

Constraining Nuclear Equations of State Using Gravitational Waves from Hypermassive Neutron Stars

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(Received 9 February 2005; published 25 May 2005)

Latest general relativistic simulations for the merger of binary neutron stars with realistic equations of states (EOSs) show that a hypermassive neutron star of an ellipsoidal figure is formed after the merger if the total mass is smaller than a threshold value which depends on the EOSs. The effective amplitude of quasiperiodic gravitational waves from such hypermassive neutron stars is $\sim 6\text{--}7 \times 10^{-21}$ at a distance of 50 Mpc, which may be large enough for detection by advanced laser interferometric gravitational wave detectors although the frequency is high, ~ 3 kHz. We point out that the detection of such signals may lead to constraining the EOSs for neutron stars.

DOI: 10.1103/PhysRevLett.94.201101

PACS numbers: 04.25.Dm, 04.30.-w, 04.40.Dg

Introduction.—Binary neutron stars [1,2] inspiral as a result of the radiation reaction of gravitational waves, and eventually merge. The most optimistic scenario based mainly on a recent discovery of binary system PSRJ0737-3039 [3] suggests that such mergers may occur approximately once per year within a distance of ~ 50 Mpc [4]. Even the most conservative scenario predicts an event rate approximately once per year within a distance of ~ 100 Mpc [4]. This indicates that the detection rate of gravitational waves by advanced laser interferometric detectors such as the advanced Laser Interferometer Gravitational Wave Observatory (LIGO) will be $\sim 40\text{--}600$ yr $^{-1}$ [4], and hence, the merger of binary neutron stars is one of the most promising sources for them [5]. Since the detection rate is likely to be very high, a detailed study for binary neutron stars and for the property of neutron stars will be possible by the data analysis of gravitational waves.

Gravitational waves in the coalescing binaries will be primarily detected for the inspiraling phase in which two stars adiabatically approach due to gravitational wave emission. In this phase, the emission time scale of gravitational waves is much longer than the orbital period, and hence, the frequency and amplitude of gravitational waves increase in the time scale much longer than the orbital period, resulting in the emission of the so-called chirp signal. The theoretical templates for the chirp signal have been computed in the post Newtonian theory with a very high accuracy [6]. They will be used for the matched filtering technique in the data analysis for the detection and possibly for determining the mass and the spin of binary components [7]. In the following argument, we assume that mass of two neutron stars will be determined from the chirp signal since we consider the events of the distance smaller than ~ 100 Mpc and, hence, the signal to noise ratio is likely to be high enough for the determination.

When the orbital separation decreases to $\sim 3R$ where R denotes the radius of neutron stars $\sim 10\text{--}15$ km, the merger

will set in [8,9]. At the onset of the merger, the nature of the gravitational waveforms changes from the chirp-type signal to the burst-type one. The frequency of gravitational waves at the transition is likely to be $f_{\text{tran}} \sim [GM/(3R)^3]^{1/2}/\pi \approx 0.9 \pm 0.3$ kHz for $M = 2.8M_{\odot}$. Here, the precise value of f_{tran} depends sensitively on the radius of neutron stars. Thus, if it is determined by the detection of gravitational waves, the equations of states (EOSs) for neutron stars may be constrained [10]. This stimulates detailed numerical simulations at the transition phase in post Newtonian gravity (e.g., [11,12]).

As mentioned above, the gravitational waveforms in the merger phase are likely to depend sensitively on the intrinsic property of neutron stars such as the mass, the radius, and the EOSs. In this Letter, we focus on gravitational waves from hypermassive neutron stars (HMNSs) which will be formed after the merger of relatively small total mass, and propose a method for constraining the EOSs using such gravitational waves. To explain the method in the following, we assume that the mass ratio is close to unity since observed binary neutron stars for which the mass is determined accurately have such mass ratios [2] (cf. Table I).

After the merger sets in, the massive merged object collapses to a black hole or settles down to a HMNS depending mainly on the total mass of binaries [13,14]. Here, we note that the “black hole formation” is referred to as the case in which a black hole is formed promptly

TABLE I. The total mass in unit of the solar mass and mass ratio of observed binary neutron stars for which each mass is determined accurately. The data are quoted from [2].

PSR	M	Mass ratio
B1913 + 16	2.828	0.963
B1534 + 12	2.678	0.991
B2127 + 11C	2.71	0.99
J0737-3039	2.59	0.94

after the onset of the merger. The HMNS is defined as a differentially rotating neutron star for which the total baryon rest mass is larger than the maximum allowed value of rigidly rotating neutron stars for a given EOS [15].

To theoretically clarify the outcome, fully general relativistic simulation is the unique approach. Over the last few years, numerical methods for solving coupled equations of the Einstein and hydrodynamic equations have been developed for this issue. Now such simulations are feasible with an accuracy high enough for yielding scientific results (e.g., [13]) that can be used for comparison with observational data.

An important finding in the latest general relativistic simulations with realistic EOSs [14] is that the threshold mass for the prompt black hole formation, M_{thr} , depends sensitively on the EOSs. In [14], the SLy and FPS EOSs [16,17] are used with a correction for the thermal pressure that plays a role in the merger in which shocks are generated (see [14] for detail). In these EOSs, the values of M_{thr} are $\sim 2.7M_{\odot}$ and $\sim 2.5M_{\odot}$ for SLy and FPS EOS, respectively. Here, we should note that the maximum mass for spherical neutron stars in the SLy and FPS EOSs are $M_{\text{max:sph}} \approx 2.04M_{\odot}$ and $1.80M_{\odot}$, respectively. Thus, M_{thr} is much larger than $M_{\text{max:sph}}$. This is due to the fact that the merged object has a large angular momentum which results from the orbital angular momentum, and hence, the large centrifugal force becomes available for sustaining the large self-gravity to yield a HMNS [15].

Also interesting is that the theoretical value of M_{thr} for the realistic EOSs is close to the mass of observed binary systems in nature [2] (cf. Table I). This suggests that the value of M_{thr} may be determined from certain observational results, and can be used for constraining the EOSs. Thus, in this Letter, we propose a method for constraining M_{thr} using the signal of gravitational waves emitted from HMNSs formed after the merger of binary neutron stars.

Method.—For binaries of mass larger than M_{thr} , most of the fluid elements with more than 99% of the total mass collapse to a black hole directly in the merger [14]. In such cases, gravitational waves associated with the quasinormal mode ringing of the formed black hole will be emitted. The latest general relativistic simulations [13,14] have shown that the nondimensional spin parameter $a \equiv cJ/GM^2$ for the formed black hole is ~ 0.7 – 0.8 . Here M , J , G , and c denote the mass and angular momentum of the black hole, the gravitational constant, and the speed of light, respectively. This suggests that the frequency of gravitational waves will be very high as 6.5 – $7(2.8M_{\odot}/M)$ kHz [18]. This value is far out of the best sensitive frequency range of the laser interferometric gravitational wave detectors [5]. This implies that the Fourier spectrum of gravitational waves will not have any peak for the frequency between 1 and ~ 6 kHz for the case of prompt black hole formation.

A HMNS is formed after the merger temporarily for $M < M_{\text{thr}}$. General relativistic simulations with the SLy

and FPS EOSs have clarified [14] that the HMNS has a highly ellipsoidal shape (see Fig. 1). Such ellipsoidal shape results from the fact that the HMNS is rapidly rotating and the adiabatic index of the EOSs is very large (≥ 2.5) [16,17].

Because of the high ellipticity, the HMNS becomes a strong emitter of quasiperiodic gravitational waves. In Fig. 2, we display the typical gravitational waveforms during the merger for the SLy EOS. In this example, two stars of the binary are identical and the mass of each is $1.3M_{\odot}$. The simulation was performed with an initial condition of a quasiequilibrium circular orbit for which the orbital frequency is slightly smaller than that for the innermost stable circular orbit [9] and the orbital period is ~ 2 ms. In the early stage of the simulation, the binary is in a quasistable circular orbit. Reflecting this fact, the chirp signal is seen for $t_{\text{ret}} \lesssim 2$ ms. The merger begins after one orbit: The signal for $t_{\text{ret}} \gtrsim 2$ ms denotes gravitational waves emitted from the ellipsoidal HMNS [14]. Figure 2 shows that waves are quasiperiodic with the characteristic frequency ~ 3 kHz and amplitude $\sim 1.5 \times 10^{-22}$ which do not vary in a short time scale. This reflects the fact that the ellipsoidal figure is preserved for a long time scale $\gg 10$ ms.

Since gravitational waves are emitted in a quasiperiodic manner, the signal of approximately identical frequency can be accumulated by a large factor. Thus, the effective amplitude $h_{\text{eff}}(f)$ can be much larger than 10^{-22} at a peak

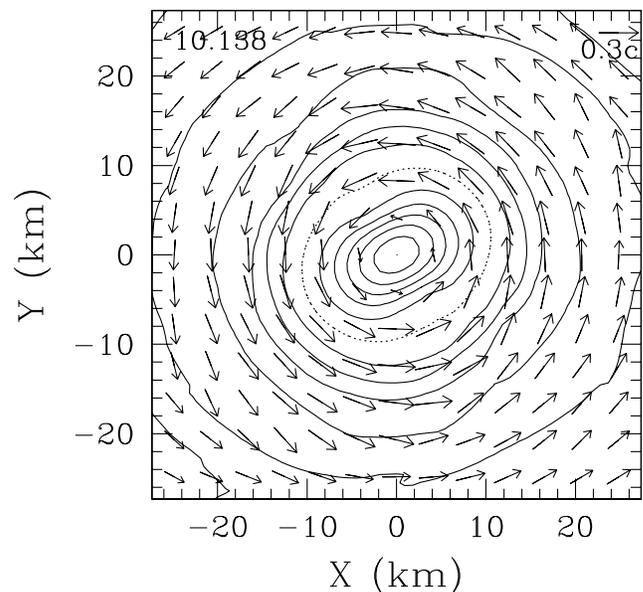


FIG. 1. The density contour curves for ρ in the equatorial plane at $t = 10.138$ ms. (The initial condition is set at $t = 0$ and the merger sets in at $t \sim 2$ ms.) The solid contour curves are drawn for $\rho = 2 \times 10^{14} \times i \text{ g/cm}^3$ ($i = 2 \sim 10$) and for $2 \times 10^{14} \times 10^{-0.5i} \text{ g/cm}^3$ ($i = 1 \sim 7$). The dotted curves denote $2 \times 10^{14} \text{ g/cm}^3$. Vectors indicate the local velocity field (v^x, v^y) , and the scale is shown in the upper right-hand corner.

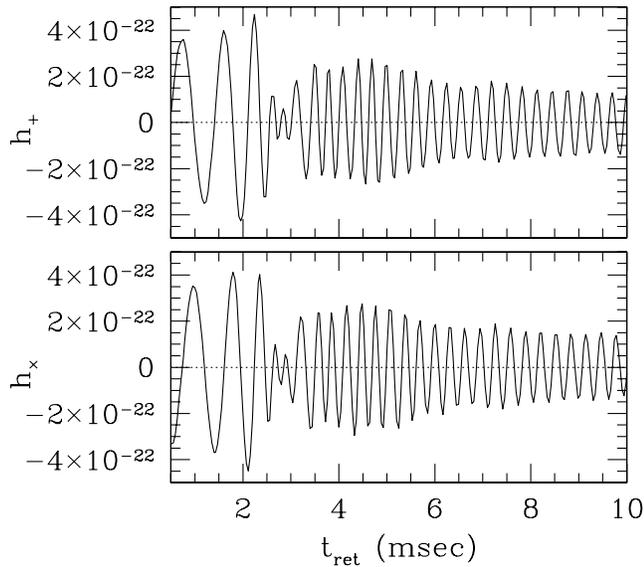


FIG. 2. h_+ and h_\times modes of gravitational waves observed along the rotational axis of the binary neutron stars at a hypothetical distance of 50 Mpc. Mass of two neutron stars is identical and $1.3M_\odot$. The SLy EOS is adopted.

frequency. Here, the effective amplitude is defined by

$$h_{\text{eff}}(f) \equiv \frac{4}{\pi r} \sqrt{\frac{dE}{df}}, \quad (1)$$

where dE/df denotes the energy power spectrum computed from the Fourier spectrum of gravitational waves and is a function of f (e.g., [11,14]). r denotes the distance to the source. In Fig. 3, we display h_{eff} as a function of f for a hypothetical distance $r = 50$ Mpc. In plotting Fig. 3, the Fourier transformation is carried out for $0 \lesssim t_{\text{ret}} \lesssim 10$ ms. The dotted line in Fig. 3 is the planned (nondimensional) noise level (h_{rms} in the notation of [5]) due to the shot noise of the laser for the advanced LIGO, $h_{\text{rms}} \approx 10^{-21.5}(f/1 \text{ kHz})^{3/2}$ for $f \geq 1$ kHz [5]. Figure 3 demonstrates that quasiperiodic gravitational waves yield a sharp peak at $f \approx 3.2$ kHz. Although the frequency is high and far out of the best sensitive region for the laser interferometric detector, the effective amplitude is larger than the noise level.

The angular momentum of the HMNS is dissipated due to gravitational radiation. Since the self-gravity is sustained by a large centrifugal force, it will eventually collapse to a black hole after a sufficient fraction of the angular momentum is dissipated. However, the emission time scale is not ~ 10 ms but much longer, and hence, the ellipsoidal figure is preserved to emit quasiperiodic gravitational waves for a longer time. This indicates that the effective amplitude of the quasiperiodic waves is in reality much larger than that shown in Fig. 3. In [14], we estimated the emission time scale from the dissipation time scale of the angular momentum. We found that the time scale is

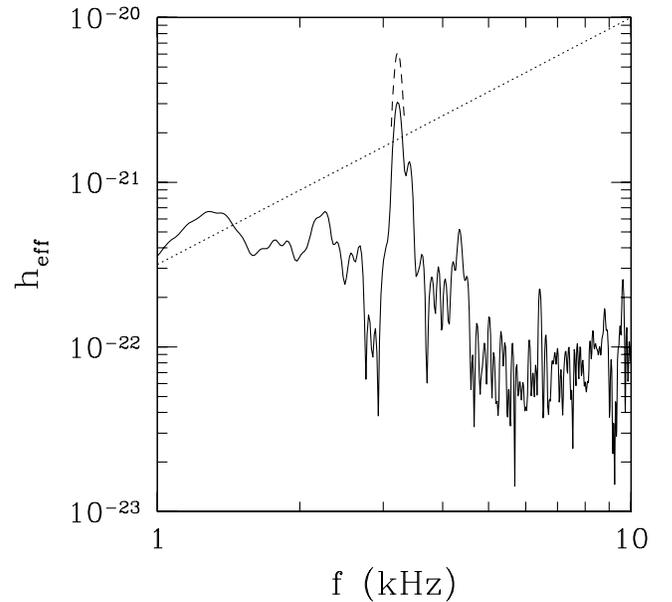


FIG. 3. The (nondimensional) effective amplitude $h_{\text{eff}}(f)$ of gravitational waves observed along the rotational axis of the binary neutron stars at a distance of 50 Mpc. The solid curve is the spectrum computed from gravitational waveforms shown in Fig. 2. The dashed curve denotes an expected peak amplitude computed for a long term integration. The dotted line denotes the planned noise level of the advanced LIGO.

~ 30 – 50 ms for the model shown in Figs. 1–3. This indicates that the number of the cycle for the quasiperiodic waves is ~ 3 – 5 times more, and hence, the effective amplitude will be by a factor of ~ 2 larger. In Fig. 3, we plot the plausible effective amplitude by the dashed curve, which is twice as large as that plotted by the solid curve. In this case, the signal to noise ratio (S/N) is ~ 3 at an event of $r = 50$ Mpc within which one event per year is expected to happen [4]. We note that the simulations were also performed for binaries of mass $(1.2$ – $1.2M_\odot)$ and $(1.25$ – $1.35M_\odot)$ with the SLy EOS, for which HMNSs are formed. It is found that the effective amplitudes are similar to those presented here [14], and the peak frequency is in the range between 3 and 3.5 kHz.

The value of S/N ~ 3 may not be large enough for the detection *in the absence of a priori information*. However, in the merger of binary neutron stars, the chirp signal of gravitational waves will be detected in the inspiral phase. Therefore, the quasiperiodic signal should be searched for in the condition that the merger indeed happened. Furthermore, the time of the arrival of the quasiperiodic signal is determined within a small uncertainty of ~ 1 ms. This information will significantly improve the signal searching in the data analysis [19]. Thus, in the following, we assume that it will be possible to determine if the quasiperiodic gravitational waves are present or absent.

The absence of the signal implies that a black hole is promptly formed after the merger, while its presence im-

plies that a HMNS is formed. Here, we assume that the mass of two neutron stars in binaries will be determined from the chirp signal in the inspiral phase for which the signal to noise ratio is likely to be very high. Then, the presence and absence of quasiperiodic gravitational waves can be used for determining the threshold mass M_{thr} for the prompt formation of black holes. Since the value of S/N is not likely to be very large, the signal can be hidden in a random noise, and hence in reality, it may be difficult to conclude that the signal is absent. However, at least its presence provides the lower limit of M_{thr} , and it will give a very important information for constraining the EOSs for neutron stars. For example, if quasiperiodic gravitational waves are detected from a HMNS of mass $M = 2.6M_{\odot}$, the FPS EOS as well as the EOSs with similar stiffness should be rejected since $M_{\text{thr}} \sim 2.5M_{\odot}$ for it [14]. As this example shows, the advantage of this method is that only one detection will significantly constrain the EOSs. The situation is in contrast to the method in which the value of f_{tran} is used to determine the radius of neutron stars. In this method, the detailed relation between f_{tran} and M is necessary for determining the EOSs, and hence, many observational data sets are required.

One of the remarkable findings in the latest general relativistic simulations [14] with realistic EOSs [16,17] is that the values of M_{thr} ($M_{\text{thr}} \sim 2.7M_{\odot}$ and $\sim 2.5M_{\odot}$ for the SLy and FPS EOSs) are very close to the total mass of the binary neutron stars observed so far (cf. Table. I) [2]. Namely, gravitational waves from the merger of two neutron stars of the total mass $M = 2.5\text{--}2.8M_{\odot}$ are likely to be observed frequently, and can be used for determining the threshold mass for the direct black hole formation. Therefore, we conclude that if the sensitivity of the detectors is improved to a planned level and if the event rate of the merger in nature agrees with a theoretical value [4], the threshold mass will be determined for constraining the EOS for neutron stars. We emphasize that it is important to search for such a signal whenever the chirp signal of gravitational waves from inspiraling binary neutron stars is detected and the masses of two neutron stars are determined.

The author thanks H. Tagoshi for explaining data analysis techniques. Numerical computations were performed on the FACOM VPP5000 machines at the data processing center of National Astronomical Observatory of Japan.

This work was in part supported by Monbukagakusho Grant (Nos. 15037204, 15740142, and 16029202).

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