Parametric Generation of Forward and Phase-Conjugated Spin-Wave Bullets in Magnetic Films

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We show experimentally as well as by numerical simulation that interaction of a linear two-dimensional spin-wave packet with quasiuniform pulsed pumping leads to the formation of strongly self-focused nonlinear spin-wave bullets propagating in both forward and reversed directions. The focusing of the reversed, phase-conjugated wave bullet is stronger than that of the forward one, because not only the nonlinear four-wave self-focusing effect but also linear focusing due to two-dimensional phase conjugation contributes to the focusing of the reversed bullet.

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The possibility of the existence of light wave bullets—stable two- or three-dimensional nonlinear light wave packets self-focused in space and time—has been suggested by Silberberg [1]. Although predicted for light, wave bullets were first experimentally observed in the system of spin waves in yttrium-iron garnet (YIG) films [2]. Then, light bullets were observed in an optical medium with quadratic nonlinearity [3] and in nonlinear plasmas [4]. In our recent experiment [5] we were able to show that spin-wave bullets can be self-generated from noise in an active ring containing a magnetic film and an external microwave amplifier. This experiment proves that wave bullets are intrinsic nonlinear multidimensional excitations of the nonlinear medium with dispersion, diffraction, and dissipation.

The main advantage of the experimental investigation of spin-wave bullets in magnetic films compared to the studies of light wave bullets [3,4] is the low group velocity of spin-wave packets compared to the speed of light. This property allows us to follow the spin-wave bullet formation and propagation in real time using the space- and time-resolved Brillouin light scattering technique [6].

An additional advantage of magnetic films as a medium for nonlinear wave studies is the fact that in these films it is possible to realize an effective parametric interaction of a propagating wave packet (having a carrier wave vector $\mathbf{k}$ and frequency w) with nonstationary (pulsed) electromagnetic pumping [7]. The conservation laws for such a parametric interaction process have the form

$$\omega + \omega' = \omega_p, \qquad \mathbf{k} + \mathbf{k}' = \mathbf{k}_p, \tag{1}$$

where ω_p, ω, ω' and $\mathbf{k}_p$, $\mathbf{k}$, $\mathbf{k}'$ are the carrier angular frequencies and carrier wave numbers of the electromagnetic pumping pulse, initial spin-wave packet, and the "idle" spin-wave packet formed in the interaction process, respectively.

It is known [8–10] that in a quasi-one-dimensional spin-wave waveguide the parametric interaction (1) may result in a substantial amplification of the initial spin-wave packet and, in the case of quasiuniform pumping ($\mathbf{k}_p \approx 0$), to the parametric generation of a phase-conjugated idle wave packet with the reversed wave front $\mathbf{k}' = -\mathbf{k}$ propagating in the opposite direction.

The goal of this Letter was to study both experimentally and theoretically parametric interaction of *two-dimensional* spin-wave packets with quasiuniform pulsed pumping. Although parametric interactions and wave-front reversal of quasi-one-dimensional spin-wave packets have been studied previously (see [8–10]), the generalization of these results to the two-dimensional case is nontrivial, as in a tangentially magnetized magnetic film the spin-wave spectrum is strongly anisotropic, i.e., the directions of phase and group velocity do not coincide for many orientations of the spin-wave vector [for details see Eqs. (5) and (6) in [11]]. In the experiments described below we demonstrate that, in spite of the anisotropy of the spin-wave spectrum, parametric interaction of a two-dimensional spin-wave packet with pumping leads to the following two effects: (i) two-dimensional phase conjugation and wave-front reversal of the incident wave packet; (ii) formation of well-pronounced nonlinear spin-wave bullets from both forward-propagating and phase-conjugated reversed wave packets.

A waveguide (size 4.1×30 mm^2) was cut out of a single crystal YIG(111) film of 5 μm thickness grown on a gallium-gadolinium garnet substrate. The saturation

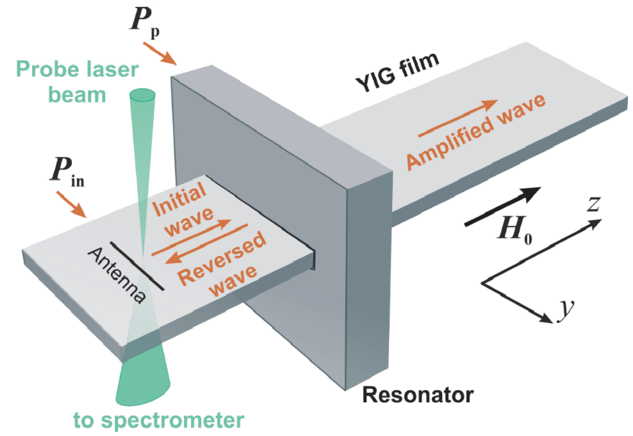

FIG. 1 (color). Experimental setup. Leads feeding the antenna and the resonator with microwave power are not shown.

magnetization of the film $4\pi M_0$ is 1750 G, and its full ferromagnetic resonance linewidth is $2\Delta H = 0.6$ Oe at $\omega/2\pi = 7$ GHz. For excitation of spin waves a microstrip antenna of 25 μm width and 2 mm length was used. The width of the waveguide was chosen to be twice the length of the antenna and several times larger than the characteristic transverse size of a spin-wave bullet (0.2–0.8 mm) (see, e.g., [2]) to guarantee a truly two-dimensional regime of spin-wave packet propagation. The magnetic field H_0 was applied in the film plane along the long side of the waveguide (z axis in Fig. 1). This geometry corresponds to the excitation of backward volume magnetostatic waves (BVMSW) (see, e.g., [2,12]).

The input BVMSW packets were excited by rectangular electromagnetic pulses of 30 ns duration with carrier frequency of $\omega/2\pi = 3.9660$ GHz. For the applied field $H_0 = 800$ Oe this frequency corresponds to the BVMSW with a carrier wave number of $k_z = 95$ rad/cm. The measured value of the group velocity of $|v_g| = 1.95 \times 10^6$ cm/s coincides well with the calculated value of 2.07×10^6 cm/s. The calculated values of the dispersion D, diffraction S, and nonlinear coefficient N are 520 $\mathrm{cm^2/s\,rad}$, 1.18×10^5 $\mathrm{cm^2/s\,rad}$, and -5.66×10^9 rad/s, respectively (see [13] for details). The coefficient of parametric coupling for the above experimental conditions was found to be $V/2\pi = 0.84$ MHz/Oe [see Eq. (2) below], while the parameter of linear dissipation $\omega_r = \gamma\Delta H$ was equal to $\omega_r = 5.4 \times 10^6$ rad/s, where $\gamma/2\pi = 2.8$ MHz/Oe.

A rectangular dielectric ($\epsilon = 80$) resonator having the size $1 \times 9.45 \times 5.85$ $\mathrm{mm^3}$ with a 0.8×4.5 $\mathrm{mm^2}$ rectangular opening in the middle was used to create a pumping magnetic field inside the YIG film. The waveguide went through the opening, and the resonator was placed at a distance of 2.5 mm from the antenna (see Fig. 1). The pumping microwave field was parallel to the applied magnetic field. The resonator created a rather long zone of pumping area along the z axis ($s = 3$ mm), giving $k_p \approx \pi/s = 10$ rad/cm and indicating that the condition $k_p \ll k_z$ formulated above was fulfilled. The loaded quality factor of the resonator was 184, and the resonance frequency was equal to $\omega_p/2\pi = 7.9322$ GHz, i.e., $\omega_p = 2\omega$ and $\omega' = \omega$ in Eq. (1).

The pumping pulse having a duration of 39 ns and a peak power $P_p = 30$ W was supplied to the dielectric resonator at the time when the propagating input pulse was close to the center of the resonator. As a result of the parametric interaction the forward-propagating amplified and the backward-propagating amplified, reversed, and phase-conjugated spin-wave packets were formed. The maximum amplification of the forward-propagating packet reached 27 dB.

Monitoring the formation and propagation of spin-wave packets was made using the space- and time-resolved Brillouin light scattering technique described in detail in [11]. Two-dimensional maps of the spin-wave intensities $I(z, y)$ proportional to the local values of the squared dynamic magnetization in the film were recorded with spatial resolution of 0.1 mm and temporal resolution of 2 ns.

A series of such maps is shown in Fig. 2. The upper panel of Fig. 2 demonstrates the evolution of a two-dimensional wave packet of duration $\tau = 30$ ns after it has been launched from the antenna and until it reaches the pumping area. The initial width of the packet (deter-

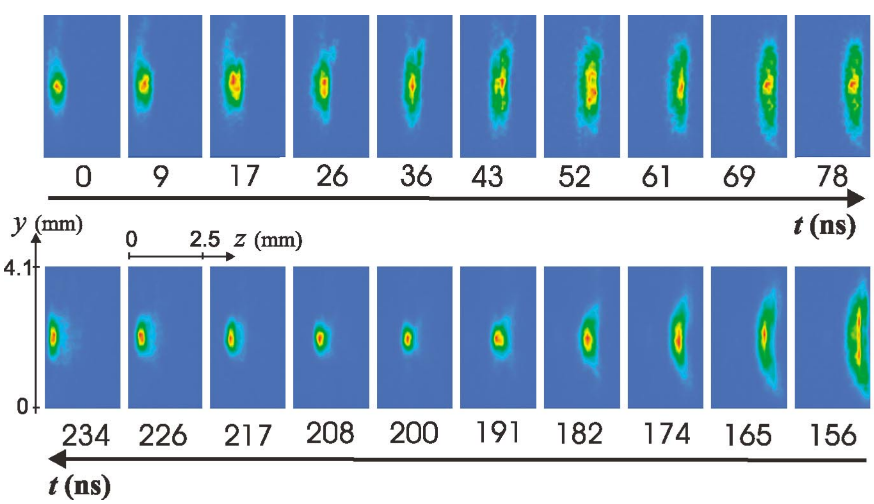

FIG. 2 (color). The spatial distributions of the spin-wave pulse intensity captured at successive moments of time both for incident (upper part) and for reversed (lower part) spin-wave packets. $P_{in} = 146$ mW.

mined by the antenna length and the carrier wave number) was $L_{y0} = 0.4$ mm, the power of the initial microwave pulse defining the intensity of the packet was $P_{\rm in} = 146$ mW. As seen in Fig. 2 the packet spreads along the transverse (y) direction during propagation due to the strong diffraction in the film. The spreading of the packet is illustrated by Fig. 3(a) showing its width versus the propagation distance. The three curves correspond to different values of the input power.

The lower panel of Fig. 2 demonstrates the propagation and evolution of the reversed ($\mathbf{k}' = -\mathbf{k}$) wave packet formed as a result of the parametric interaction. It is clear that the amplified wave packet experiences a strong nonlinear two-dimensional self-focusing, which leads to the formation of a well-pronounced spin-wave bullet (see frames corresponding to 191, 200, and 208 ns). The evolution of the bullet width L_y in this reverse propagation is shown by curves (1) and (2) in Fig. 3(c). Curve (3) shows the linear case for reference. Curve (1) in Fig. 3(c) obtained in a strongly nonlinear regime ($P_{\rm in} = 226$ mW) demonstrates the formation of a wave bullet in the reversed wave packet for a large region of propagation distances (from 1.7 to 1.1 mm) where L_y is almost constant.

One can see from Fig. 2 that the reversed packet undergoes a wave-front reversal (phase conjugation). The wave front of the incident pulse that is slightly concave immediately before the interaction with pumping (frame corresponding to 78 ns) becomes clearly convex after the interaction (156 ns) (note the change of direction of propagation between these two frames). As it is seen from Fig. 3(d), the effect of two-dimensional wave-front reversal is even more pronounced for a linear, low-power input wave packet.

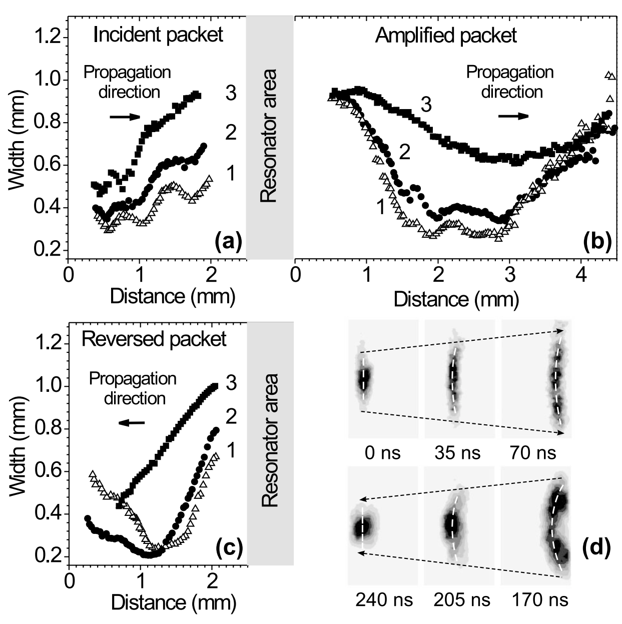

FIG. 3. Behavior of the spin-wave packet characteristics with propagation distance z and time t. (a),(b) The evolution of the transversal width L_y of the spin-wave packet before its interaction with pumping and after its parametric amplification, respectively. (c) The evolution of the width L_y of the reversed spin-wave packet. Curves (1), (2), and (3) correspond to the input powers $P_{\rm in}$ of 226, 146, and 22 mW, respectively. (d) A spin-wave packet at given instants as indicated illustrating the two-dimensional phase conjugation and front reversal process of a linear (22 mW) pulse due to interaction with the pumping.

Figure 3(b) demonstrates that formation of a spin-wave bullet takes place also in the forward-propagating packet amplified by pumping. In this case, however, the bullet is formed at a larger distance from the pumping resonator (1.5 mm compared to 0.8 mm for the reversed packet) and it exists in a larger range of propagation distances (from 1.5 to 3.1 mm).

These differences can be attributed to the fact that two mechanisms contribute to the bullet formation from the reversed packet. The first mechanism is the same nonlinear four-wave self-focusing effect that is responsible for the bullet formation from the forward-propagating packet, as discussed in our previous work [2]. The second mechanism is the linear focusing effect characteristic for the process of two-dimensional phase conjugation [10,14]. This linear focusing effect can be observed separately if the power of the input signal is low enough [see Fig. 3(d) and curve (3) for $P_{\rm in} = 22$ mW in Fig. 3(c)].

Thus, due to the combined action of linear and nonlinear effects, the focusing and compression of the reversed wave packet is more pronounced than that of the forward-propagating amplified wave packet, although the amplitude of the forward packet is larger.

It has been demonstrated previously [2,11] that formation and propagation of a two-dimensional spin-wave bullet can be described by a two-dimensional parabolic (or nonlinear Schrödinger) equation with dissipation [see, e.g., Eq. (1) in [2]]. To describe the parametric interaction of a two-dimensional spin-wave packet with pumping, we shall use a system of two coupled parabolic equations for the envelopes of forward-propagating and reversed wave packets,

$$i\left(\frac{\partial U_1}{\partial t} + v_g \frac{\partial U_1}{\partial z} + \omega_r U_1\right) + \frac{D}{2}\frac{\partial^2 U_1}{\partial z^2} + \frac{S}{2}\frac{\partial^2 U_1}{\partial y^2} - N(|U_1|^2 + 2|U_2|^2)U_1 = iVh_p(z,t)U_2^*, \tag{2a}$$

$$-i\left(\frac{\partial U_2^*}{\partial t} - v_g \frac{\partial U_2^*}{\partial z} + \omega_r U_2^*\right) + \frac{D}{2}\frac{\partial^2 U_2^*}{\partial z^2} + \frac{S}{2}\frac{\partial^2 U_2^*}{\partial y^2} - N(|U_2|^2 + 2|U_1|^2)U_2^* = -iV^* h_p^*(z,t)U_1, \tag{2b}$$

where U_1 and U_2^* are the amplitudes of the forward-propagating and reversed spin-wave packet envelopes, $v_g = \partial\omega/\partial k_z|_{k_{0z}}$ is the group velocity, $D = \partial^2\omega/\partial k_z^2|_{k_{0z}}$ and $S = \partial^2\omega/\partial k_y^2|_{k_{0z}}$ are the dispersion and diffraction coefficients, respectively, $N = \partial\omega/\partial|U|^2|_{k_{0z}}$ is the non-

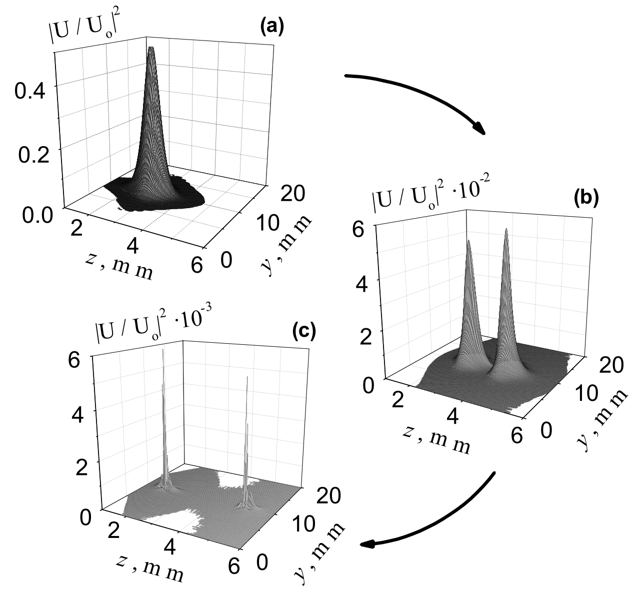

FIG. 4. Numerical modeling of bullet formation by parametrically amplified and conjugated spin-wave packets. (a) The initial spin-wave packet. (b) The amplified and conjugated spin-wave packets just after their parametric interaction with an electromagnetic pumping. (c) A pronounced two-dimensional compression of both spin-wave packets.

linear coefficient, ω_r is the relaxation parameter, $k_0 = k_{0z}$ is the carrier wave number, $\omega(k_y, k_z, |U|^2)$ is the nonlinear wave dispersion law, and V is the coupling coefficient between the parametric waves and the pumping field of the amplitude h_p.

The above described processes of spin-wave packet propagation and interaction with parametric pumping were modeled numerically using the system (2). It was assumed that the input linear wave packet has the following profile:

$$U_1^2(y, z, t = 0) = U_0^2 \cosh^{-2}\left[\frac{2(z - z_0)}{L_{z_0}}\right]\cos^2\left[\frac{\pi(y - y_0)}{L_{y_0}}\right], \tag{3}$$

where L_{y0} and L_{z0} were the same as in the experiment Fig. 3. We used the above determined values of v_g, D, S, N, ω_r, and V as coefficients. The pumping field of $h_p =$ 60 Oe was assumed to be uniform in space and was acting for 40 ns starting at the time when the maximum of the input propagating pulse was at a distance of 3 mm from the antenna.

The two-dimensional distributions of the normalized spin-wave intensity $|U(y, z)/U_0|^2$ calculated for three different times (before, immediately after, and 30 ns after the end of the pumping pulse) are shown in Fig. 4. The numerical model correctly reproduces all the qualitative features of the observed parametric interaction process: immediately after the interaction with pumping two counterpropagating wave packets [Fig. 4(b)] are formed from the initial linear wave packet [Fig. 4(a)]. This is followed by the formation of bullets in both forward-propagating and reversed wave packets [Fig. 4(c)]. We note that the numerical results also show that self-focusing of the reversed spin-wave bullet is stronger than that of the forward-propagating packet. Thus, the model successfully describes the above observed effect of linear focusing due to the two-dimensional parametric front reversal and phase conjugation.

In conclusion, we have shown that spin-wave bullets being two-dimensional intrinsic excitations of a nonlinear diffractive and dispersive medium with dissipation can be generated parametrically in magnetic films from linear input wave packets. They are formed from both the forward-propagating and reversed wave packets, and their properties are similar to the properties of wave bullets generated by other means [2,5].

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[1] Y. Silberberg, Opt. Lett. **15**, 1282 (1990).
[2] M. Bauer, O. Büttner, S. O. Demokritov, B. Hillebrands, V. Grimalsky, Yu. Rapoport, and A. N. Slavin, Phys. Rev. Lett. **81**, 3769 (1998).
[3] X. Liu, L. J. Qian, and F. W. Wise, Phys. Rev. Lett. **82**, 4631 (1999).
[4] Nadja I. Vogel and N. Kochan, Phys. Rev. Lett. **86**, 232 (2001).
[5] A. A. Serga, S. O. Demokritov, B. Hillebrands, and A. N. Slavin, Phys. Rev. Lett. **92**, 117203 (2004).
[6] S. O. Demokritov, B. Hillebrands, and A. N. Slavin, Phys. Rep. **348**, 441 (2001).
[7] G. A. Melkov, A. A. Serga, A. N. Slavin, V. S. Tiberkevich, A. N. Olejnik, and A. V. Bagada, Sov. Phys. JETP **89**, 1189 (1999).
[8] A. V. Bagada, G. A. Melkov, A. A. Serga, and A. N. Slavin, Phys. Rev. Lett. **79**, 2137 (1997).
[9] P. A. Kolodin, P. Kabos, C. E. Patton, B. A. Kalinikos, N. G. Kovshikov, and M. P. Kostylev, Phys. Rev. Lett. **80**, 1976 (1998).
[10] G. A. Melkov, A. A. Serga, V. S. Tiberkevich, A. N. Oliynyk, and A. N. Slavin, Phys. Rev. Lett. **84**, 3438 (2000).
[11] O. Büttner, M. Bauer, S. O. Demokritov, B. Hillebrands, Yuri S. Kivshar, V. Grimalsky, Yu. Rapoport, and A. N. Slavin, Phys. Rev. B **61**, 11 576 (2000).
[12] R. W. Damon and J. R. Eshbach, J. Phys. Chem. Solids **19**, 308 (1961).
[13] A. N. Slavin, B. A. Kalinikos, and N. G. Kovshikov, in *Nonlinear Phenomena and Chaos in Magnetic Materials*, edited by P. E. Wigen (World Scientific, Singapore, 1994), Chap. 9.
[14] B. Ya. Zel'dovich, R. F. Pilipetskii, and V. V. Shkunov, *Principles of Phase Conjugation* (Springer, Berlin, 1985).