Observation of Two-Dimensional Classical Wave Localization: Third Sound on Superfluid ^{4}He Films on a Disordered Substrate

D. R. Luhman, J. C. Herrmann, and R. B. Hallock

Laboratory of Low Temperature Physics, Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003, USA

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We present the results of measurements of the propagation of third sound waves on superfluid ^{4}He adsorbed to two-dimensional ordered and disordered substrates. In the disordered case we compare the experimental results to theoretical predictions of classical wave localization in such systems and conclude that classical wave localization is present in our system.

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Wave localization is a phenomenon of widespread importance. Anderson first predicted the existence of localization in quantum electronic systems [1] and later wave localization was predicted to be achievable in a variety of classical systems [2–6]. The presence of localization in diverse physical systems is due to the ubiquitous existence of waves in nature and because localization is in its essence a purely wave phenomenon. Localization is the spatial confinement of a wave and its energy; extended states are not possible in a system that is localized. The signature of localization is that the time averaged wave amplitude, $|\psi|$, exponentially decreases as $|\psi| \sim e^{-x/\xi}$, where x is the distance from the wave source. The localization length, ξ, characterizes the length scale of the wave confinement and is a quantity of primary interest.

Localization occurs in systems where wave scattering sites are arranged in a random, disordered manner. The ability of a system to exhibit localized states depends on the characteristics of the random media. In conducting electron systems the mean free path is often much larger than the electron wavelength and the systems are said to exhibit weak localization. In classical systems [7] it is possible to achieve the opposite situation where the excitation wavelength is commensurate with or larger than the scattering mean free path, a situation termed strong localization. We report here the results of experiments that use third sound surface excitations of thin superfluid ^{4}He films to investigate the strongly localized regime for a two-dimensional classical system.

Third sound is a thickness and temperature fluctuation that can occur on atomically thin superfluid helium films. The speed of a third sound wave on a thin helium film of thickness d on a smooth surface is given by [8] $C_3^2 = (\langle\rho_s\rangle/\rho)Fd$, where $(\langle\rho_s\rangle/\rho)$ is the effective superfluid fraction in the film [9]. The restoring force, F, is dominated by the van der Waals attraction, which for thin films can be approximated as $F = 3\alpha_v/d^4$, where the van der Waals constant is $\alpha_v = 27$ (layers) ^{3}K for helium on glass [10] and one layer of helium is defined [8] as 0.36 nm. The speed of third sound on a rough surface, such as thermally deposited CaF_2 [11], is less than that on a smooth surface [12] and therefore patches of rough CaF_2 deposited on a smooth glass substrate can effectively scatter third sound [13]. The effectiveness of using third sound to investigate wave phenomena in one-dimension, including localization, has been shown previously [14–16].

Much theoretical effort has been expended to calculate the localization length, ξ, for a variety of systems. Cohen and Machta [5] have used a course grained approach to calculate ξ for third sound propagating in a two-dimensional disordered medium. By assuming that the characteristic times and lengths associated with the scatterers are much shorter than those of third sound they have shown [5] that the only quantity needed for scatterer characterization is $\kappa = \partial \Delta V/\partial d$, where ΔV is the excess volume of fluid adsorbed to a single scatterer and d is the equilibrium helium film thickness on a smooth surface away from the scatterer. The system is characterized by the index of refraction of third sound through the system, which is given by $n_s = C_0/C_s = (1 + \bar{\rho}_0\kappa)^{1/2}$, where C_0 is the speed of third sound on a clean smooth substrate without scatterers and C_s is the speed of third sound on the disordered substrate with an average areal density of scatterers of $\bar{\rho}_0$ [17].

Using these assumptions Cohen and Machta [5] have shown that the frequency-dependent localization length, $\xi(E)$, is given by

$$\xi(E) = l(E)e^{(E_0/E)^2}, \qquad (1)$$

which is valid in the low frequency regime ($E < E_0$), where $E = 2\pi f$, f is the third sound frequency, E_0 is a constant, and $l(E)$ is the scattering mean free path. $\xi(E)$ is strongly divergent as $E \to 0$, but rapidly decreases toward the scattering length as $E \to E_0$. Writing E_0 and $l(E)$ in terms of experimentally accessible quantities for third sound Cohen and Machta [5,18] find that

$$\xi(f) = \frac{C_0^3 n_s \bar{\rho}_0}{4\pi^3 f^3 (n_s^2 - 1)^2} \exp\left\{\frac{\bar{\rho}_0}{2\pi}\left(\frac{C_0 n_s}{f(n_s^2 - 1)}\right)^2\right\}. \qquad (2)$$

In this Letter, we present third sound data consistent with the predictions of Eq. (2).

The substrate used to investigate localization was a plain glass substrate of dimensions 5.08×2.54 cm with two Ag third sound generators and six thin-film Al superconducting transition-edge detectors deposited on it and arranged as shown in Fig. 1(a). The superconducting detectors record the presence of third sound by detecting the associated temperature fluctuation. Scatterers were then deposited on the surface of the substrate covering a lateral area 2 cm wide and decorating the full length of the substrate. The scatterers were formed by evaporating 300 nm of CaF_2 onto the substrate through a thin brass shim stock mask that was perforated by 1514 randomly drilled holes of diameter $a = 0.2505$ mm. The scatterer positions were determined using a random number generator and do not overlap. The scatterers have a packing fraction of $\bar{\rho}_0 \pi a^2 = 0.3$ with a mean scatterer density of $\bar{\rho}_0 = 152.2\ \mathrm{cm}^{-2}$. A substrate with a periodic (i.e., ordered) arrangement of scatterers was also fabricated to provide a direct comparison to the disordered data, as shown in Fig. 1(b). A plain glass substrate with no CaF_2 and a fourth substrate uniformly coated with 300 nm of CaF_2 were also used in the experiment, each with a single third sound driver and detector.

Transmission data [19] were collected at four different ^{4}He film thicknesses that spanned the range $5.43 \leq d \leq 11.41$ layers. The nominal ^{4}He film thickness was determined using the approximate relation $d^3 = \alpha_v/[T \ln(P_0/P)]$, where P is the vapor pressure in the sample cell and P_0 is the saturated vapor pressure at temperature T. The samples were mounted in a brass sample chamber, placed in a standard ^{4}He pumped-bath Dewar, and operated at a temperature of $T = 1.650$ K. The temperature was regulated to $\sim$1 mK during the course of the measurements.

The transmission spectrum for third sound propagation across the substrates was determined by applying a continuous sinusoidal voltage to a selected third sound generator on a substrate and monitoring the response at a specific detector. The amplitude was measured as a function of frequency by monitoring the voltage drop across the current-biased superconducting detector with a lock-in amplifier operating in "$2f$" mode. Since the third sound was generated thermally, the frequency of the waves was twice the drive voltage frequency. All data reported here are given in terms of the third sound frequency. In this way the amplitude transmission spectrum was measured as a

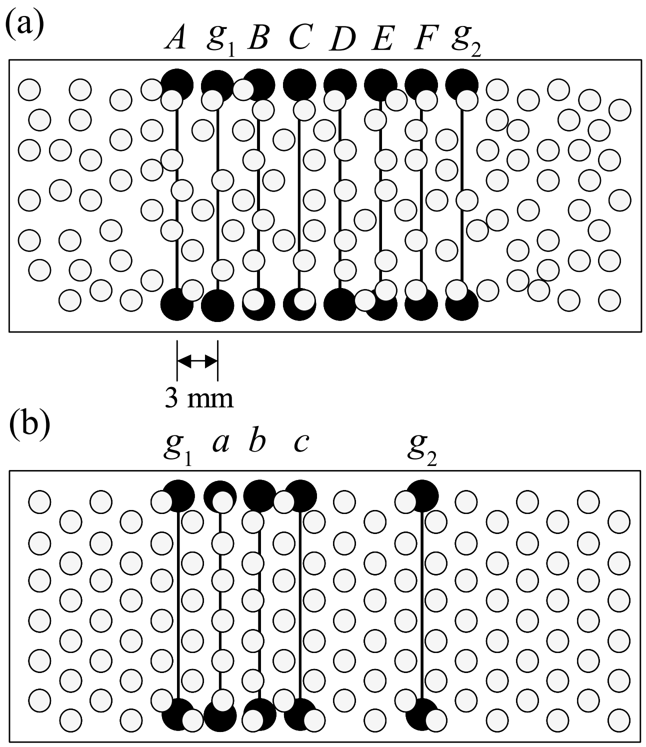

FIG. 1. Schematic representation (not to scale) of the (a) disordered and (b) ordered substrates. The solid lines shown with solid black circular contact pads located at each end are the third sound generators and detectors. g_1 and g_2 indicate the third sound generators on each substrate and uppercase (lowercase) letters indicate the third sound detectors on the disordered (ordered) substrate. The lightly shaded circles represent the CaF_2 scatterers.

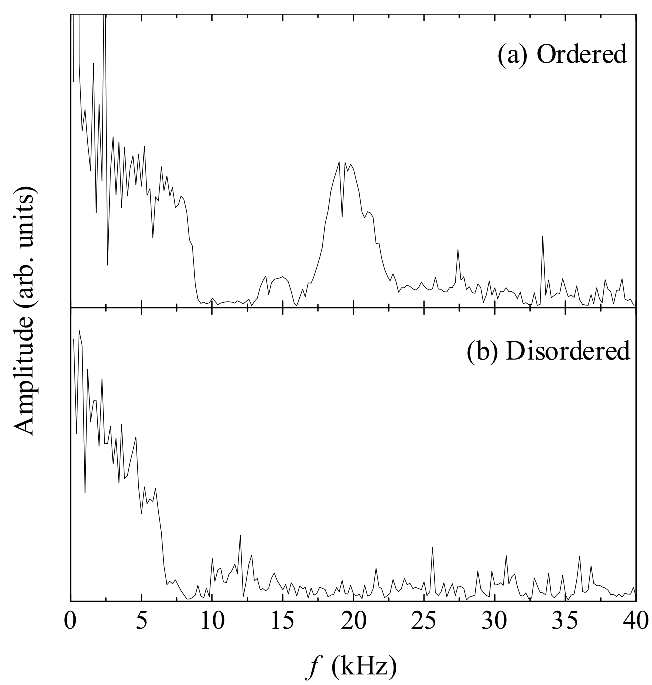

FIG. 2. Third sound amplitude in arbitrary units for (a) detector a on the ordered substrate [Fig. 1(b)] and (b) detector B on disordered substrate [Fig. 1(a)] versus frequency. The generator-detector distance was 3 mm. The waves originated at g_1 in both cases. Instrumental noise is typically small as seen in the gap near 10 kHz for the ordered case. Data for an unpatterned substrate have been shown previously [13]. The helium film thickness is $d = 5.43$ layers for these data.

function of frequency for each of the third sound generator-detector pairs on the ordered and disordered substrates.

Pulsed third sound was also measured on the plain and uniformly coated substrates by applying a 50 μs-wide square-wave voltage pulse to the third sound generator and measuring the time it takes for the pulse to arrive at the detector. This allows us to determine the speed of third sound across these surfaces.

The most obvious evidence for the presence of localization is the comparison between the third sound amplitude on the disordered and ordered substrates. Figure 2 shows these amplitudes for a ^{4}He film thickness of $d = 5.43$ layers as observed at a detector located at a distance of 3 mm from the wave source. The passband present on the ordered substrate near 20 kHz is clearly absent in the disordered data indicating that the disorder effects the frequency dependence of the amplitude in a manner consistent with localization. The amplitude on the disordered substrate falls to zero as frequency increases and does not increase significantly at higher frequency. These features are typical of the data collected at all ^{4}He film thicknesses and all third sound generator-detector separations. The general behavior is similar to that seen previously for a different type of scattering site in the one-dimensional case [14]. While the data for one dimension and two dimensions look similar, the frequency dependence of the localization length is very different, which alters the behavior along the frequency axis. The jagged nature of the amplitude curves is likely caused by resonant modes induced by the geometry of the substrate [14]. Instrumental noise also likely contributes somewhat to the sharp features.

In order to do a quantitative analysis of the disordered substrate data we must account for ordinary third sound attenuation, which can mimic localization effects. Attenuation decreases the measured third sound amplitude with distance as $|\psi| \sim e^{-\alpha x}$, where α is the attenuation coefficient. For the data presented here we can use the thin-film approximation for α predicted by Bergman [20] and corrected to fit our data. To estimate the experimental attenuation coefficient for our disordered substrate we use the prediction of Bergman and compare that to the ordered substrate data. We find that the frequency dependence of Bergman's attenuation coefficient, α_B, is sufficient to describe the frequency dependence of the data. However, we need to multiply α_B by a thickness dependent coefficient, $A_0(d)$, to describe the thickness dependence of the data. Therefore our experimentally determined estimate of the attenuation coefficient on the disordered substrates is $\alpha = A_0(d)\alpha_B$, where $A_0(d)$ is found to have the form of $A_0(d) \approx 0.01d^3$. Bergman [20] predicts that $\alpha_B \sim d^{-5/2}$, in contrast to our results that suggest that $\alpha \sim d^{1/2}$.

To compare our experimental results to the theory of Cohen and Machta [5] we have calculated the time averaged amplitude of the wave as a function of frequency using [21]

$$|\psi| = e^{-x(\xi^{-1}+\alpha)}, \tag{3}$$

where we have included both localization and attenuation effects. ξ was calculated using Eq. (2) with $\bar{\rho}_0 = 152.2$ cm^{-2} and C_0 was determined experimentally for each value of d with n_s a free parameter. α was determined as discussed in the above paragraph. We compare our data, normalized such that in each case the amplitude trend is to unity as $f \to 0$, to the prediction of Eq. (3). Figure 3, which is representative of all of our data, shows the comparison for $x = 3$ mm and $x = 6$ mm for $d = 5.43$ layers and $d = 10.67$ layers. The smooth lines calculated from Eq. (3) have two main features. The initial gentle drop in amplitude at low frequencies is due entirely to the attenuation term, while the sudden more abrupt drop in amplitude with frequency at somewhat higher frequencies is due to the localization term in the exponent of Eq. (3). The position of the sudden drop in frequency is determined to some degree

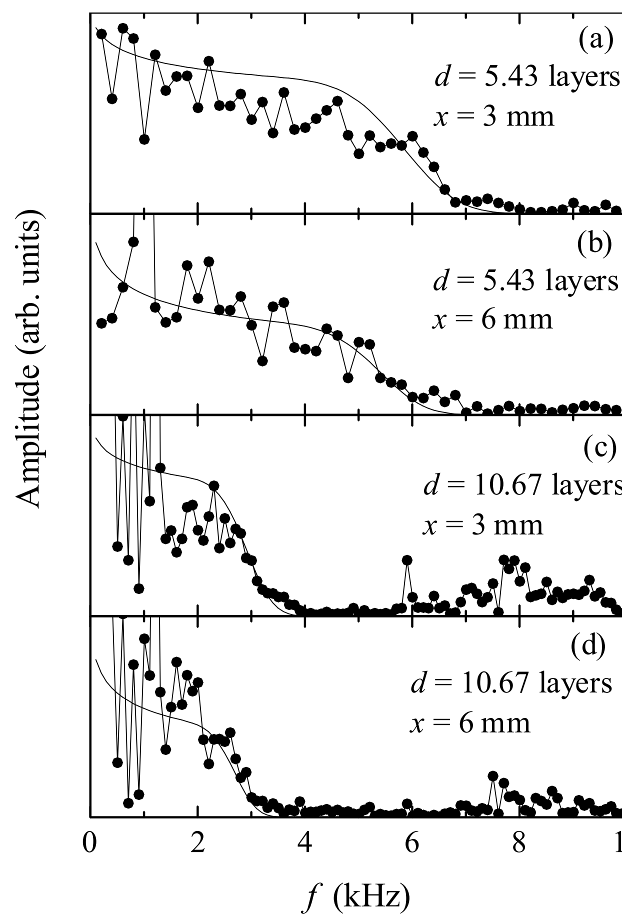

FIG. 3. Third sound amplitude as measured at detectors B and C as a function of frequency for generator g_1 for (a) $d = 5.43$ layers and $x = 3$ mm, (b) $d = 5.43$ layers and $x = 6$ mm, (c) $d = 10.67$ layers and $x = 3$ mm, (d) $d = 10.67$ layers and $x = 6$ mm. The lines connecting adjacent data points are a guide to the eye and the smooth solid lines are the calculated results of Eq. (3).

TABLE I. Values of the deduced experimental parameters for each ^{4}He film thickness. The values of n_s are as determined by the comparison of the data and Eq. (2), and $E_0/(2\pi)$ is calculated using the so determined values of n_s.

d (layers)	C_0 (m/s)	n_s	$E_0/(2\pi)$ (kHz)
5.43	21.7	1.70	9.6
6.32	18.2	1.63	8.8
8.36	12.8	1.60	6.5
10.67	9.3	1.58	4.8
11.41	8.5	1.57	4.5

by our free parameter n_s. n_s was found to be constant for each value of d (i.e., C_0) and the results are given in Table I. The agreement between the experimental data and the theory shown in Fig. 3 is quite good, particularly with regard to the features due to localization. This sudden decrease in received amplitude cannot be explained by attenuation. The agreement with theory provides compelling evidence that localization is present in this two-dimensional system.

The valid application of the theory of Cohen and Machta [5] requires that a scatterer be in local equilibrium with the fluid associated with the propagating third sound wave. Basically this requires that the third sound wavelength be longer than the length scale of the scatterers. Since the theory is valid only for $f < E_0/(2\pi)$, for the region of parameter space investigated here the minimum third sound wavelength would be $n_s C_0/(E_0/2\pi) \approx 3.4$ mm compared to the scatterer radius $a = 0.2505$ mm. The features in the data that are relevant to localization occur roughly at a wavelength of twice this. In addition, the theory of Cohen and Machta [5] requires that an extra amount of adsorbed fluid, ΔV, be associated with each scatterer to provide the scattering mechanism. In the case of CaF_2 it has been shown that for 300 nm CaF_2, the areal density of adsorbed ^{4}He is, depending on the value of the chemical potential, up to 10 times greater than that of a smooth surface [12]. Therefore we expect that both of these criteria are satisfied.

We have shown that high frequency propagating third sound modes are not present on our two-dimensional disordered substrate, in direct contrast to what is observed on the ordered substrate. In addition, when experimentally determined attenuation is taken into account, we have shown the correspondence between our data and expectations based on the theory of Cohen and Machta [5]. These results are strong evidence that two-dimensional classical wave localization is present in this system.

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