Magnetic-Field-Induced Electric Polarization in Multiferroic Nanostructures

Ce-Wen Nan, Gang Liu, and Yuanhua Lin

State Key Laboratory of New Ceramics and Fine Processing, and Department of Materials Science and Engineering, Tsinghua University, Beijing 100084, China

Haydn Chen

Department of Physics and Materials Science, City University of Hong Kong, Kowloon, Hong Kong, China (Received 2 July 2004; published 20 May 2005)

Magnetic-field-induced electric polarization in nanostructured multiferroic composite films was studied by using the Green's function approach. The calculations showed that large magnetic-field-induced polarization could be produced in multiferroic nanostructures due to enhanced elastic coupling interaction. Especially, the 1-3 type films with ferromagnetic nanopillars embedded in a ferroelectric matrix exhibited large magnetic-field-induced polarization responses, while the 2-2 type films with ferroelectric and ferromagnetic nanolaminates showed much weaker magnetoelectric coupling and lower magnetic induced polarization due to large in-plane constraint effect, which was in agreement with the recent observations.

DOI: 10.1103/PhysRevLett.94.197203

PACS numbers: 75.80.+q, 75.70.Ak, 75.75.+a, 77.84.-s

Multiferroic materials have drawn increasing interest due to their multifunctionality [1-3], which provides significant potentials for applications as the next-generation multifunctional devices. In these multiferroic materials, the coupling interaction between multiferroic orders could produce some new effects, such as magnetoelectric [1-3]or magnetodielectric effect [4]. The magnetoelectric response is the appearance of an electric polarization upon applying a magnetic field and/or the appearance of magnetization upon applying an electric field. This magneticfield-induced electric polarization (MIEP) has been observed as an intrinsic effect at low temperature and high magnetic field in some natural material systems (e.g., in $RMnO_3$ with R = Tb, Dy [5]). Alternatively and with greater design flexibility, multiferroic composites made by a combination of ferromagnetic and ferroelectric substances, such as combinations of piezoelectric ceramics [e.g., BaTiO₃ and lead-zirconate-titanate (PZT)] and ferrites or rare-earth-iron alloys (e.g., Terfenol-D), have been recently found to exhibit large magnetoelectric response at room temperature and low magnetic field [6-8]. The magnetoelectric behavior in these multiferroic composites is dependent on their microstructure and coupling interaction across ferromagnetic-ferroelectric interface [9].

Most recently, nanostructured $BaTiO_3/CoFe_2O_4$ multiferroic composites have been deposited in a film-onsubstrate geometry and magnetoelectric coupling has been observed for the first time in multiferroic nanostructures [10]. The coupling interaction between $BaTiO_3$ and $CoFe_2O_4$ in the multiferroic nanostructures has been found to be due to elastic interaction as was the case in bulk composites. However, the mechanical constraint arising from the film-on-substrate structures and the good bonding between the ferromagnetic and ferroelectric phases in the nanostructured films could significantly affect the coupling interactions, thereby the magnetoelectric effect and/or MIEP. Currently, a theoretical description on such multiferroic nanostructured films is lacking. In this Letter, we calculate the magnetoelectric effect in the nanostructured ferroelectric-ferromagnetic composite films (e.g., $BaTiO_3/CoFe_2O_4$ films [10]) based on the Green's function approach which had been successfully applied to bulk multiferroic composites [2,7]. The present work provides the first phenomenological understanding of the MIEP in multiferroic nanostructures.

As the coupling interaction between ferroelectric and ferromagnetic phases in the multiferroic nanostructures is still an elastic interaction [10], the constitutive equations for the coupling magnetic-mechanical-electric interactions in the nanostructured films can be expressed by direct notation for tensors as

$$\sigma = \mathbf{c}\varepsilon - \mathbf{e}^{T}\mathbf{E} - \mathbf{c}\varepsilon^{ms} - \sigma_{s},$$

$$\mathbf{D} = \mathbf{e}\varepsilon + \kappa \mathbf{E} + \alpha \mathbf{H} + \mathbf{P}_{s},$$
 (1)

$$\mathbf{B} = \mu(\varepsilon, \mathbf{E}, \mathbf{H})\mathbf{H} + \mathbf{M}_{s},$$

where σ , ε , **D**, **E**, **B**, and **H** are the stress, strain, electric displacement, electric field, magnetic induction, and magnetic field, respectively; **c** and κ are, respectively, the stiffness at constant fields and the dielectric constant at constant strain; the permeability μ strongly depends on ε and electric and magnetic fields; **e** (\mathbf{e}^{T} being the transpose of **e**) is the piezoelectric coefficient; and ε^{ms} is the magnetostrictively induced strain related with the magnetic-field dependent magnetostriction constants (e.g., λ_{001} and λ_{111} [11]) of the ferromagnetic phase; α is the magneto-electric coefficient. These are the same as the case for bulk multiferroic composites [2,7]. However, in comparison to the bulk composites, there exist remarkable residual stress

 σ_s (or residual strain ε_s), spontaneous polarization \mathbf{P}_s and magnetization \mathbf{M}_s in the multiferroic films.

The effective properties (denoted by the starred quantities below) of the multiferroic films can still be defined as usual [7] in terms of the averaged fields (denoted by $\langle \rangle$), e.g., $\langle \mathbf{D} \rangle = \mathbf{e}^* \langle \varepsilon \rangle + \kappa^* \langle \mathbf{E} \rangle + \alpha^* \langle \mathbf{H} \rangle + \bar{\mathbf{P}}_s$. We considered the MIEP of such nanostructured films. Under only applied magnetic field, the effective polarization in the mechanically clamped films is

$$\bar{\mathbf{P}} = \alpha^* \langle \mathbf{H} \rangle + \bar{\mathbf{P}}_{\mathbf{s}}.$$
 (2)

In a state of static equilibrium,

$$\partial \sigma_{ij}(\mathbf{x}) / \partial x_j = 0, \qquad \partial D_i(\mathbf{x}) / \partial x_i = 0,$$

 $\partial B_i(\mathbf{x}) / \partial x_i = 0.$ (3)

By solving the equilibrium equations (3) under the homogeneous magnetic-mechanical-electric ($\mathbf{H}^0 \cdot \boldsymbol{\varepsilon}^0 \cdot \mathbf{E}^0$) boundary conditions in terms of the Green's function approach [7], the local fields within the composite films can be obtained as

$$\varepsilon = \varepsilon^{0} + \mathbf{G}^{u}(\mathbf{c} - \mathbf{c}^{0})\varepsilon - \mathbf{G}^{u}\mathbf{e}^{T}\mathbf{E} - \mathbf{G}^{u}(\mathbf{c}\varepsilon^{ms} + \sigma_{s}),$$

$$\mathbf{E} = \mathbf{E}^{0} + \mathbf{G}^{\phi}\mathbf{e}\varepsilon + \mathbf{G}^{\phi}(\kappa - \kappa^{0})\mathbf{E} + \mathbf{G}^{\phi}(\alpha\mathbf{H} + \mathbf{P}_{s}), \quad (4)$$

$$\mathbf{H} = \mathbf{H}^{0} + \mathbf{G}^{\psi}(\mu - \mu^{0})\mathbf{H} + \mathbf{G}^{\psi}\mathbf{M}_{s},$$

where \mathbf{c}^0 , κ^0 , and μ^0 represent the constitutive constants of a homogeneous reference medium; \mathbf{G}^{μ} , \mathbf{G}^{ϕ} , and \mathbf{G}^{ψ} are the modified displacement, electric and magnetic potential Green's functions for the homogeneous medium [2,7,12]. Substitution of Eq. (4) into (1) directly gives explicit solutions for the local σ , **D**, and **B**, also as a function of ε^0 , \mathbf{E}^0 , and \mathbf{H}^0 . By averaging these solutions for local field quantities and eliminating ε^0 , \mathbf{E}^0 , and \mathbf{H}^0 from them, we can get the effective polarization in the mechanically clamped films as

$$\bar{\mathbf{P}} = \langle \mathbf{P}_{\mathbf{s}} + (\kappa - \kappa^*) \mathbf{T}^{33} \mathbf{G}^{\phi} [\mathbf{P}_{\mathbf{s}} - \mathbf{e} \mathbf{T}^{66} \mathbf{G}^u (\mathbf{c} \varepsilon^{ms} + \sigma_s)] + (\mathbf{e}^* - \mathbf{e}) \mathbf{T}^{66} \mathbf{G}^u (\mathbf{c} \varepsilon^{ms} + \sigma_s + \mathbf{e}^T \mathbf{T}^{33} \mathbf{G}^{\phi} \mathbf{P}_{\mathbf{s}}) \rangle, \quad (5)$$

where \mathbf{T}^{66} and \mathbf{T}^{33} are two so-called *t*-matrix tensors as given before [7,12]. This equation is quite general and independent of the models assumed for the topology of the phases in the composite film.

For the (00*l*)-oriented multiferroic nanostructures [10], only the volume average is necessary in Eq. (5). We take the ferroelectric phase as the homogeneous reference medium as usual, since the ferroelectric phase with high volume fraction is a matrix phase [10]. Further due to the fact that $\mathbf{e}(\text{and}\mathbf{e}^T) = 0$ and $\mathbf{P}_s = 0$ for the ferromagnetic phase, $\varepsilon^{ms} = 0$ and $\mathbf{M}_s = 0$ for the ferroelectric phase, and $\alpha = 0$ for both phases, Eq. (5) can be rewritten as

$$\bar{\mathbf{P}} = (1 - f) \{ \mathbf{P}_{\mathbf{s}} + (\kappa^{p} - \kappa^{*}) \mathbf{G}^{\phi} (\mathbf{P}_{\mathbf{s}} - \mathbf{e}^{p} \mathbf{G}^{u} \sigma_{s}^{p})
+ (\mathbf{e}^{*} - \mathbf{e}^{p}) \mathbf{G}^{u} (\sigma_{s}^{p} + \mathbf{e}^{T} \mathbf{G}^{\phi} \mathbf{P}_{s}) \}
+ f \mathbf{e}^{*} [\mathbf{I} - \mathbf{G}^{u} (\mathbf{c}^{m} - \mathbf{c}^{p})]^{-1} (\mathbf{c}^{m} \varepsilon^{ms} + \sigma_{s}^{m}), \quad (6)$$

where f is the volume fraction of the ferromagnetic phase, I is the unit tensor; and the quantities denoted using the superscripts p and m refer to those for the ferroelectric and ferromagnetic phases. Here e^* and κ^* were given as before [12].

For the 1-3 type (using the terminology introduced by Cross and co-workers [13]) nanostructured composite films with (00*l*) ferromagnetic nanopillars embedded in a ferroelectric matrix [Fig. 1(a)] [10], the effective electric polarization \bar{P}_3 along the symmetric direction is easily obtained from Eq. (6) as

$$\bar{P}_{3} = (1-f)P_{s3} + \frac{2f(1-f)e_{31}^{p}}{\bar{k} + c_{11}^{p} - c_{12}^{p}}(\sigma_{11}^{p} - \sigma_{11}^{m}), \quad (7)$$

in the case that the height of pillars is much larger than their diameter (e.g., 20–30 nm in diameter and ~400 nm in height for nanopillars [10]), where $\bar{k} = f(c_{11}^p + c_{12}^p) + (1 - f)(c_{11}^m + c_{12}^m)$; σ_{11}^p and σ_{11}^m are in-plane total stresses including residual stresses and magnetostrictively induced stresses, respectively, in the ferroelectric and ferromagnetic phases, and are expressed as

$$\sigma_{11}^{p(m)} = (c_{11}^{p(m)} + c_{12}^{p(m)})\varepsilon_{11}^{p(m)} + c_{13}^{p(m)}\varepsilon_{33}^{p(m)}, \qquad (8)$$

where $\varepsilon_{11}^{p(m)}$ and $\varepsilon_{33}^{p(m)}$ are in-plane and out-of-plane total strains including residual strains (ε_{s11} and ε_{s33}) and magnetostrictively induced strains (ε_{ij}^{ms}) upon applying the magnetic field, respectively, in the ferroelectric and ferromagnetic phases, e.g., $\varepsilon_{11}^m = \varepsilon_{11}^{ms} + \varepsilon_{s11}^m$ and $\varepsilon_{33}^m = \varepsilon_{33}^{ms} + \varepsilon_{s33}^m$. At zero magnetic field, ε_{ij}^p and ε_{ij}^m are equal to their residual strains, i.e., $\varepsilon_{ij}^p(H=0) = \varepsilon_{sij}^p$ and $\varepsilon_{ii}^m(H=0) = \varepsilon_{sij}^m$.

For the 2-2 type [13] nanostructured composite films with (00*l*) ferroelectric and ferromagnetic nanolaminates as shown in Fig. 1(b), the effective electric polarization \bar{P}_3 can also be directly obtained from Eq. (6) as



FIG. 1. Schematic illustration of the nanostructured multiferroic $BaTiO_3/CoFe_2O_4$ films in (a) the 1-3 type where the $CoFe_2O_4$ nanopillars (shaded) are embedded in the $BaTiO_3$ matrix, and in (b) the 2-2 type where the $BaTiO_3$ layer is first deposited on the substrate $SrRuO_3$ and then the $CoFe_2O_4$ layer (shaded) on the piezoelectric layer.

$$\bar{P}_{3} = (1-f)\frac{\kappa_{33}^{m}}{\bar{\kappa}_{33}}P_{s3} + \frac{f(1-f)e_{33}^{p}\kappa_{33}^{m}}{\bar{c}_{33}\bar{\kappa}_{33}}(\sigma_{33}^{p} - \sigma_{33}^{m}),$$
(9)

in the case that the thickness of a layer (e.g., ~30 nm [10]) is much less than the macroscopic size of the film plane, where $\bar{c}_{33} = fc_{33}^p + (1 - f)c_{33}^m$, $\bar{\kappa}_{33} = f\kappa_{33}^p + (1 - f)\kappa_{33}^m$; σ_{33}^p and σ_{33}^m are out-of-plane total stresses including residual stresses and magnetostrictively induced stresses, respectively, in the ferroelectric and ferromagnetic phases, and are expressed as

$$\sigma_{33}^{p(m)} = 2c_{13}^{p(m)}\varepsilon_{11}^{p(m)} + c_{33}^{p(m)}\varepsilon_{33}^{p(m)}.$$
 (10)

At zero magnetic field, $\varepsilon_{11}^{p(m)}(H=0) = \varepsilon_{s11}^{p(m)}$ and $\varepsilon_{33}^{p(m)}(H=0) = \varepsilon_{s33}^{p(m)} = 0$.

These explicit equations show that the effective electric polarization is sensitive to residual strains and spontaneous polarization in the films, materials constants of the two phases and the ways of their combination, and applied



FIG. 2. Dependence of the magnetic-field-induced polarization $\Delta \bar{P}_3$ on the magnetic field for (a) the nanostructured films and (b) the bulk composites of BaTiO₃/CoFe₂O₄ with f = 0.35 at room temperature. The inset in (b) is the magnetostrictive behavior taken for CoFe₂O₄. (c) Effect of the magnetostriction of the ferromagnetic phase on $\Delta \bar{P}_3$ for this 1-3 type films.

magnetic field. For the 1-3 type films, after applying the magnetic field, the strains meet $(1 - f)\varepsilon_{11}^p + f\varepsilon_{11}^m = 0$ and $\varepsilon_{33}^p = \varepsilon_{33}^m$ due to the constraint in the 1-3 film-on-substrate structures. However, for the 2-2 type films, the magnetostrictively induced in-plane strains are zero, i.e., $\varepsilon_{11}^p = \varepsilon_{11}^m = 0$, due to gigantic constraint effect, and the out-of-plane strains also meet $(1 - f)\varepsilon_{33}^p + f\varepsilon_{33}^m = 0$.

For quantitative comparison purposes, the effective electric polarization of the 1-3 and 2-2 type nanostructured multiferroic BaTiO₃/CoFe₂O₄ films [(00l)-oriented films on SrTiO₃ substrate] [10] were calculated [14]. The magnetostrictive behavior of CoFe₂O₄ is taken as shown in the inset of Fig. 2(b) [10,11]. For such (001)-oriented films, $\varepsilon_{33}^{ms} = \lambda_{001}$. Figure 2(a) shows the MIEP, i.e., $\Delta \bar{P}_3 = \bar{P}_3 - \bar{P}_3$ $\bar{P}_3(H=0)$, calculated for the two multiferroic films with f = 0.35 at room temperature. $\Delta \bar{P}_3$ of the films nonlinearly increases with the magnetic field due to the nonlinear dependence of the magnetostriction of $CoFe_2O_4$ on the applied magnetic field, and begins to approach its saturation at high magnetic fields where the magnetostriction gets saturated. Of particularly interesting to note is that a large MIEP is obtained in the 1-3 films at room temperature. When the applied magnetic field increases from zero up to 240 kA/m (≈ 0.3 T), the MIEP of the 1-3 multiferroic BaTiO₃/CoFe₂O₄ film is about 400 μ C/m², comparable with that in monophase RMnO₃ observed at low temperature and high magnetic field [5]. In comparison, the MIEP in the 2-2 BaTiO₃/CoFe₂O₄ film is quite low and much less than that in the 1-3 type. The main reason is attributed to the tremendous clamping effect in the 2-2 type film, where the in-plane gigantic constraint effect greatly inhibits the magnetostriction. In actual experiment [10], the 2-2 BaTiO₃/CoFe₂O₄ film was indeed found to exhibit much weaker magnetoelectric coupling than the 1-3 type. For comparison, the 1-3 and 2-2 bulk composites [15] of $BaTiO_3/CoFe_2O_4$ were calculated as well, as shown in Fig. 2(b), where the residual stress σ_s and the spontaneous polarization P_s were neglected. As seen, the 1-3 bulk composites exhibit lower MIEP than the 1-3 nanostructured counterparts, and the 2-2 bulk composites also exhibit lower MIEP than the 1-3 type.

As shown in Fig. 2, the MIEP of the nanostructured composite films is closely related to the magnetostrictive behavior of the ferromagnetic phase. For further illustration, we take a few different values for the saturation magnetostriction λ_{001} . As seen from Fig. 2(c), the MIEP strongly depends on the magnetostriction. In the multiferroic composite films, the effective spontaneous polarization decreases with the increase in the volume fraction f of the ferromagnetic phase [Fig. 3(a)] as expected. The effective polarization degrades gradually with increasing f for the 1-3 film but sharply drops around f = 0.1 for the 2-2 film. However, on the other hand, the MIEP increases from zero to the maxima with increasing f and then drops to zero at f = 1, as shown in Fig. 3(b). Especially, the variation in $\Delta \bar{P}_3$ with f is remarkable for the 1-3 nano-



FIG. 3. The variation of (a) polarization \bar{P}_3 and (b) $\Delta \bar{P}_3$ with the volume fraction f of magnetostrictive phase at $H_3 = 250$ kA/m for both the 1-3 and 2-2 type nanostructured films.

structured BaTiO₃/CoFe₂O₄ films. Again, much smaller $\Delta \bar{P}_3$ in the 2-2 nanostructured BaTiO₃/CoFe₂O₄ films indicates a much weaker magnetoelectric coupling. As seen from Eqs. (7) and (9), the magnetically induced polarization in the 1-3 film is associated with the in-plane stresses while that in the 2-2 film is associated with the out-of-plane stresses. Thus much larger in-plane stresses and their variation with the applied magnetic field than the out-of-plane stresses in the film-on-substrate structures lead to much larger the magnetically induced polarization in the 1-3 film than in the 2-2 film. Moreover, the totally different constraint effect in the these two nanostructures as discussed above also contributes to such a large difference between their MIEP results.

In summary, the explicit formula for the magnetic-field induced polarization in the nanostructured multiferroic films have been given based on the Green's function technique. The calculations showed that the 1-3 nanostructured films exhibit a large MIEP, but the 2-2 films exhibit much weaker magnetoelectric effect due to large in-plane constraint, which is in agreement with the observations. Large MIEP could be produced in multiferroic nanostructures due to enhanced elastic coupling interaction.

This work was supported by NSF of China (Grants No. 50318002 and No. 50232030), the State Key Project of Fundamental Research of China (Grants No. 2002CB613303 and No. G2000067108), and RGC of Hong Kong (Grant No. N_CityU104/03).

- [1] N.A. Hill, Annu. Rev. Mater. Res. 32, 1 (2002).
- [2] C. W. Nan, Phys. Rev. B 50, 6082 (1994); C. W. Nan, Y. H. Lin, and J. H. Huang, Ferroelectrics 280, 153 (2002).
- [3] J. Wang et al., Science 299, 1719 (2003).
- [4] G. Lawes, A.P. Ramirez, C.M. Varma, and M.A. Subramanian, Phys. Rev. Lett. 91, 257208 (2003).
- [5] T. Kimura *et al.*, Nature (London) **426**, 55 (2003); T. Goto *et al.*, Phys. Rev. Lett. **92**, 257201 (2004); N. Hur *et al.*, Nature (London) **429**, 392 (2004).
- [6] J. van Suchtelen, Philips Res. Rep. 27, 28 (1972);
 A. M. J. G. van Run *et al.*, J. Mater. Sci. 9, 1710 (1974);
 G. Harshe *et al.*, Int. J. Appl. Electromagn. Mater. 4, 145 (1993);
 M. I. Bichurin *et al.*, Ferroelectrics 204, 289 (1997);
 K. K. Patankar *et al.*, Mater. Chem. Phys. 65, 97 (2000);
 G. Srinivasan *et al.*, Phys. Rev. B 65, 134402 (2002).
- [7] C. W. Nan, M. Li, X. Q. Feng, and S. Yu, Appl. Phys. Lett.
 78, 2527 (2001); C. W. Nan, M. Li, and J. H. Huang, Phys. Rev. B 63, 144415 (2001).
- [8] J. Ruy *et al.*, J. Am. Ceram. Soc. **84**, 2905 (2001); K. Mori and M. Wuttig, Appl. Phys. Lett. **81**, 100 (2002); S. X. Dong, J. F. Li, and D. Viehland, Appl. Phys. Lett. **83**, 2265 (2003); S. X. Dong, J. R. Cheng, J. F. Li, and D. Viehland, Appl. Phys. Lett. **83**, 4812 (2003).
- C. W. Nan, G. Liu, and Y. H. Lin, Appl. Phys. Lett. 83, 4366 (2003); G. Liu, C. W. Nan, N. Cai, and Y. H. Lin, J. Appl. Phys. 95, 2660 (2004).
- [10] H. Zheng et al., Science 303, 661 (2004).
- [11] V. J. Folen, in *Magnetic and Other Properties of Oxides and Related Compounds*, edited by K. H. Hellwege and A. M. Hellwege, Landolt-Bornstein Group III, Vol. 4, Pt. b (Springer, Berlin, 1970), p. 366.
- [12] C. W. Nan and F. Jin, Phys. Rev. B 48, 8578 (1993); C. W. Nan, J. Appl. Phys. 76, 1155 (1994).
- [13] R.E. Newnham, D.P. Skinner, and L.E. Cross, Mater. Res. Bull. 13, 525 (1978).
- [14] The properties of BaTiO₃ and CoFe₂O₄ used in the numerical calculations were given as $(c_{11}^p, c_{12}^p, c_{13}^p, c_{33}^p) =$ (166.2, 76.5, 77.4, 161.4) GPa, $\kappa_{33}^p/\kappa_o = 1350$, $(e_{31}^p, e_{33}^p) =$ (-4.22, 18.6) C/m², and $P_{s3} = 26 \ \mu$ C/cm² for BaTiO₃, and $(c_{11}^m, c_{12}^m, c_{13}^m, c_{33}^m) =$ (286.0, 173.0, 170.5, 269.5) Gpa and $\kappa_{33}^m/\kappa_o = 10$ for CoFe₂O₄. From Ref. [10], the out-of-plane residual strains in the 1-3 nanostructured film were given as $\varepsilon_{s33}^p = -\varepsilon_{333}^m = 0.8\%$, and the in-plane residual strains were determined as $\varepsilon_{s33}^p = -1\%$ and $\varepsilon_{333}^m = -1.89\%$ from the lattice constant mismatches between BaTiO₃ and the substrate and between CoFe₂O₄ and the substrate. In the 2-2 type film, however, there exists only the in-plain residual strains, and $\varepsilon_{s11}^p = -1\%$ and $\varepsilon_{s11}^m = -1.6\%$ estimated from the lattice constant mismatches.
- [15] The 1-3 and 2-2 type bulk composites of $BaTiO_3/CoFe_2O_4$ are normal $BaTiO_3/CoFe_2O_4$ ceramic composites, i.e., $CoFe_2O_4$ ceramic rods embedded in $BaTiO_3$ ceramic matrix for the 1-3 type, and laminated $BaTiO_3$ and $CoFe_2O_4$ ceramic layers for the 2-2 composite [2].