Application of Radiative Electron Capture for the Diagnostics of Spin-Polarized Ion Beams at Storage Rings

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It is proposed to apply the radiative electron capture into high-Z projectiles as a *probe* process for measuring the spin polarization of the hydrogenlike ions at storage rings. We argue that such polarization measurements are possible since the linear polarization of emitted x-ray photons is greatly sensitive to the spin states of incoming ions with nuclear spin $I > 1/2$. In particular, for K-shell electron capture into the hydrogenlike ions, the linear polarization of light as measured out of the reaction plane is found to be proportional to the degree of ion polarization. Detailed computations for the dependence of the photon polarization on the ion spin states and projectile energies are carried out for the electron recombination into hydrogenlike Bi^{82+} ions.

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During the last two decades, experiments with spin-polarized electrons and protons have stimulated considerably many areas in basic research and applications. In nuclear physics, for instance, the use of spin-polarized particles in nuclear reactions had a profound impact on the design of new storage rings. While for protons and light ions, however, there are various techniques known to obtain particle beams with a high degree of polarization, no polarized beams with heavy nuclei are yet available. For high-Z ions, intensive and polarized beams are anticipated at the new GSI heavy-ion facility which is currently under construction. In the future, the operational access and control of such beams is needed for investigating parity nonconservation effects in high-Z ions [1], the existence of a permanent electric dipole moment [2], or for testing the standard model [3].

In practice, however, there are two key issues which need first to be solved before collision experiments with polarized high-Z ions will become accessible. Apart from the generation of polarized ion beams, an efficient control or measurement of the beam polarization is required. A possible scheme for producing polarized ions has been discussed recently by Prozorov and co-workers [4], who proposed an optical pumping of the hyperfine levels of hydrogenlike ions in order to obtain the predominant population of some particular hyperfine state $|FM_F\rangle$. Since, for a nonzero nuclear spin, this hyperfine state results from the coupling of the electron in the (one-particle) state $|j_b \mu_b\rangle$ and the nucleus,

$$|FM_F\rangle = \sum_{\mu_b M_I} \langle IM_I, j_b\mu_b|FM_F\rangle |j_b\mu_b\rangle |IM_I\rangle, \qquad (1)$$

it requires that both the electron as well as the atomic nucleus be polarized [4].

The spin polarization of heavy ions at storage rings by optical pumping or any another technique is of little help for future studies if the degree of polarization cannot be controlled experimentally. In this contribution, therefore, we investigate which physical process is sensitive enough to the spin state of the high-Z projectiles and is sufficiently simple to be measured. From our theoretical analysis, we suggest to use the *radiative capture* of a free (or quasifree) electron by the projectiles. This radiative electron capture (REC) of the ions, which is a time-reversed photoeffect and hence is accompanied by the emission of photons, is *efficient* as it is the dominant process in relativistic collisions of high-Z projectiles with low-Z target atoms. In the past, the REC of highly charged ions has been explored in great detail in a number of experiments [5–7]. Making use of recent advances in the design of x-ray detection techniques, a new position-sensitive germanium detector was developed and applied successfully to measure the linear polarization of the recombination photons [8]. When compared with theory, such polarization measurements are quantitatively well described by means of the density matrix formalism, based on Dirac's relativistic equation [9–11]. As recently shown, the linear polarization of the emitted photons is strongly influenced by the spin polarization of the (incident) electrons [12]. Moreover, since the electrons and ions occur rather symmetrically in the recombination theory, a similar effect is expected also for the capture of electrons by spin-polarized projectiles. Following the recent proposal by Prozorov and co-workers [4], a detailed analysis of the electron capture by spin-polarized ions has been carried out and is presented, in particular, for the capture into *hydrogenlike* ions.

To study the polarization properties of the incident and/or emitted particles in ion-atom collisions, the density matrix approach was found appropriate at many places before in order to "accompany" the ensemble through the various regions of interaction in the course of the collision [9,12,13]. Below, therefore, we will restrict ourselves to a rather short account of the theory and recall only a few basic relations. From these relations, we will then derive how the polarization of emitted light is affected by the spin polarization of the incident ion beam. Computations have been performed for the electron capture into the ground state of polarized (hydrogenlike) bismuth Bi^{82+} projectile ions, and are discussed below to illustrate that the *polarization transfer is a valuable tool for determining the spin polarization of the ion beam.*

Experiments on the linear polarization of hard x-ray radiation are no longer unfeasible. During the last few years, for example, a series of measurements were first carried out at the GSI storage ring on the photon polarization of the emitted x-ray photons, following the capture of an electron into the K shell of bare uranium [8]. These measurements became possible, in particular, owing to the design of new position-sensitive germanium detectors which help determine not only the degree but also the direction of the photon polarization. As usual, here the polarization of the recombination photons is described most conveniently in terms of the *Stokes parameters*, i.e., the intensity I_χ (ratios) of the light, linearly polarized at different angles χ with respect to the reaction plane (as formed by the directions of the incident ion beam and emitted photons). While the first Stokes parameter

$$P_1 = \frac{I_0 - I_{90}}{I_0 + I_{90}} \tag{2}$$

is derived from the intensities of light, polarized *in parallel* and *perpendicular* to the reaction plane, the parameter P_2 follows from a similar ratio, taken at $\chi = 45°$ and $\chi = 135°$, respectively:

$$P_2 = \frac{I_{45} - I_{135}}{I_{45} + I_{135}}. \tag{3}$$

The Stokes parameters are also directly related to the photon spin-density matrix of the recombination light by [13,14]

$$\langle \mathbf{k}\lambda|\hat{\rho}_\gamma|\mathbf{k}\lambda'\rangle = \frac{1}{2}\begin{pmatrix} 1 + P_3 & P_1 - iP_2 \\ P_1 + iP_2 & 1 - P_3 \end{pmatrix}, \tag{4}$$

where $\mathbf{k}$ denotes the wave vector and $\lambda = \pm 1$ the helicity of the recombination photons, that is, their spin projection onto the direction of propagation. A third Stokes parameter P_3 finally reflects the degree of the circular polarization of the light.

As seen from Eq. (4), the Stokes parameters can be expressed in terms of the (matrix) elements of the photon spin-density matrix. For the radiative capture of a free unpolarized electron with asymptotic momentum $\mathbf{p}$ and spin projections $m_s = \pm 1/2$ into the hyperfine bound state $|F'M_F'\rangle$ of the (subsequently heliumlike) projectile, these matrix elements are obtained by standard techniques [15],

$$\langle \mathbf{k}\lambda|\hat{\rho}_\gamma|\mathbf{k}\lambda'\rangle = \frac{1}{2}\sum_{M_F m_s}\sum_{F'M_F'} M_{\mathbf{p}}^*(m_s, M_F; \lambda, F', M_F') \times M_{\mathbf{p}}(m_s, M_F; \lambda', F', M_F') n_{FM_F}. \tag{5}$$

They show that the spin state of the emitted photons depend on both the amplitudes $M_{\mathbf{p}}(m_s, M_F; \lambda, F', M_F')$ for the capture of the electron *as well as* the (relative) population n_{FM_F} of the hyperfine sublevels $|FM_F\rangle$ of the initially hydrogenlike ions (1). In the computation of the transition amplitudes, Dirac's equation has been utilized together with the minimal coupling of the radiation field. Moreover, the two-electron wave functions in these amplitudes were described in an independent particle approximation, using screened hydrogenic functions and a proper set of Slater determinants.

By inserting the spin-density matrix (5) into Eq. (4), we are able to express the Stokes parameters for the recombination photons in terms of the (reduced) transition matrix elements. For the sake of brevity, let us omit here the details of this derivation and note just that it follows similar lines as for the capture of spin-polarized electrons by bare ions [12]. For the capture by hydrogenlike ions, the Stokes parameter P_1 remains (again) unaffected by the spin polarization of the projectiles but merely depends on the nuclear charge Z, the projectile energy T_p, and the angle θ:

$$P_1 = f_1(Z, T_p; \theta), \tag{6}$$

under which the recombination photons are observed with respect to the ions beam.

The second Stokes parameter P_2, in contrast, is proportional to the degree of the ion polarization,

$$P_2 \sim \lambda_F = \frac{1}{F}\sum_{M_F} n_{FM_F} M_F, \tag{7}$$

and thus, due to Eq. (1), depends also on the nuclear spin I. For the capture into 1 s ground state of the (initially) hydrogenlike ions, for instance, the Stokes parameter P_2 can be written as

$$P_2 = \lambda_F \frac{I - 1/2}{I + 1/2} f_2(Z, T_p; \theta), \tag{8}$$

where the function $f_2(Z, T_p; \theta)$ is independent of both the polarization of the ions and the nuclear spin. As seen from Eq. (8), therefore, the parameter P_2 is affected by the ion polarization for all nuclei with spin $I > 1/2$.

Figure 1 displays the behavior of the Stokes parameters (6) and (8) as a function of the angle θ for the electron capture into hydrogenlike bismuth Bi^{82+} ($I = 9/2$) ions. For *unpolarized* ions ($\lambda_F = 0$), P_2 is identically zero while the parameter P_1 is nonzero and positive. When compared with the definitions (2) and (3) of the Stokes parameters,

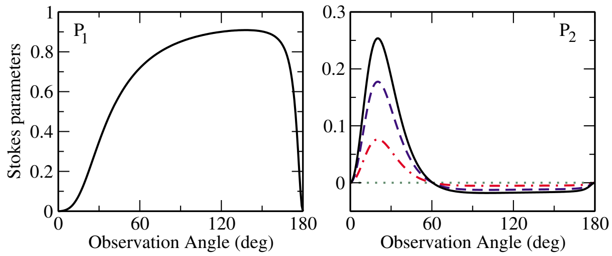

FIG. 1 (color online). The Stokes parameters P_1 and P_2 of the recombination photons which are emitted in the electron capture into the K shell of hydrogenlike bismuth ions with projectile energy $T_p = 420$ MeV/u. The Stokes parameter P_2 is shown for four degrees of ion polarization: $\lambda_F = 1$ (solid line), $\lambda_F = 0.7$ (dashed line), $\lambda_F = 0.3$ (dash-dotted line), and $\lambda_F = 0.0$ (dotted line). The Stokes parameter P_1 (solid line) is independent of the ion polarization. Calculations are presented in the laboratory frame (i.e., rest frame of the electron target).

this implies that for unpolarized ions the (linear) polarization of the x-ray emission always lies *within* the reaction plane. The second Stokes parameter P_2 becomes nonzero only if the ions and/or the electrons were *polarized* before; in contrast, the parameter P_1 is not influenced by the polarization properties of the particles involved in the recombination. A nonzero P_2 parameter, however, leads to an overall *rotation* of the linear polarization (of the recombination photons) out of the reaction plane and hence can be used as a signature for the polarization of the ion beam. In the experiment, such a rotation of the linear polarization of the photons is visualized most naturally by means of a *polarization ellipse* in place of the explicit use of the two Stokes parameters.

In the theory of light, the polarization ellipse is defined in the plane perpendicular to the photon momentum **k** and is characterized by the relative length $P_L = \sqrt{P_1^2 + P_2^2}$ of the principal axis as well as the angle χ_0 with respect to the reaction plane [cf. Fig. 2]. When expressed in terms of the Stokes parameters, this angle is given by the ratio [13]

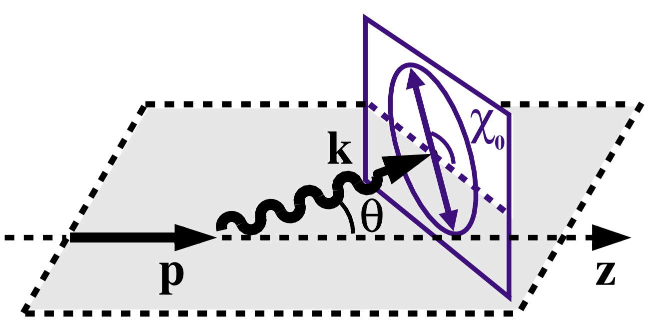

FIG. 2 (color online). Definition of the polarization ellipse; its principal axis is characterized by χ_0, the angle with respect to the reaction plane which is formed by the directions of incoming ion beam and emitted photons.

$$\tan 2\chi_0 = \frac{P_2}{P_1}, \tag{9}$$

and thus can be used as a single parameter for analyzing the polarization of the emitted light. Making use of the Stokes parameters (6) and (8) in expression (10), this angle becomes

$$\tan 2\chi_0 = \lambda_F \frac{I - 1/2}{I + 1/2} \mathcal{R}(Z, T_p; \theta), \tag{10}$$

where the abbreviation $\mathcal{R}(Z, T_p; \theta) = f_2(Z, T_p; \theta)/f_1(Z, T_p; \theta)$ is introduced. As seen from Eq. (10), the capture of (unpolarized) electrons by unpolarized hydrogenlike ions, $\lambda_F = 0$, always leads to an emission of light which is polarized either within or perpendicular to the reaction plane ($\chi_0 = 0°$ or $\chi_0 = 90°$), while any contribution from a nonzero λ_F parameter will rotate the polarization ellipse ($\chi_0 \neq 0°$ and $\chi_0 \neq 90°$) out of the reaction plane. The measurement of the rotation angle χ_0 therefore provides direct access to the degree λ_F of the polarization of the incoming ions without a further analysis of the Stokes parameters or the shape of polarization ellipse.

To measure the polarization (ellipse) of the recombination photons, of course, enough intensity and a sufficiently large degree of linear polarization P_L are required experimentally. In Fig. 3, therefore, the angular dependence of the rotation angle χ_0 (top row) is shown together with the Stokes parameters P_1 (solid line) and P_2 (dashed line) of the recombination light (middle row) and the photon angular distributions (bottom row). In Fig. 3, a capture of unpolarized electrons into completely polarized

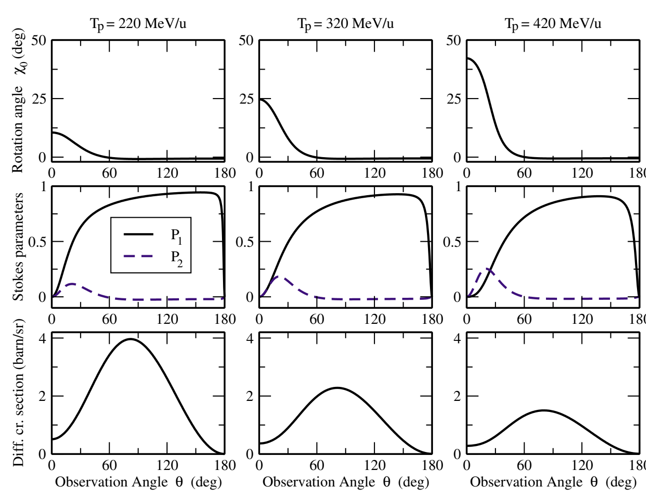

FIG. 3 (color online). Rotation angle χ_0 of the polarization ellipse (top row) and the Stokes parameters (middle row) of the emitted photons following the capture of unpolarized electrons into the K shell of hydrogenlike bismuth ions. In the bottom row, the REC angle-differential cross sections are shown. Calculations are presented for completely polarized projectile ions ($\lambda_F = 1$) and in the laboratory frame.

($\lambda_F = 1$) bismuth ions has been supposed for the three projectile energies $T_p = 220$, 320, and 420 MeV/u, respectively. At such energies, the angle χ_0 is large for a forward emission of the recombination photons so that a rotation of the polarization ellipse is most easily observed for emission angles $\theta < 60°$. Note, however, that χ_0 is not defined at $\theta = 0°$ since there is no reaction plane at this angle and hence no linear polarization for an emission of photons in parallel (or antiparallel) to the beam direction (cf. middle row of Fig. 3). In the forward direction ($\theta < 5°$), moreover, the degree of linear polarization can hardly be measured due to the low photon intensity. At larger angles of, say, $10° < \theta < 60°$, however, the (degree) of linear polarization as well as the photon emission become large enough for experiments and preferable for first investigations on the polarization of ion beams.

In conclusion, the linear polarization of the emitted photons has been studied for the radiative capture of electrons into high-Z, hydrogenlike ions. For this capture process, emphasis was placed especially on the question of how the polarization of the emitted photons is affected by the *spin polarization* of the projectile ions. It is shown that for hydrogenlike ions with nuclear spin $I > 1/2$ the spin polarization of the ion beam gives rise to a linear polarization of the photons *out of* the reaction plane, while for the capture into unpolarized ions in contrast the photon polarization is always either *within* or *perpendicular* to this plane. Moreover, the angle of this *out-of-plane* polarization of the photons is uniquely defined by the degree of polarization of the incoming ions. Therefore the measurement of the rotation angle may serve as a tool for determining the polarization properties of the ion beam. Calculations on this polarization transfer have been carried out for the electron capture into the 1 s ground state of (the initially hydrogenlike) Bi^{82+} projectile ions, by using the independent particle approximation. Although such an approach, in which the initial and final two-electron states are described both by a single Slater determinant, seems to be justified for fast collisions [7], interelectronic (or correlation) effects may become important for decelerated ions. These effects may slightly modify the transition amplitudes for the capture of electron but keep the linear dependence (10) of the rotation angle on the polarization of the ion beam untouched.

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