opt. Lett. **27**, 966 (2002); Ph. Grelu *et al.*, J. Opt. Soc. Am. B **20**, 863 (2003); J. M. Soto-Crespo *et al.*, Opt. Lett. **28**, 1757 (2003).
[21] Ph. Grelu, J. Béal, and J. M. Soto-Crespo, Opt. Express **11**, 2238 (2003).