Rectification in Luttinger Liquids

D. E. Feldman,[1] S. Scheidl,[2] and V. M. Vinokur[3]

[1]Department of Physics, Brown University, Providence, Rhode Island 02912, USA
[2]Institut für Theoretische Physik, Universität zu Köln, Zülpicher Strasse 77, D-50937 Köln, Germany
[3]Materials Science Division, Argonne National Laboratory, 9700 South Cass, Argonne, Illinois 60439, USA

(Received 5 October 2004; published 12 May 2005)

We investigate the rectification of an ac bias in Luttinger liquids in the presence of an asymmetric potential (the ratchet effect). We show that a strong repulsive electron interaction enhances the ratchet current in comparison with Fermi-liquid systems, and the dc I-V curve is strongly asymmetric in the low-voltage regime even for a weak asymmetric potential. At higher voltages the ratchet current exhibits an oscillatory voltage dependence.

DOI: 10.1103/PhysRevLett.94.186809 PACS numbers: 73.63.Nm, 71.10.Pm

Asymmetric conductors have asymmetric I-V curves. This phenomenon is known as the diode or ratchet effect and plays a major role in electronics. Recently transport asymmetries in single-molecule devices and other mesoscopic systems have attracted a lot of interest. The idea that asymmetric molecules can be used as rectifiers is rather old [1]; however, it was implemented experimentally [2] only in the 1990s. Another experimental realization of a mesoscopic rectifier is an asymmetric electron waveguide constructed within the inversion layer of a semiconductor heterostructure [3]. Transport asymmetry has been observed in Luttinger liquid systems such as carbon nanotubes [4] and for the tunneling in the quantum Hall edge states [5]. These experimental advances have stimulated much theoretical activity [6–10] with the main focus on the simplest Fermi-liquid systems [11].

Transport in one-channel quantum wires, where electrons form a Luttinger liquid, differs significantly from the Fermi-liquid case. In particular, impurity effects are stronger in Luttinger liquids, and even a weak impurity potential may render the linear conductance zero at low temperatures [12]. In this Letter we investigate the ratchet effect in Luttinger liquids. We show that strong repulsive electron interaction enhances the ratchet current, and the low-voltage part of the I-V curve is strongly asymmetric even in quantum wires with weak asymmetric potentials.

We consider the ratchet effect in a one-channel quantum wire with repulsive electron interaction in the presence of a weak potential barrier $U(x)$, asymmetric with respect to spatial inversion. We assume that electrons are spin polarized and $U(x) \ll E_F$, where E_F is the bandwidth. We define the ratchet current as the response to a low-frequency square voltage wave of amplitude V, $I_r(V) = [I(V) + I(-V)]/2$. The ratchet current vanishes for systems with symmetric I-V curves. First, we consider voltages $V < V_0 = \hbar v_F/(ea)$, where v_F is the Fermi velocity, e the electron charge, and a the size of the region containing the asymmetric potential. We find a weak ratchet effect in the interval $eV_0 > eV > \sqrt{UE_F}$ for both Fermi and Luttinger liquids, $I_r \sim (e/h)U^2(eV)^{2g}/E_F^{2g+1}$, where $g = 1$ for Fermi liquids and $g < 1$ for Luttinger liquids with repulsive interaction. However, at strong repulsive interaction (the Luttinger liquid parameter $g \ll 1$) and sufficiently low voltages, the ratchet current $I_r(V)$ *grows* as the voltage decreases until $I_r(V)$ becomes comparable with the total current $I(V)$ at $eV = eV^* \sim (UE_F^{-g})^{1/(1-g)}$. At $E_F \gg eV > eV_0$ the ratchet current oscillates as a function of the voltage and can become comparable with the total current $I(V)$ for any repulsive interaction strength. We also briefly discuss the ratchet effect in the presence of a strong asymmetric potential $U > E_F$. The complicated ratchet-current behavior is caused by the energy dependence of the effective impurity strength in Luttinger liquids [12]. This introduces an additional energy scale V^* absent in Fermi-liquid systems.

One-channel quantum wires can be described by the Tomonaga-Luttinger model with the Hamiltonian

$$H = \int dx \Big\{ -\hbar v_F [\psi_R^\dagger(x) i\partial_x \psi_R(x) - \psi_L^\dagger(x) i\partial_x \psi_L(x)] + U(x)\rho(x) + \int dy K(x-y)\rho(x)\rho(y) \Big\}, \quad (1)$$

where $\psi_R^\dagger$ and $\psi_L^\dagger$ are the creation operators for right- and left-moving electrons, $\psi^\dagger = \psi_R^\dagger + \psi_L^\dagger$ gives the conventional electron creation operator, $\rho = \psi^\dagger\psi$ is the electron density, $U(x)$ is the asymmetric potential, and $K(x-y)$ the interaction strength. Our aim is to calculate the current I as a function of the applied voltage V. We assume that the long-range Coulomb interaction is screened by the gates so that $K(x-y)$ decreases rapidly for large $(x-y)$. Electric fields of external charges are also assumed to be screened. Thus, the applied voltage reveals itself only as the difference of the electrochemical potentials E_L and E_R of the particles injected from the left and right reservoirs.

We assume that one lead is connected to the ground so that its electrochemical potential $E_R = E_F$ is fixed. The electrochemical potential of the second lead $E_L = E_F + eV$ is controlled by the voltage source. In such situations a

symmetric potential $U(x)$ is sufficient for rectification. For example, in a noninteracting system $I(V) \sim \int_{E_R}^{E_L}[1 - R(E)]dE$, where $R(E)$ is the reflection coefficient. If the only relevant scale for the energy dependence of the reflection probability is the bandwidth $\sim E_F$ then the ratchet current is $I_r \sim \int_0^{eV} dE[R(E_F - E) - R(E_F + E)] \approx -2\int_0^{eV} dER'(E_F)E \sim R(E_F)(eV)^2/E_F \sim U^2(eV)^2/E_F^3$ for small U and V, and any coordinate dependence $U(x)$.

A "nontrivial" ratchet effect can be observed not only in the setup with $E_R = E_F, E_L = E_F + eV$, but also when the injected charge density is voltage independent, $E_{L/R} = E_F \pm eV/2$. Symmetry considerations require an asymmetric $U(x)$ for a nonvanishing ratchet current in the latter case. Also an electron interaction must be present. Indeed, for free particles the reflection coefficient $R(E)$ is independent of the electron propagation direction [13] and hence $I(V) = -I(-V)$.

The ratchet effect is absent in the first two orders in $U(x)$, if the injected charge density is voltage independent. Indeed, in the lowest two orders the nontrivial ratchet current is $I_r^{(1,2)} = \int dxC(x)U(x) + \int dxdyD(x, y)U(x)U(y)$. Since the nontrivial ratchet current is zero for any symmetric potential, $I_r^{(1,2)}$ must be zero for any symmetric potential. Substituting $U(x) = U\delta(x - x_0)$ we find that $C(x_0), D(x_0, x_0) = 0$. Substituting $U(x) = U\delta(x - x_1) + U\delta(x - x_2)$ we see that $D(x_1, x_2) + D(x_2, x_1) = 0$. Hence, $I_r^{(1,2)} = 0$ for any $U(x)$.

We first consider the nontrivial ratchet effect and then check what changes after the voltage dependence of the injected charge density is taken into account. Let us begin with a qualitative explanation before we make a rigorous calculation. The origin of the ratchet current can be understood from a simplified Hartree-Fock picture. In this approximation, electrons are backscattered off a combined potential $\tilde{U}(x) = U(x) + W(x)$, where $W(x)$ is a self-consistent electrostatic potential created by the average local charge density. To obtain $W(x)$ we use the following approximation in the last term of Eq. (1): $\rho(x)\rho(y) \approx [\psi_R^+(x)\psi_R(x) + \psi_L^+(x)\psi_L(x)][\psi_R^+(y)\psi_R(y) + \psi_L^+(y)\psi_L(y)] + [\langle\rho(x)\rangle\psi_R^+(y)\psi_L(y) + \langle\rho(y)\rangle\psi_R^+(x)\psi_L(x) + \text{H.c.}] + \text{const}$. Thus, the relation between W and ρ is linear. The combined potential $\tilde{U}(x)$ is different for the opposite voltage signs.

In the model (1) the electron interaction is short ranged due to the screening gates, and hence, the relation between the potential $W(x)$ and the electron density $\rho(x)$ is local, $W(x) \sim \rho(x)$. The simplest choice of $U(x)$ is a two-impurity asymmetric potential $U(x) = U_1\delta(x + a/2) + U_2\delta(x - a/2)$. The charge density profile [14] in the presence of a two-impurity potential and the voltage drop V is sketched in Fig. 1. Depending on the voltage sign, the charge density decreases or grows as a function of the coordinate x. So does the electrostatic potential $W(x)$. Hence, $\tilde{U}(x)$ is different for the opposite voltage signs. The density is essentially independent of the coordinate between the impurities [14], as well as on the left and on the right of the impurities, since no backscattering occurs in those regions. The charge density and the electrostatic potential drop at the positions of the impurities. The magnitude of a drop is proportional to the electric charge backscattered off the impurity. Indeed, if the incident charge densities of the electrons approaching the impurity from the left and from the right are $\rho_L^{\rightarrow}$ and $\rho_R^{\leftarrow}$, and the backscattered charge densities are $\rho_L^{\leftarrow}$ and $\rho_R^{\rightarrow}$, then the density drops across the impurity $\Delta\rho = (\rho_L^{\rightarrow} + \rho_L^{\leftarrow} + \rho_R^{\leftarrow} - \rho_R^{\rightarrow}) - (\rho_R^{\leftarrow} + \rho_R^{\rightarrow} + \rho_L^{\rightarrow} - \rho_L^{\leftarrow}) = 2(\rho_L^{\leftarrow} - \rho_R^{\rightarrow}) \sim I_{\text{bs}}$, where I_{bs} is the current backscattered off the impurity. Thus $W(x) = \tilde{U} - U \sim I_{\text{bs}}$. From Ref. [12] we know that for a weak potential U

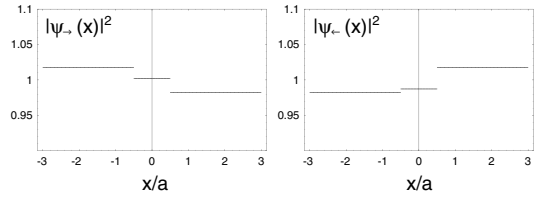

FIG. 1. Density profiles averaged over the period of Friedel oscillations for a potential with $U_1 < U_2$. The averaged densities show drops at the impurity positions.

$$I_{\text{bs}} \sim |U_{2k_F}|^2|V|^{2g-1}\,\text{sign}V/E_F^{2g}, \tag{2}$$

where $U_{2k_F} \sim k_F \int dx \exp(2ik_Fx)U(x)$, k_F is proportional to the mean electron density, and the dimensionless constant g characterizes the interaction strength; $g = 1$ for noninteracting electrons (in which case $W(x) = 0$).

Now we can substitute the renormalized potential $\tilde{U} = U + W$ for U in Eq. (2). The Fourier component W_{2k_F} is different for the opposite voltage signs. Hence, we obtain the asymmetric part of the I-V characteristics $I_r \sim eU^3|eV|^{4g-2}/(hE_F^{4g})$. The ratchet effect is strongest for $g \to 0$ when the ratchet current grows as the voltage decreases.

The above Hartree-Fock argument provides a qualitatively correct picture at small g, but underestimates fluctuations in Luttinger liquids. As shown below, the ratchet-current growth at small voltages differs from our estimate: $I_r \sim U^3|V|^{6g-2}$, $E_F \gg V > V^* \sim (UE_F^{-g})^{1/(1-g)}$, $g \ll 1$. We will see that the growth terminates at $V = V^*$. At such voltage $I_r(V^*)/I(V^*) \sim [(V^*)^{3g+1}/E_F^{3g}]/[V^*] \sim (V^*/E_F)^{3g} \sim 1$ as $g \ll 1$. Fluctuations are less important in many-channel systems and the Hartree-Fock picture gives exact results for some two-channel systems and for Fermi liquids.

We use the bosonization technique [15] to calculate the ratchet current. After an appropriate rescaling of the time variable, the system can be described by the action [12]

$$S = \int dtdx\Big\{\frac{1}{8\pi}[(\partial_t\Phi)^2 - (\partial_x\Phi)^2] - \delta(x)\sum_{n\geq 1} 2\tilde{U}_{2nk_F}\cos(n\sqrt{g}\Phi + \alpha_n)\Big\}, \quad (3)$$

where the bosonic field Φ is related to the charge density as $\rho = e(\sqrt{g}\partial_x\Phi + 2k_F)/(2\pi)$, and $\tilde{U}_{2nk_F}\exp(i\alpha_n)$ are of the order of the Fourier components of the asymmetric potential, $k_F\int\exp(2ink_Fx)\tilde{U}(x)dx$. We assume that the charge density $\sim k_F$ is independent of the voltage. The operator $\cos(n\sqrt{g}\Phi + \alpha_n)$ describes scattering events involving n electrons. We assume that $\alpha_1 = 0$. Indeed, we can always set $\alpha_1 = 0$ by a constant shift of the bosonic field Φ. For a general asymmetric potential, α_n with $n > 1$ remains nonzero after this shift. On the other hand, for a symmetric potential $U(x) = U(-x)$ all $\alpha_n = 0$. In most problems it is sufficient to keep only the $n = 1$ term. The $n = 2$ contribution is relevant in the theory of resonant transmission in Luttinger liquids [12]. This term is also important for the ratchet effect.

We use the standard model [16–18] for Fermi-liquid leads adiabatically connected to the wire. We assume that the action (3) is applicable for $|x| < L$ only. At large $|x|$ the interaction strength $K(x - y)$, Eq. (1), is zero. This model can be interpreted as a quantum wire with electron interaction completely screened by the gates near its ends. Electric fields of external charges are assumed to be screened in all parts of the wire. A simple modification of this model describes electrically neutral leads [18]. All results coincide for our setup and the model [18].

The current injected from the noninteracting 1D regions is given by the Landauer formula $I_0 = e^2V/h$ [16]. Indeed, left- or right-movers entering the noninteracting region from the central part of the wire cannot affect the current of right- or left-movers in the noninteracting region. Hence, the current of right- or left-moving particles in the left or right noninteracting region is determined by the chemical potential of the left or right reservoir. The total current is the sum [17,18] of the injected current and the current backscattered off the asymmetric potential: $I = I_0 + I_{\rm bs}$. Only $I_{\rm bs}$ contributes to the ratchet effect. To find the backscattered current we employ the Keldysh formalism [19]. We assume that at $t = -\infty$ there is no backscattering in the Hamiltonian, and then the backscattering is gradually turned on. Thus, at the initial moment of time the numbers N_L and N_R of left- and right-moving electrons conserve separately. Hence, at $t = -\infty$ the system can be described by a partition function with two chemical potentials $\mu_R = E_F$ and $\mu_L = E_F + eV$ conjugated with the particle numbers N_R and N_L. This initial state determines bare Keldysh Green functions.

We will consider only zero temperature. It is convenient to switch [17] to the interaction representation $H \to H - \mu_R N_R - \mu_L N_L$. This transformation induces time dependence in the electron creation and annihilation operators. As a result, $\sum_n 2\tilde{U}_{2nk_F}\cos(n\sqrt{g}\Phi + \alpha_n)$ in the action should be modified as $\sum_n 2\tilde{U}_{2nk_F}\cos[n\sqrt{g}\Phi + \alpha_n + nA(t)]$, where $A(t) = eVt/\hbar$ [12,17,18]. The backscattered current operator equals [12,18]

$$I_{\rm bs}(t) = dN_L/dt = i[H, N_L]/\hbar = -\delta S/\delta A(t), \quad (4)$$

where we omit dimensional factors such as e, $\hbar$, and v_F for brevity. We need to calculate

$$\langle\hat{I}_{\rm bs}(t=0)\rangle = \langle 0|S(-\infty;0)\hat{I}_{\rm bs}(0)S(0;-\infty)|0\rangle, \quad (5)$$

where $|0\rangle$ denotes the initial state and S is the evolution operator. In the weak impurity case this can be done with the perturbation theory in $\tilde{U}_{2nk_F}$ using the bare Green function [15] $\langle 0|\Phi(t_1, x_1 = 0)\Phi(t_2, x_2 = 0)|0\rangle = -2\ln(\delta + i[t_1 - t_2])$, where δ is an infinitesimal positive constant.

If all $\alpha_k = 0$ then the ratchet current is zero. Indeed, at $\alpha_k = 0$ the action (3) is invariant under the transformation $\Phi \to -\Phi$, $V \to -V$ while the current operator (4) changes its sign. As discussed above, for an asymmetric potential we expect $\alpha_2 \neq 0$. Then a ratchet current I_r emerges in the order $\tilde{U}^2_{2k_F}\tilde{U}_{4k_F}$. Before the calculation of I_r let us determine its voltage dependence with a heuristic argument similar to Ref. [12]. As one changes the energy scale E, the backscattering amplitudes $\tilde{U}_{2nk_F}$ in the action (3) scale as $\tilde{U}_{2nk_F}(E) \sim \tilde{U}_{2nk_F}E^{n^2g-1}$ [12]. This renormalization stops at the energy scale V. Assuming that a scattering matrix approach could be applied for an estimation of the current, we write $I_{\rm bs}(V) \sim VR_{\rm eff}(V)$, where $R_{\rm eff}(E) = \sum{\rm const}\tilde{U}^2_{2nk_F}(E) + \sum{\rm const}\tilde{U}_{2nk_F}(E)\tilde{U}_{2mk_F}(E)\tilde{U}_{2lk_F}(E) + \ldots$ is an effective reflection coefficient. Quadratic terms do not contribute to the ratchet current. The leading contribution emerges in the order $\tilde{U}^2_{2k_F}\tilde{U}_{4k_F}$. One gets $I_r \sim V\tilde{U}^2_{2k_F}(V)\tilde{U}_{4k_F}(V) \sim V^{6g-2}$. Below we obtain the same result rigorously from Eqs. (4) and (5).

Expanding Eq. (5) to the order $\tilde{U}^2_{2k_F}\tilde{U}_{4k_F}$ gives, after tedious but straightforward manipulations,

$$I_r = 2\sin\alpha_2\tilde{U}^2_{2k_F}\tilde{U}_{4k_F}\Big\{\int_{-\infty}^{\infty} dtd\tau\cos[V(\tau - t)]P(-(t+\tau), -\tau, -t) - \int_{-\infty}^{\infty} dtd\tau\cos[V(\tau + t)]P(\tau - t, \tau, t)\Big\}, \quad (6)$$

where $P(t, s, q) = (\delta - it)^{2g}(\delta - is)^{-4g}(\delta - iq)^{-4g}$. Dimensional analysis shows that $I_r \sim V^{6g-2}$ in agreement with our previous estimate. The first integral in (6) is zero as seen from the location of the branching points of the function P. The second integral yields

$$I_r = -\sin\alpha_2\tilde{U}^2_{2k_F}\tilde{U}_{4k_F}\cos(\pi g) \times \frac{2^{2+2g}\pi^{3/2}\Gamma(g + 1/2)}{\Gamma(4g)\Gamma(3g)}|V|^{6g-2}. \quad (7)$$

This expression becomes 0 at $g = 1/2$. We also get a zero ratchet current for noninteracting electrons, $g = 1$,

because the Hamiltonian (1) is quadratic in Fermi operators in the noninteracting case and hence no operators which backscatter more than one electron can appear, $\tilde{U}_{4k_F} = 0$. At small g the ratchet current (7) is proportional to a negative power of the voltage. This denotes an unusual behavior: the dc response to an ac voltage grows as the ac voltage decreases.

So far we ignored the voltage dependence of the injected charge density. At $g \ll 1$, Eq. (7) gives the main contribution to the ratchet current only for $eV < \sqrt{UE_F}$. For g close to 1 the result (7) is always exceeded by another contribution. This contribution emerges in the second order in U and is related to the voltage dependence of the injected charge density. The density is proportional to k_F, which enters the expression for U_{2k_F} in Eq. (2). At small $V \ll E_F$ the correction to U_{2k_F} is a linear function of V. The substitution of this correction into Eq. (2) gives an additional ratchet current

$$I_r^{(\text{density})} \sim \frac{eU_{2k_F}^2 (eV)^{2g}}{hE_F^{2g+1}}. \tag{8}$$

For $g > 1/3$ and $V > V^*$ the contribution (8) always exceeds (7). At $g < 1/3$ the current (8) is greater than (7) above a threshold voltage that depends on U and g. As we already discussed, I_r (7) is comparable with the total current $I(V) \sim e^2V/h$ at small g near the border of the perturbatively accessible region $UV^{g-1}/E_F^g < 1$. On the other hand, Eq. (8) provides only a small correction to the total current for any g. Still a repulsive interaction of any strength enhances the ratchet effect as seen from the comparison of the current (8) for $g < 1$ and for the noninteracting case $g = 1$.

What happens beyond the perturbative region when $V < V^* \sim U^{1/(1-g)}$? As the energy scale decreases the effective impurity strength grows. Hence, we need to consider a strong $U > E_F$ limit. In this limit we have a weak tunneling between the left and right halves of the wire. The current is $I(V) \sim t^2V^{(2/g)-1}/E_F^{2/g}$, where t is the tunneling amplitude [12]. Inserting the voltage dependence of the tunneling amplitude in the expression above we estimate $I_r(V) \sim V^{2/g}$.

A single impurity model (3) can be used only when the potential $U(x)$ is confined in a small space region of size $a < a_V \sim \hbar v_F/(eV)$. If the potential changes slowly at the scales $x > a_V \gg 1/k_F$ it cannot backscatter electrons since backscattering involves high momentum transfers, $\Delta k \geq k_F$. Interesting interference effects are possible for a two-impurity potential $U_1\delta(x) + U_2\delta(x-a)$ and other $U(x)$ which significantly change at the scale $1/k_F$ but are nonzero in a region of size $a \sim a_V$. In the two-impurity case the current oscillates as a function of the voltage bias [20]. For $U_1, U_2 \ll E_F$, $I - e^2V/h \sim [U_1^2 + U_2^2 + 2U_1U_2\cos(2k_Fa)H(geVa/[\hbar v_F])]|V|^{2g-1}\,\text{sign}V$, where $H(x) = \sqrt{\pi}\Gamma(2g)J_{g-1/2}(x)/[\Gamma(g)(2x)^{g-1/2}]$ and $J_{g-1/2}(x)$ is the Bessel function of the first kind [20]. The main contribution to the ratchet current at $a \sim a_V$ comes from the shift of k_F due to the change of the electrochemical potential of the left reservoir by eV. From the minimum of the quadratic part of the bosonized Hamiltonian one finds the charge density shift [21]. This gives $k_F = k_F^{(0)} + g^2eV/(2\hbar v_F)$. After the substitution to the expression for the total current I we find

$$I_r(V) \sim U_1U_2\sin(2k_F^{(0)}a)|V|^{2g-1}\sin(g^2e|V|a/[\hbar v_F]) \times H(geVa/[\hbar v_F]). \tag{9}$$

Thus, $I_r(V)$ oscillates. Notice that for $V \sim V^* \ll E_F$, $a \sim a_{V^*}$ the ratchet current (9) is of the order of the total current $\sim e^2V/h$.

In conclusion, we have found the ratchet current for strong and weak asymmetric potentials. It exhibits a set of universal power dependencies on the voltage and can grow as the voltage decreases.

This work was supported by the US DOE Office of Science under Contract No. W31-109-ENG-38.

[1] A. Aviram *et al.*, Chem. Phys. Lett. **29**, 277 (1974).
[2] N. J. Geddes *et al.*, Appl. Phys. Lett. **56**, 1916 (1990); A. S. Martin *et al.*, Phys. Rev. Lett. **70**, 218 (1993); C. Joachim *et al.*, Nature (London) **408**, 541 (2000).
[3] H. Linke *et al.*, Science **286**, 2314 (1999); A. Löfgren *et al.*, Phys. Rev. B **67**, 195309 (2003).
[4] H. W. C. Postma *et al.*, Science **293**, 76 (2001).
[5] S. Roddaro *et al.*, Phys. Rev. Lett. **90**, 046805 (2003).
[6] P. Reimann *et al.*, Phys. Rev. Lett. **79**, 10 (1997).
[7] J. Lehmann *et al.*, Phys. Rev. Lett. **88**, 228305 (2002).
[8] S. Scheidl *et al.*, Phys. Rev. B **65**, 195305 (2002).
[9] A. Komnik *et al.*, Phys. Rev. B **68**, 235323 (2003).
[10] B. Spivak *et al.*, Phys. Rev. Lett. **93**, 226801 (2004).D. Sanchez *et al.*, Phys. Rev. Lett. **93**, 106802 (2004).
[11] V. I. Belinicher *et al.*, Sov. Phys. Usp. **23**, 199 (1980).
[12] C. L. Kane *et al.*, Phys. Rev. B **46**, 15 233 (1992).
[13] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Addison-Wesley, Reading, MA, 1965).
[14] We do not take Friedel oscillations into account since they do not change our estimate qualitatively.
[15] A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelik, *Bosonization and Strongly Correlated Systems* (Cambridge University Press, Cambridge, England, 1998).
[16] D. L. Maslov *et al.*, Phys. Rev. B **52**, R5539 (1995); V. V. Ponomarenko, *ibid.* **52**, R8666 (1995); I. Safi *et al.*, *ibid.* **52**, R17 040 (1995).
[17] D. E. Feldman *et al.*, Phys. Rev. B **67**, 115337 (2003).
[18] B. Trauzettel *et al.*, Phys. Rev. Lett. **92**, 226405 (2004); Phys. Rev. B **71**, 165309 (2005).
[19] J. Rammer *et al.*, Rev. Mod. Phys. **58**, 323 (1986).
[20] C. de C. Chamon *et al.*, Phys. Rev. B **55**, 2331 (1997).
[21] H. Grabert, cond-mat/0107175.