Complete destructive interference of partially coherent sources of acoustic waves

Lorenzo Basano* and Pasquale Ottonello
Dipartimento di Fisica Università di Genova Via Dodecaneso 33 16146 Italy
(Received 13 January 2005; published 3 May 2005)

Theoretical analysis has recently shown that the optical fields from several point sources may exhibit complete destructive interference even if the sources are not fully coherent with respect to each other. The experimental verification of this statement in the optical domain is not easy. In this Letter we demonstrate the effect using acoustical waves instead of light waves.

DOI: 10.1103/PhysRevLett.94.173901 PACS numbers: 42.25.-p, 43.20.+g

The logic of this Letter is similar to that of a work published some time ago in this Journal by Bocko *et al.* [1]; in that Letter, the existence of *optical* spectral shifts previously predicted by Wolf [2] was demonstrated using *acoustical* waves. In Bocko's experiment, which was widely referenced in articles and textbooks [3,4], each spectrum consisted of two parts, an anticorrelated higher frequency component and a correlated lower frequency one. Only the correlated component was preserved in the superposition, so that the acoustical spectrum measured in the far zone appeared to be significantly shifted toward the lower frequencies.

Now, in a recent Letter [5], Gbur, Visser, and Wolf have examined the possibility of complete destructive interference of partially coherent optical fields in a "three-pinhole interferometer." In particular, we quote from Ref. [5]: "*... Surprisingly, the fields from several point sources may exhibit complete destructive interference in isolated regions even if the sources are not fully coherent with respect to each other.*"

Here we follow the philosophy of Ref. [1] and recall that the explanation of this interesting effect relies only on the *wave properties* of optical fields; hence an *acoustical* counterpart of the *optical* case described by Gbur *et al.* is expected to exist. Indeed, by properly feeding a system of three loudspeakers (each emitting a single line plus random noise) we have been able to implement a three-point source whose emitted signals are only partially coherent, and we also found a zone in which *complete* cancellation of the fields occurred. Owing to the geometrical symmetry of the source (the loudspeakers are located at the vertices of an equilateral triangle) complete cancellation is obtained along the "acoustical axis."

So far, an optical experiment has been proposed by the authors of Ref. [5] and is based on the superposition of two Laguerre-Gauss ($LG_0^{\pm 1}$) beams aimed at a mask pierced by three holes. In more general cases (e.g., when the optical field is more complex or noise is present) it is not easy to do the experiment using optical means.

We are aware that our work is just a particular experimental confirmation of a general theoretical prediction, but it demonstrates quite simply the possibility of achieving complete destructive interference of partially coherent fields. In the case described here, signal cancellation is obtained thanks to the fact that the *noise components of the sources are mutually correlated*; on the other hand, the presence of noise is necessary to reduce the mutual coherences of the sources to values below unity. Another advantage of our acoustical implementation is that it allows use of a whole range of degrees of coherences and not just some particular values.

We now present a description of the experiment and of the results we obtained. Three loudspeakers are positioned at the vertices of an equilateral triangle lying in a vertical plane. Precautions were taken to minimize direct mechanical coupling between the sources due to the fact that they are fastened to a common framework.

The signal emitted by the jth loudspeaker ($j = 1, 2, 3$) contains two components. The first is a deterministic signal $D_j(t)$, consisting of a narrow line centered at 4000 Hz, whose phase and amplitude are set as explained below.

In addition to this, the jth source may emit an appropriate random noise signal $N_j(t)$ so that the total emitted signal is $S_j(t) = D_j(t) + N_j(t)$. It is easy to see [5] that with *two* sources we could obtain complete destructive interference only by requiring *full* coherence of the signals $S_1(t)$ and $S_2(t)$; regarding this point, see also the remark following Eq. (1). When a third source is added, more degrees of freedom come into play and it is then possible to obtain complete cancellation even if the sources are only partially coherent with respect to each other.

In Ref. [5] the authors paid special attention to the case in which the spectral degrees of coherence μ are the same for all pairs of sources; they proved that, besides the trivial full-coherence solution $\mu = 1$, one can obtain complete cancellation also when $\mu = -\frac{1}{2}$. The existence of solutions corresponding to nonunitary absolute values is the most original and interesting mathematical result of the paper by Gbur *et al.*. It is worth noting that the solution $\mu = -\frac{1}{2}$ *for all pairs of sources* is a particular case because it corresponds to a perfect ternary symmetry of the system. In general, however, all sets of correlation coefficients satisfying Eq. (20) of Ref. [5] can lead to complete cancellation.

In the following we use conventional signal analysis rather than optical coherence theory. To this end, we consider the superposition of three signals $S_j(t)$ ($j = 1, 2, 3$), without preliminarily assuming the existence of any relationship between their mutual correlations; for the moment, the sources are equally distant from the receiver, which is located on the acoustical axis (this insures that the delays of the signals are equal).

Let $S(t) = S_1(t) + S_2(t) + S_3(t)$ be the amplitude of the signal recorded by the microphone. The null condition for all values of t is obviously

$$S_1(t) + S_2(t) + S_3(t) = 0. \tag{1}$$

Incidentally, as mentioned above, this equation confirms that when only two sources are active we must have $S_1(t) = -S_2(t)$, an expression that requires full correlation between the two signals.

Let us now multiply Eq. (1) by $S_1(t)$ and take an ensemble average (denoted by the $< >$):

$$<S_1(t)S_1(t)> + <S_1(t)S_2(t)> + <S_1(t)S_3(t)> = 0. \tag{2}$$

Each addend in Eq. (2) can now be expressed using the normalized correlation coefficient:

$$C_{jk} = \frac{\langle S_j(t)S_k(t)\rangle}{\sqrt{\langle S_j^2(t)\rangle}\sqrt{\langle S_k^2(t)\rangle}},$$

so that, by defining $\sigma_j = \sqrt{\langle S_j^2(t)\rangle}$ Eq. (2) becomes

$$C_{11}\sigma_1 + C_{12}\sigma_2 + C_{13}\sigma_3 = 0. \tag{3}$$

In the same manner, sequentially using $S_2(t)$ and $S_3(t)$ in place of $S_1(t)$, we obtain the system of equations:

$$\sum_{k=1}^{k=3} C_{jk}\sigma_k = 0 \qquad (j = 1, 2, 3), \tag{4}$$

where the diagonal elements of the correlation matrix C_{jk} have unitary values.

In the end, by requiring the determinant of the homogeneous system (4) to be zero, we obtain the equation:

$$1 + 2C_{12}C_{13}C_{23} - ({C_{12}}^2 + {C_{13}}^2 + {C_{23}}^2) = 0. \tag{5}$$

This is the real analogue of Eq. (20) of Ref. [5] from which, to tell the truth, our analysis could have started. We decided to present our own derivation of Eq. (5), however, for the benefit of those readers who may be more acquainted with the fundamentals of signal processing than with the advanced theory of optical coherence.

We subsequently used Eq. (5) to perform several computer simulations and tried to make them as close to reality as possible by introducing a few delayed and randomly phased echoes from nearby structures. In so doing, we systematically obtained simulation graphs (Fig. 1) very similar to the experimentally recorded lower trace reported in Fig. 2. In the echo free part of Fig. 1 (abscissa values $<$ 350) the signal appears to be completely destroyed; in the right part of the trace, echoes are superposed on the direct signal and complete cancellation can no longer take place.

This explains why, upon shifting from computer simulations to real acoustical experiments, we ran into a serious difficulty. Even in the largest spaces available to us, reflections from walls, floor, and nearby structures produced so many random interferences that it was virtually impos-

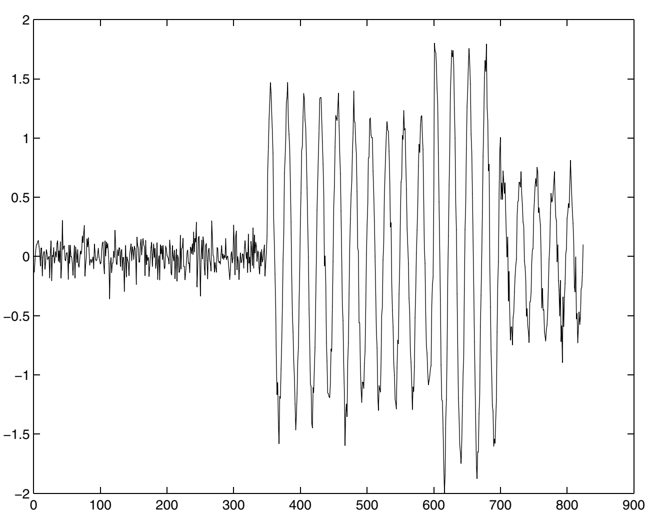

FIG. 1. Computer simulation (arbitrary units): the signal consists of three noisy sinusoids and three randomly phased (and delayed) echoes from nearby structures. Prior to abscissa 350 the signal is echo free and only ambient noise is detected; soon afterwards, the echo arrivals render signal cancellation impossible. Compare with lower trace in Fig. 2.

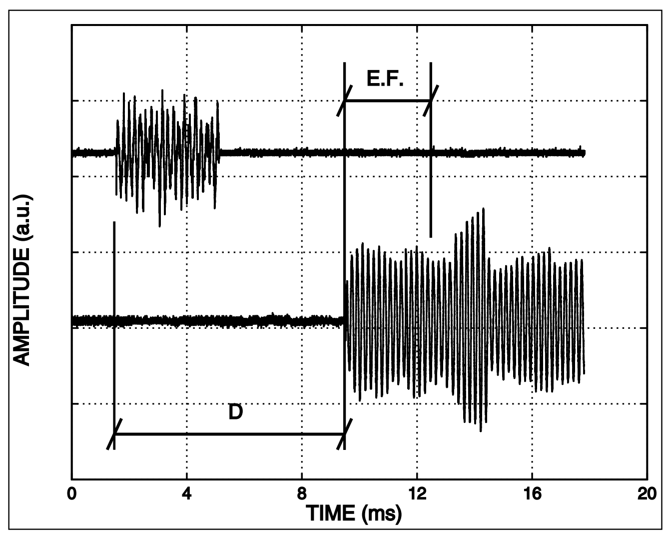

FIG. 2. In this case only loudspeakers 1 and 2 are transmitting. Upper trace: signal transmitted by one of the loudspeakers (4 ms duration). Lower trace: signal received by the microphone (after the 8 ms delay D, equal to the transit time). The interval labeled as EF (echo free) represents the temporal window during which only the signals from the loudspeakers, but not the reflected echoes, are reaching the microphone.

sible to obtain an acceptable cancellation of the signal. In other words, the *effective signal sensed by the microphone* was much more complex than that emitted by the 3-point source system we had managed to implement.

Owing to the complexity of building a completely anechoic room, we tried to circumvent the problem by means of the following strategy. We set the distance between the source and the receiver at about 270 cm, with the nearest reflecting surface at more than 1 m from the acoustical axis. This implies a transit time of about 8 ms for the *direct* signal to reach the microphone and an additional delay of at least 3 ms for the arrival of the earliest *reflected* signals.

Now, rather than being continuously transmitted, the signal is periodically chopped with a properly determined duty cycle (see below). Each time a new cycle is started, the direct signal reaches the detector 3 ms before the arrival of the earliest echoes. The microphone is synchronously activated with the transmitted train, and its activation starts after a fixed delay of 8 ms, to allow for the transit time of the direct signal. It follows that the signal picked up by the microphone is completely echo free during the first 3 ms of each receiving cycle. This is the temporal window within which complete cancellation may be detected.

In other words, if the instrumentation is properly set and the effect is real, we should expect nearly complete signal cancellation lasting for 3 ms at the beginning of each cycle. After this lapse of time, *random* superpositions of primary signals and echoes will cooperate to eliminate the cancellation. Apart from the expected smoothing of the rising time, due to the finite response of the speakers, this is what we actually measured, thereby confirming the complete negative interference of *partially* coherent signals.

In order to obtain the maximum allowable cancellation we carefully ascertained the following: (i) the three loudspeakers are selected out of a larger group of nominally identical items. (ii) Their coils are fed by three identical current generators; in this way, the trailing edge of each train is quite independent of the speaker's parameters and is reduced to its shortest possibile value. (iii) Once the correct amplitudes and phases are chosen and the microphone is positioned on the acoustical axis, small adjustments of the sound levels of the sources allow us to increase the sharpness of the destructive interference: the signal amplitude turned out to be less than 10% of the value measured when any one of the three speakers was switched off and was almost indistinguishable from ambient noise.

Figures 2 and 3 illustrate typical oscilloscopic records for a single trasmission cycle. In Fig. 2 only speakers 1 and 2 are activated and complete signal cancellation cannot occur because the two sources are only partially correlated. In Fig. 3 all three speakers are simultaneously fed and cancellation is successfully achieved. The upper traces in Figs. 2 and 3 represent the typical noisy signal (of 4 ms duration) transmitted by one of the speakers, while the lower traces represent the signals picked up by the microphone. If the lower trace in Fig. 2 were extended towards longer times, additional echoes of progressively decreasing intensity would come in from the right. On the whole, the signal decays towards ambient noise in about 300 ms (*the reverberation time of the room*, a parameter that enables us to set a proper value for the repetition rate of the transmitted signal).

For completeness, it is fair to point out that a closer inspection of Fig. 3 reveals that the cancellation of the signal appears to be somewhat less satisfactory *at the very beginning* of the 3 ms interval, where a few oscillations slightly higher than elsewhere can be seen. But this is just the moment when the transmitted signal builds up most abruptly: even slight and unavoidable differences in the *dynamic* response of the speakers are then likely to produce their maximum negative effects against cancellation. Generally, in our opinion, it is encouraging that the total noise in the echo free 3 ms interval is only slightly greater than the ambient noise, as is visually displayed in Fig. 3 by the thickness of the lower trace before and after activation of the microphone.

The case $C = [-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}]$ considered in Ref. [5] was obtained as follows, without making recourse to noise. We first generated two independent sinusoids: $\sin(\omega t)$ and $\sin[\omega t + \phi(t)]$ where $\phi(t)$ is a slowly varying random phase term which accounts for the superposition being incoherent. The *sum* of these two sinusoids was sent to loudspeaker 1. Loudspeakers 2 and 3 were fed with similar signals in which the sinusoids were delayed, respectively, by 120° and 240°, while the random phase term $\phi(t)$ was the same for all loudspeakers. It is easy to show that the normalized correlation coefficients of these loudspeaker signals are $[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}]$ and we also experimentally verified that complete destruction occurred precisely at

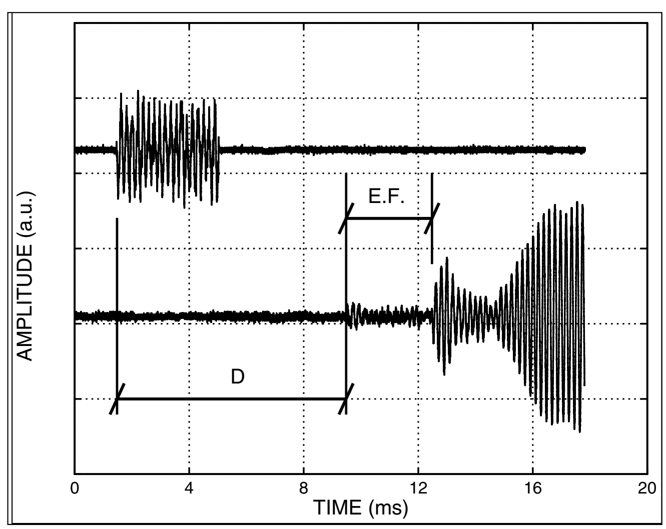

FIG. 3. In this case all three loudspeakers are transmitting. Comparison with Fig. 2 shows that during the 3 ms echo free window *EF* almost complete cancellation of the signal is achieved.

the points predicted by theory as in Fig. 4 of Ref. [5]. We also made computer simulations using sets of coefficients that do *not* satisfy Eq. (5); in these cases total cancellation could not be found. A special example of this class was the symmetric set $C = [+\frac{1}{2}, +\frac{1}{2}, +\frac{1}{2}]$.

*Corresponding author.
E-mail: basano@fisica.unige.it

[1] M. F. Bocko, D. H. Douglass, and R. S. Knox, Phys. Rev. Lett. **58**, 2649 (1987).
[2] E. Wolf, Nature (London) **326**, 363 (1987).
[3] M. Born and E. Wolf, *Principles of Optics* (Cambridge University Press, Cambridge, Massachusetts, 1999), 7th edition, p. 588.
[4] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, Massachusetts, 1995), p. 312.
[5] G. Gbur, T. D. Visser, and E. Wolf, Opt. Commun. **239**, 15 (2004).