Modified Jeans Instability for Dust Grains in a Plasma

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An investigation of the properties of linear stability is conducted for a system consisting of particles having mass m and charge q, interacting through the gravitational and electrostatic force (Jeans instability). However, in light of recent works showing that dust particles in a plasma can have a Lennard-Jones-like shielding potential, a new set of equations has been derived, where the electrostatic interaction among the dust particles is Lennard-Jones-like instead of Coulomb-like. A new condition for the gravitational instability is derived, showing a broader spectrum of unstable modes with faster growth rates.

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Introduction.—It is well known that a plasma acts on a dust grain immersed in it in two ways: first, the grain is charged by plasma particles collection and, second, the shielding potential around the grain is governed by the screening due to the plasma. In the absence of other processes such as electron emission from the dust grain, the grain is negatively charged owing to the higher mobility of the electrons with respect to the ions. In this case, known as primary charging, the shielding potential is monotonic [1].

However, Delzanno *et al.* have shown in a recent Letter [2] how the scenario described for primary charging changes dramatically if the grain is an electron emitter. In fact, if the electron current from the grain is sufficiently high not only the polarity of the grain can be reversed, but the shielding potential no longer shows the typical monotonic behavior. Instead, a potential well appears, reminiscent of a Lennard-Jones potential. Note that the work of Ref. [2] considered the case of thermionic emission, motivated by the fact that in meteor physics the role of thermionic emission has been shown to be very important [3]. However, the same qualitative results have been recently obtained for photoemission and secondary emission as well [4].

The presence of a potential well in the shielding potential has extremely important consequences. In fact, it can lead to attractive forces on other grains even if they have the same sign of charge. This novel mechanism for the attraction is not the only instance where grains with the same sign of charge can attract each other but it might be particularly interesting for astrophysical situations, for example, to study dust aggregation in astrophysical systems.

Focusing then on astrophysical scenarios, the simplest theory of aggregation of masses in space is certainly the Jeans instability [5]. The system consists of particles that can aggregate together depending on the relative magnitude of the gravitational force to the pressure force. In its basic form, the theory of the Jeans instability does not include charged particles, but it is easy to show that the electrostatic repulsion reduces or even cancels the threshold for the gravitational instability.

The idea presented in this Letter consists of investigating a system of gravitating particles, all sharing the same charge, that do not interact electrostatically with each other through the Coulomb repulsive potential. Instead, the electrostatic interaction is governed by the Lennard-Jones-like potential that was discovered in Ref. [2].

Mathematical model.—The fundamental starting point for our study is Poisson's equation. Poisson's equation determines the electrostatic potential from a continuous distribution of charges and hides in itself the fact that all the particles constituting the medium interact through Coulomb's law. In other words, the potential for a continuous charge distribution $\rho_e(\mathbf{x})$ defined as

$$\phi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0} \int \rho_e(\mathbf{x}') \frac{d\mathbf{x}'}{|\mathbf{x}' - \mathbf{x}|} \qquad (1)$$

leads to Poisson's equation and it is a direct consequence of Coulomb's law [6]. Therefore, if one arbitrarily changed the electrostatic force acting at particle level, the consequence would be a different relation between the continuous charge distribution and the potential or, in other words, a modified Poisson's equation.

On the other hand, it is well known that in a dusty plasma, dust grains do not interact through Coulomb's law. The presence of a plasma surrounding the grains is such that the charge on the grains is screened by the plasma. In its simplest form, for a grain with a radius that is small compared to the Debye length, the screening potential is represented well by the Debye-Huckel potential [1]. However, recent works by Delzanno *et al.* [2,4] have shown that different scenarios are possible. In particular, in the presence of an electron emitting grain, the shielding potential can show a nonmonotonic behavior, with a potential well. We have recently developed an exact theory for the shielding potential around an emitting dust [4]. However, the expression for the screening potential is obtained numerically. We find that an accurate fit of the theoretical result can be obtained with the following simple expression:

$$\phi(r) = \frac{q}{4\pi\varepsilon_0 r}(\alpha_1 e^{-r/\lambda_1} - \alpha_2 e^{-r/\lambda_2}), \quad (2)$$

where q is the particle charge. The parameters α_1, α_2, λ_1, and λ_2 depend on the physical properties of the system, i.e., the plasma temperatures and densities, as well as the grain. Note that there are many functional dependencies that can be used to parametrize a potential well; among them, however, expression (2) has the advantage of including pure Coulomb potentials (if $\lambda_1, \lambda_2 \to \infty$ and $\alpha_1 - \alpha_2 = 1$) and the Debye shielding (for instance if $\alpha_2 = 0$). It also leads to a fairly simple derivation of a modified Jeans instability (see below).

The goal here is to derive a model that describes the evolution of a system of grains, all sharing the same charge q and mass m, where the interaction among the grains is due to the gravitational force and to the electrostatic force. The grains however, do not interact through Coulomb's law: the interaction potential of each grain is given by Eq. (2). We derive a model for a system where the plasma is present, but implicitly, through the potential of Eq. (2). The new system has the advantage of being reasonably simple while retaining the physics necessary to study the linear properties of the Jeans instability. Moreover, our choice could be justified by considering that the grains and the plasma evolve on completely different time scales: owing to the large mass of the grains compared to the plasma particles, the grains are much slower than the plasma and on the grain's time scale the plasma can be considered adiabatic.

Below, we build a modified Poisson's equation containing the Lennard-Jones-like potential of Eq. (2). In order to do so, we use the Cartesian geometry and physical quantities are expressed in the International System units. It is useful to start from the following definition of the potential [obtained from expression (2) by setting $\alpha_1 = 1$ and $\alpha_2 = 0$]:

$$\phi_1(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\int \rho_e(\mathbf{x}')\frac{e^{-|\mathbf{x}'-\mathbf{x}|/\lambda_1}}{|\mathbf{x}'-\mathbf{x}|}d\mathbf{x}'. \quad (3)$$

By applying the Laplacian operator to Eq. (3), one obtains

$$\begin{aligned}\nabla^2\phi_1(\mathbf{x}) = {} & \frac{1}{4\pi\varepsilon_0}\int \rho_e(\mathbf{x}')\frac{e^{-|\mathbf{x}'-\mathbf{x}|/\lambda_1}}{|\mathbf{x}'-\mathbf{x}|^2}\left(\frac{-3}{\lambda_1} + \frac{-3}{|\mathbf{x}'-\mathbf{x}|}\right) \\ & \times d\mathbf{x}' + \frac{1}{4\pi\varepsilon_0}\int \rho_e(\mathbf{x}')e^{-|\mathbf{x}'-\mathbf{x}|/\lambda_1}(\mathbf{x}'-\mathbf{x}) \\ & \cdot(\mathbf{x}'-\mathbf{x})\left(\frac{3}{\lambda_1|\mathbf{x}'-\mathbf{x}|^4} + \frac{3}{|\mathbf{x}'-\mathbf{x}|^5}\right)d\mathbf{x}' \\ & + \frac{\phi_1(\mathbf{x})}{\lambda_1^2}. \end{aligned} \quad (4)$$

With simple algebra, it follows immediately that the integral terms of Eq. (4) vanish whenever $\mathbf{x}' \neq \mathbf{x}$. Therefore, we are left only with the contribution from $\mathbf{x}' = \mathbf{x}$ which can be evaluated as follows [5]. Let us consider a sphere of radius ε centered at $\mathbf{x}$. In the limit of vanishing ε, the charge density is constant within the sphere and Eq. (4) becomes

$$\nabla^2\phi_1(\mathbf{x}) = \frac{\phi_1(\mathbf{x})}{\lambda_1^2} - \frac{\rho_e(\mathbf{x})}{4\pi\varepsilon_0}\int_{|\mathbf{x}'-\mathbf{x}|\le\varepsilon}\frac{(\mathbf{x}'-\mathbf{x})\cdot\mathbf{n}}{|\mathbf{x}'-\mathbf{x}|^3}dS. \quad (5)$$

In Eq. (5), we have used the divergence theorem; $\mathbf{n} = \mathbf{x}' - \mathbf{x}$ is the unit vector normal to the sphere surface and $dS = \varepsilon d\Omega$ is the surface element ($d\Omega$ being the solid angle element). It is easy to show that in the limit of vanishing ε, the integral term in Eq. (5) reduces to $-\rho_e/\varepsilon_0$. Finally, we can conclude that the following Poisson's equation relates the potential and the distribution of charge, if the electrostatic potential due to an isolated particle is $qe^{-r/\lambda_1}/(4\pi\varepsilon_0 r)$:

$$\nabla^2\phi_1(\mathbf{x}) = \frac{\phi_1(\mathbf{x})}{\lambda_1^2} - \frac{\rho_e(\mathbf{x})}{\varepsilon_0}. \quad (6)$$

Having established relation (6), it is obvious that the contribution of the term proportional to α_2 in Eq. (2) can be modeled in the same way, with the replacement of λ_1 with λ_2 (in this case the potential is labeled with the index $_2$ in the remaining text of this Letter).

The definition of our model is completed by the Vlasov equation for the dust grains and by Poisson's equation for the gravitational potential, thus our model consists of the following equations:

$$\begin{aligned} & \frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{x}}f(\mathbf{x},\mathbf{v},t) + \frac{\mathbf{F}(\mathbf{x})}{m}\cdot\nabla_{\mathbf{v}}f(\mathbf{x},\mathbf{v},t) = 0, \\ & \nabla^2\phi_g(\mathbf{x}) = 4\pi G\int f(\mathbf{x},\mathbf{v},t)d\mathbf{v}, \\ & \nabla^2\phi_1(\mathbf{x}) = \frac{\phi_1(\mathbf{x})}{\lambda_1^2} - \frac{q}{m\varepsilon_0}\int f(\mathbf{x},\mathbf{v},t)d\mathbf{v}, \\ & \nabla^2\phi_2(\mathbf{x}) = \frac{\phi_2(\mathbf{x})}{\lambda_2^2} - \frac{q}{m\varepsilon_0}\int f(\mathbf{x},\mathbf{v},t)d\mathbf{v}. \end{aligned} \quad (7)$$

Finally, the forces acting on the grains are gravitational and electrostatic:

$$\frac{\mathbf{F}(\mathbf{x})}{m} = -\nabla\phi_g - \frac{q}{m}(\alpha_1\nabla\phi_1 - \alpha_2\nabla\phi_2) \quad (8)$$

and the mass density distribution is related to the distribution function through $\rho(\mathbf{x},t) = \int f(\mathbf{x},\mathbf{v},t)d\mathbf{v}$.

Summarizing, the model consisting of Eqs. (7) and (8) describes a system of dust grains, each having mass m and charge q, interacting through the gravitational force and through the electrostatic force described by Eq. (2). The system is therefore similar to the one usually considered for studies of the gravitational Jeans instability, the only difference being the different electrostatic force acting among the grains. Below, we study the properties of linear stability of the system.

Dispersion relation.—Following the classic Jean's swindle hypothesis [5], we consider an initial configuration where the particle density is constant, $\rho = \rho_0$, and there is no macroscopic flow, $\mathbf{V} = 0$. The kinetic distribution function is consequently dependent only upon the microscopic velocities, $f_0(\mathbf{v})$.

System (7) is linearized by assuming each unknown to be composed by the equilibrium quantity (labeled with the index $_0$) plus a perturbation (labeled with the index $_1$) whose magnitude is small compared to the unperturbed value: $f(\mathbf{x}, \mathbf{v}, t) = f_0(\mathbf{v}) + f_1(\mathbf{x}, \mathbf{v}, t)$. Neglecting second order terms and assuming each unknown to depend as $f_1(\mathbf{x}, \mathbf{v}, t) = \hat{f}_1(\mathbf{v}) e^{i(\omega t - \mathbf{k}\cdot\mathbf{x})}$, straightforward algebra leads to the dispersion relation:

$$1 = \left[\frac{4\pi G}{k^2} - \frac{q^2/m^2}{\varepsilon_0}\left(\frac{\alpha_1 \lambda_1^2}{1 + \lambda_1^2 k^2} - \frac{\alpha_2 \lambda_2^2}{1 + \lambda_2^2 k^2}\right)\right] \times \int \frac{\mathbf{k}\cdot\frac{\partial f_0}{\partial \mathbf{v}}}{\omega - \mathbf{k}\cdot\mathbf{v}} d\mathbf{v}. \tag{9}$$

It is immediately seen that if we set $q = 0$ in Eq. (9), the dispersion relation of the classical Jeans instability is recovered [5]. The analysis of Eq. (9) obviously depend on the choice of f_0. A natural choice, which allows the link between a fluid treatment and the kinetic approach presented here, is a Maxwellian distribution function:

$$f_0(\mathbf{v}) = \rho_0 \left(\frac{m}{2\pi k T}\right)^{3/2} e^{-[m(v_x^2 + v_y^2 + v_z^2)/2kT]}, \tag{10}$$

with T the grain kinetic temperature. This is the choice that will be adopted here.

In order to investigate the properties of Eq. (9), let us set $\omega = 0$ and derive a threshold criteria for the instability [5]. For the Maxwellian distribution function (10), one obtains

$$\frac{k_J^2}{k^2} = 1 + \frac{\omega_{\rm pd}^2}{\omega_J^2}\left[\alpha_1 \frac{\lambda_1^2 k_J^2}{1 + \lambda_1^2 k^2} - \alpha_2 \frac{\lambda_2^2 k_J^2}{1 + \lambda_2^2 k^2}\right], \tag{11}$$

where $k_J^2 = 4\pi G \rho_0 / v_{th}^2$ is the Jeans wave vector, $\omega_J^2 = 4\pi G \rho_0$ is the Jeans frequency, $\omega_{\rm pd}^2 = \rho_0 q^2/m^2/\varepsilon_0$ is the dust plasma frequency, and the thermal velocity is defined by $v_{\rm th} = \sqrt{kT/m}$. Equation (11) gives the value of $k^2 = k^{*2}$ such that a perturbation travels in the system with $\omega = 0$. Unstable modes occur for $k < k^*$. The same Eq. (11) can also be obtained by a fluid treatment. The fluid treatment has been used by several authors in the past (e.g., [7] and references therein) for theoretical investigations of the Jeans instability in dusty plasmas. It is interesting to point out that if we had adopted the fluid equivalent of system (7) and set $\alpha_2 = 0$ (namely pure Debye shielding), we would have obtained exactly the same low frequency root of the dispersion relation of Pandey *et al.* [8]. Thus our simplified system (7), in which the behavior of the plasma is just parametrized with some coefficients, is indeed the actual representation of the system for low frequency perturbations (mathematically this is equivalent to $\omega \ll k v_{\rm thi}, k v_{\rm the}$; $v_{\rm thi}$ and $v_{\rm the}$ being ion and electron thermal velocities).

By considering Eq. (11) and setting $\alpha_2 = 0$, it follows immediately that the presence of the plasma reduces the threshold for the instability with respect to the pure gravitational case. On the other hand, the term proportional to $\alpha_2(>0)$ acts to broaden the spectrum of unstable k. In order to make a quantitative comparison, we have determined the shielding potential around a spherical grain for parameters relevant to the densest cores of molecular clouds [9]. Specifically, we have assumed a plasma with electron and ion unperturbed densities $n_e = n_i = 10^{11}$ part/m^3, temperatures $T_e = T_i = T = 0.67$ eV, mass ratio $m_i/m_e = 1836$ and calculated the shielding potential (through the theory developed in Ref. [4]) for a dust grain of radius $a = 1$ mm, work function $W = 4.8$ eV (corresponding to carbon) that emits photoelectrons in response of a UV field of wavelength $\lambda = 256.8$ nm and intensity $I = 2 \times 10^{-5}$ photons/cm^2/s. The latter value is consistent with the UV field created in the vicinity of a star. The value of the dust size is chosen for convenience and should not be taken as typical of any astrophysical system. The shielding potential calculated for these parameters is plotted in Fig. 1 with the solid line, while the dashed line is obtained from expression (2) for $\alpha_1 = 2.01$, $\alpha_2 = 1.14$, $\lambda_1 = 0.0024$ m, and $\lambda_2 = 0.0124$ m (these parameters have been obtained with the least squares method). While the best fit of the functional form chosen represents correctly the potential well depth, the asymptotic behavior presents some differences.

With these parameters, we have calculated the solution of Eq. (11) as a function of $y_J = \lambda_1^2 k_J^2$ and $\omega_{\rm pd}^2/\omega_J^2$. Results are shown in the left panel of Fig. 2, while the right panel shows the case where we have set $\alpha_2 = 0$ (the other parameters unchanged) in order to see the effect of the Debye-Huckel shielding potential, considered in previous works. For reference, the purely gravitational case has $k^* = k_J$. Immediately one can see that indeed the gravitational range of unstable modes is reduced at smaller $k < k^*$

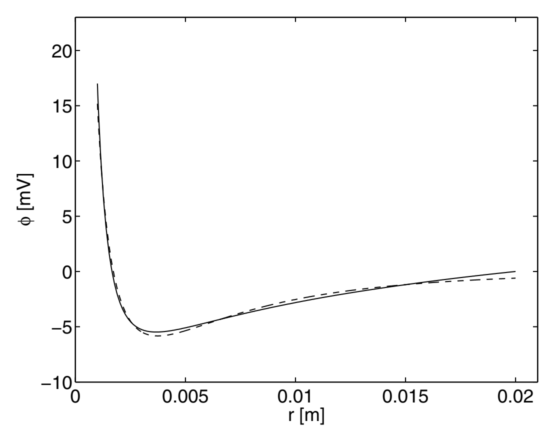

FIG. 1. Shielding potential obtained for a carbon dust grain for parameters relevant to the densest cores of molecular clouds.

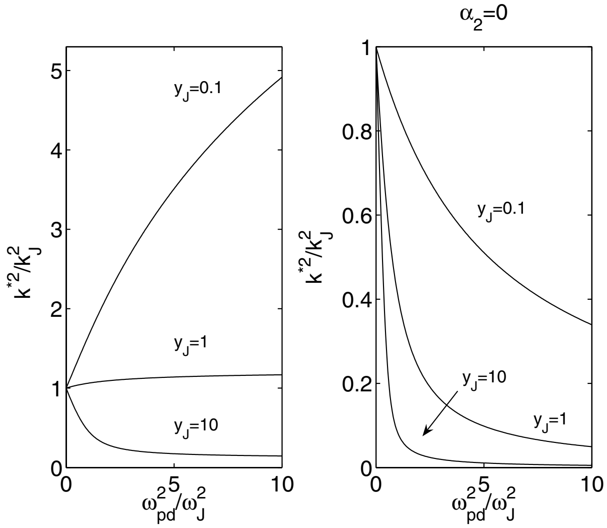

FIG. 2. Solution of Eq. (11) for the interaction potential shown in Fig. 1.

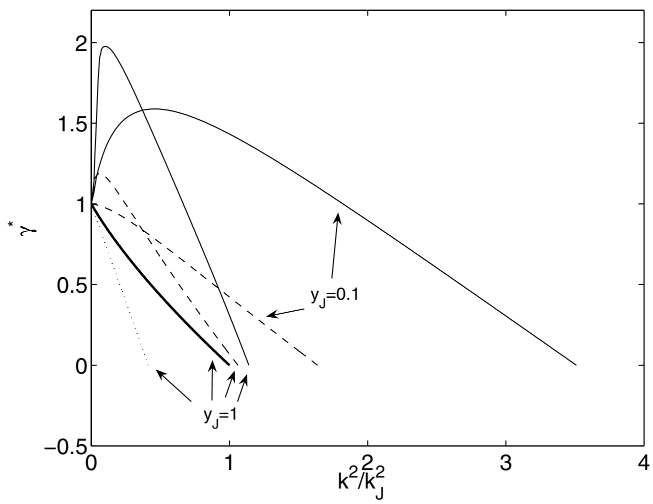

FIG. 3. Normalized growth rates according to Eq. (12).

(i.e., longer wavelengths) by the electrostatic force modeled by the Debye-Huckel potential. On the other hand, the Lennard-Jones interaction potential acts in the opposite way and, depending on y_J, can broaden the spectrum of unstable k even with respect to the pure gravitational analogue.

The same considerations can be inferred by calculating the growth rate for $k < k^*$. For simplicity, we consider the wave vector $\mathbf{k}$ in the x direction: $\mathbf{k} = k\mathbf{e}_x$. First of all, it is known from the theory of the Jeans instability that the unstable modes of the system are of the form $\omega = -i\gamma$, i.e., the real part of the frequency is zero [5]. Furthermore, setting $\omega = -i\gamma$ the integral in Eq. (9) is calculated easily [5], leading to

$$1 = \left[\frac{k_J^2}{k^2} - \frac{\omega_{\rm pd}^2}{\omega_J^2}\left(\alpha_1 \frac{\lambda_1^2 k_J^2}{1 + \lambda_1^2 k^2} - \alpha_2 \frac{\lambda_2^2 k_J^2}{1 + \lambda_2^2 k^2}\right)\right] \times \left\{1 - \sqrt{\frac{\pi}{2}}\frac{k_J}{k}\gamma^* e^{\frac{k_J^2}{2k^2}\gamma^{*2}}\left[1 - \mathrm{Erf}\left(\frac{k_J \gamma^*}{\sqrt{2}k}\right)\right]\right\}, \quad (12)$$

where we have introduced $\gamma^* = \gamma/\omega_J$. Results are plotted in Fig. 3 for various y_J. The bold line represents the pure gravitational case ($\omega_{\rm pd}^2/\omega_J^2 = 0$); the solid lines are obtained setting $\omega_{\rm pd}^2/\omega_J^2 = 5$ while the dashed and dotted lines correspond to $\omega_{\rm pd}^2/\omega_J^2 = 1$. Furthermore, the dotted line corresponds to a pure Debye-Huckel interaction potential ($\alpha_2 = 0$). Indeed, the Lennard-Jones-like interaction potential can determine higher growth rates even with respect to the pure gravitational case, while a pure Debye-Huckel potential always reduces them.

Conclusions.—In summary, we presented a theory of the modified Jeans instability, in which particles interact through the gravitational force and the electrostatic force, the latter modeled by expression (2) that includes the presence of the attractive potential wells recently discovered for emitting dust grains [2]. We have shown that in certain conditions that can be met in astrophysical scenarios, the electrostatic interaction potential can have a potential well (see Fig. 1). The well acts to broaden the spectrum of unstable modes and to enhance their growth rates. We suggest that this effect could have a strong impact on the collapse of gravitational systems, offsetting the stabilizing effect of charges of the same sign on the grains, and allowing the gravitational collapse to take place as indeed has been observed in many instances [9].

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