Initial Decoherence in Solid State Qubits

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(Received 20 September 2004; published 27 April 2005)

We study decoherence due to low frequency noise in Josephson qubits. Non-Markovian classical noise due to switching impurities determines inhomogeneous broadening of the signal. The theory is extended to include effects of high-frequency quantum noise, due to impurities or to the electromagnetic environment. The interplay of slow noise with intrinsically non-Gaussian noise sources may explain the rich physics observed in the spectroscopy and in the dynamics of charge based devices.

DOI: 10.1103/PhysRevLett.94.167002 PACS numbers: 85.25.Cp, 03.65.Yz, 03.67.Lx, 05.40.−a

Considerable progress has been recently achieved in implementing qubits with superconducting nanocircuits. Coherent oscillations [1–3] and entanglement of coupled charge qubits [4] have been observed. Limitations in the performances arise from noise due to material and device dependent sources [5–10]. Noise due to individual impurities behaving as bistable fluctuators (BF) is a severe source of dephasing for charge based devices. Sets of BFs determine $1/f$ noise [11,12], and effects due to individual BFs have been observed both in spectroscopy and in time resolved dynamics [13,14]. Observations show a variety of features, such as the drastic reduction of the amplitude of the coherent signal [1,2,14] or relaxation limited decoherence [3], strongly dependent on the particular device and on details of the protocol [2,15,16]. Theories of BF environments [6–8,16,17] allow us to understand several physical aspects, although a quantitative framework embedding the variety of phenomena is still missing. Phenomenological models of the environment as a suitable set of harmonic oscillators [5,9,10] have also been studied. While they are unable to describe aspects related to the discrete nature of noise [6–8,16,17], Gaussian environments may sometimes provide useful information.

In this work we study numerically a model of discrete noise which potentially explains the experimental features due to $1/f$ noise, and seek a classification of the possible effects on the basis of simple theoretical arguments. In particular, we study inhomogeneous broadening due to slow noise and its interplay with additional noise sources, pointing out that the presence of BFs may pose reliability problems for charge based devices.

We consider a qubit anisotropically [5] coupled to classical stochastic process $\xi(t)$. The Hamiltonian is

$$H = H_Q - \frac{1}{2}\xi(t)\sigma_z, \tag{1}$$

where $H_Q = -\frac{1}{2}\vec{\Omega}\cdot\vec{\sigma}$ refers to the qubit. Both the operating point, i.e., the angle θ between $\hat{z}$ and $\vec{\Omega}$, and the splitting Ω are tunable. This also modulates sensitivity to noise. For weak coupling, the relaxation $T_1^{-1} = s^2 S(\Omega)/2$ and the dephasing rate $T_2^{-1} = (2T_1)^{-1} + T_2'^{-1}$, $T_2'^{-1} = c^2 S(0)/2$ being the adiabatic term which gives secular broadening [18], are tuned by $c = \cos\theta$ and $s = \sin\theta$. Only the power spectrum of noise, $S(\omega) = \langle \xi\xi\rangle_\omega$, enters; therefore, in weak coupling the qubit is sensitive only to properties of the environment at the level of two point correlations. This picture breaks down if the environment extends to low frequencies [19]. For instance, random telegraph noise (RTN) due to a single BF, $\xi(t) = \{0, v_0\}$, switching at a rate γ_0 is slow if $g_0 = (\Omega' - \Omega)/\gamma_0 > 1$ [7], where the qubit frequencies Ω and $\Omega' = \Omega[(v_0/\Omega + c)^2 + s^2]^{1/2}$ correspond to the two values of ξ. This model describes an incoherently switching charged impurity close to a qubit. For $g_0 > 1$ features of the discrete nature of the BF become apparent [7].

A set of $N_{\rm BF}$ BFs (ξ_i) switching at rates γ_i, coupled with the qubit via $\xi(t) = \sum_i \xi_i(t)$, models $1/f$ noise if γ_i are distributed [11] with $P(\gamma) \propto 1/\gamma$. The BF-$1/f$ spectrum is $S^{1/f}(\omega) = \sum_i \frac{1}{2} v_i^2 \gamma_i/(\gamma_i^2 + \omega^2)$, and if $\gamma_i \in [\gamma_m, \gamma_M]$, in the same interval of frequencies is approximated by $S^{1/f}(\omega) \approx [(\pi/4)N_{\rm BF}\overline{v}^2/\ln(\gamma_M/\gamma_m)]\omega^{-1}$. Noise extends for several decades and, in particular, slow BFs ($g_i > 1$), an environment with memory, make unstable the calibration of the device. Hence the qubit dynamics will depend on details of the protocol. Decoherence due to BFs $1/f$ noise for various protocols has been studied for $\theta = 0$, where exact solutions are available [6]. On the other hand, the splitting is less sensitive to fluctuations at optimal working point [2], $\theta = \pi/2$ (parameters g_i become smaller), and part of the effects of the slow noise is eliminated (at lowest order $T_2'^{-1}$ vanishes).

Ideal quantum protocols assume measurements of individual members of an ensemble of identical (meaning that preparation is controlled) evolutions of the qubit, defocusing occurring only *during* the time evolution. In practice for solid-state devices one collects several qubit evolutions in an overall measurement time t_m. Lack of control on the environment preparation determines defocusing of the signal, analogous to inhomogeneous broadening in NMR

[18]. This is also true for single-shot measurements [20]. In our case, BFs active in additional broadening have $\gamma_i > \gamma^* \sim \min\{\overline{v}/10, t_m^{-1}\}$ [16].

We first study Hamiltonian (1) by simulating the stochastic Schrödinger equation for the qubit in a BF-$1/f$ environment. We generate $\xi(t)$ as a sum of $N_{\rm BF} \leq 2000$ RTN processes with proper distribution of parameters. In order to minimize errors in generating $\xi(t)$ we use a "waiting time" algorithm [21], which also reduces the computational time. The qubit propagator is evaluated as the product of the propagators between successive switches. Finally, we perform the statistical average. We study an ensemble of time evolutions of the qubit, each lasting for a time t. During the overall time t_m of the protocol, the environment evolves in an uncontrolled way, so BFs with $\gamma_i \gg 1/t_m$ average, whereas BFs with $\gamma_i \ll 1/t_m$ are frozen. Thus for the simulation we consider $\geq 10^5$ realizations of $\xi(t')$ for $0 < t' < t$. For the individual BFs at $t' = 0$ we choose *the same* initial $\xi_i(0) = 0, 1$ if $\gamma_i < 1/t_m$ whereas if $\gamma_i > 1/t_m$ we take a distribution with $0 < \overline{\xi_i(0)} < 1$. This prescription has been checked against more accurate ones in Ref. [16].

Results at $\theta = \pi/2$ for an adiabatic $1/f$ environment, $\gamma_M \ll \Omega$, show the presence of several time scales (Fig. 1). Coherent oscillations of $\langle \sigma_y \rangle$ are initially suppressed with a power law. Relaxation occurs on much longer time scales, given by the weak coupling result. The initial suppression is due to inhomogeneous broadening. This is apparent if we compare with results with a feedback protocol simulated by resetting $\xi(0)$ at the same value for each realization of $\xi(t')$.

Negligible relaxation allows us to treat $\xi(t)$ in the adiabatic approximation. Observables are then given by path integrals over a weight $P[\xi(t)]$ of the stochastic process. We study the averaged phase shift $\Phi(t)$, defined as

$$\mathrm{e}^{-i\Omega t - i\Phi(t)} = \int \mathcal{D}\xi(t') P[\xi(t')] e^{-i\int_0^t dt' \Omega[\xi(t')]}, \qquad (2)$$

which gives the decay of the qubit coherences, $\langle \sigma_y \rangle \propto \exp[\Im\Phi(t)]$. Here $\Omega[\xi(t)] = \Omega\{[\xi(t)/\Omega + c]^2 + s^2\}^{1/2}$ is the instantaneous qubit splitting. Numerical evaluation of the path integral Eq. (2) fully agrees with the simulations. Further insight is obtained by approximating Eq. (2). The static-path approximation (SPA), $\xi(t') = \xi_0$, accounts for lack of control on the environment preparation via a statistically distributed ξ_0. This blurs the overall signal, an effect analogous to the rigid lattice line breadth in NMR [18]. For a set of BFs, if $N_{\rm BF}$ is large enough ξ_0 is Gaussian distributed with variance $\sigma_\xi^2 = \overline{v}^2 N_{\rm BF}/4 = \int_0^\infty (d\omega/\pi) S(\omega)$, where it is intended that we consider only active BFs, $\gamma_i > \gamma^*$. The result, plotted in Figs. 1 and 2 accounts for the initial suppression of the signal, showing that this latter is entirely due to inhomogeneous broadening. Therefore, the analysis of the initial suppression may give information on the amplitude of $1/f$ noise at intermediate frequencies $1/t_m < \nu < 1/t$. A good estimate of the SPA is obtained by a quadratic expansion of $\Omega(\xi_0)$ in ξ_0 [see Fig. 2(a)]

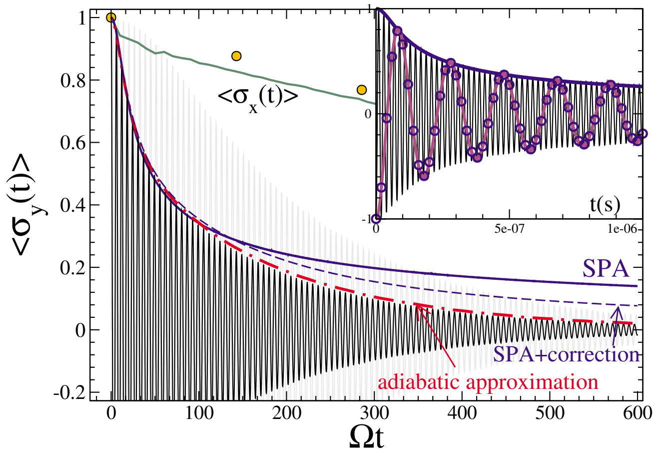

FIG. 1 (color online). Simulations of an adiabatic BF-$1/f$ environment at $\theta = \pi/2$. Relaxation studied via $\langle \sigma_x \rangle$ (green line) is well approximated by the weak coupling theory (dots). Dephasing in repeated measurement damps the oscillations (thin black line). Part of the signal is recovered if the environment is recalibrated (thin gray line). Noise is produced by $n_d = 250$ BFs per decade, with $1/t_m = 10^5$ rad/s $\leq \gamma_i \leq \gamma_M = 10^9$ rad/s $< \Omega = 10^{10}$ rad/s. The coupling $\bar{v} = 0.02\ \Omega$ is appropriate to charge devices, and corresponds to $S(\omega) = 16\pi A E_C^2/\omega$ with $A \sim 10^{-6}$ [12]. The adiabatic approximation Eq. (2) fully accounts for dephasing (red dotted-dash line). The static-path approximation (SPA) Eq. (3) (blue solid line) and the first correction (blue dashed line) account for the initial suppression, and it is valid also for times $t \gg 1/\gamma_M$. In the inset Ramsey fringes with parameters appropriate to the experiment [2] (thin black lines). The SPA (blue solid line), Eq. (3), is in excellent agreement with observations [14], and also predicts the correct phase shift of the Ramsey signal (blue dots, compared with simulations for small detuning $\delta = 5$ Mhz, violet line), which tends to $\approx \pi/4$ for large times.

$$-i\Phi(t) = -\frac{1}{2}\frac{(c\sigma_\xi t)^2}{1 + is^2\sigma_\xi^2 t/\Omega} - \frac{1}{2}\ln\left(1 + is^2\frac{\sigma_\xi^2 t}{\Omega}\right), \quad (3)$$

which is accurate close to $\theta = 0$ and $\theta = \pi/2$ for σ_ξ/Ω small enough. The resulting suppression factor $\exp(\Im\Phi)$ turns from a $\exp(-\frac{1}{2}c^2\sigma_\xi^2 t^2)$ behavior at $\theta \approx 0$ to a power law, $[1 + (s^2\sigma_\xi^2 t/\Omega)^2]^{-1/4}$, at $\theta \approx \pi/2$. These limits reproduce known results for Gaussian $1/f$ environments, namely, at $\theta = 0$ the $t \ll 1/\gamma_M$ limit of the exact result [22] and for $\theta = \pi/2$ the short-time result of the diagrammatic approach of Ref. [9]. This is not surprising since the SPA does not require knowledge about the *dynamics* of the noise sources, provided they are slow [23].

Equation (2) can be systematically approximated by sampling better $\xi(t')$ in $[0, t]$. For the first correction $P[\xi(t)]$ is approximated by the joint distribution $P(\xi_t t; \xi_0 0)$, where $\xi_t = \xi(t)$. At $\theta = \pi/2$ for generic

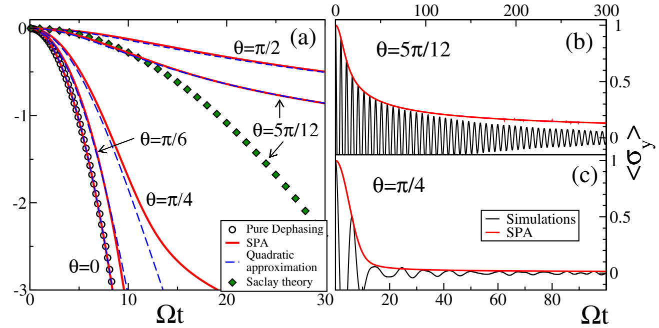

FIG. 2 (color online). Signal decay at nonoptimal points. (a) The SPA (solid red lines) is compared with the quadratic approximation Eq. (3) (dashed blue lines) at different bias points. Equation (3) works near $\theta = 0$ and $\theta = \pi/2$. The exact result at $\theta = 0$ is also indicated (dots). This result multiplied by $\cos^2\theta$ is often used also at $\theta \neq 0$ for interpreting experiments ("Saclay theory," diamonds). Out of the optimal point the SPA agrees with results of the simulations with BF-$1/f$ noise, for the set of Fig. 1, at $\theta = 5\pi/12$ (b) and $\theta = \pi/4$ (c).

Gaussian noise we find

$$i\Phi(t) = \frac{1}{2}\ln\left[1 + i\frac{\sigma_\xi^2[1-\pi(t)]t}{\Omega}\right] + \frac{1}{2}\ln\left[1 + i\frac{\sigma_\xi^2\pi(t)t}{3\Omega}\right],$$

where $\pi(t) = \frac{1}{2\sigma^2}\int_0^\infty (d\omega/\pi)S(\omega)(1 - e^{-i\omega t})$ is a transition probability, depending on the stochastic process. For Ornstein-Uhlembeck processes it reduces to the result of Ref. [10]. The first correction suggests that the SPA, in principle valid for $t < 1/\gamma_M$, may have a broader validity (see Fig. 1). For $1/f$ noise due to a set of BFs it is valid also for $t \gg 1/\gamma_M$ if $\gamma_M \lesssim \Omega$. Of course, the adiabatic approximation is tenable if $t < T_1 = 2/S(\Omega)$.

The main effect of faster BFs in the $1/f$ spectrum is the decrease of T_1. Relaxation is due only to the fast part of the spectrum $\omega \sim \Omega$ and well reproduced by the Golden Rule. This is not true for decoherence; for instance, in our example (Fig. 3) $T_2 \approx T_1$, as observed in the experiments [3] of Astafiev *et al.* We study the interplay of fast and slow noise by a two-stage elimination. We first decompose $\xi(t) \to \xi(t) + \xi_f(t)$. Here $\xi(t)$ represents all BFs having switching rates small enough to be treated in the adiabatic approximation, $\gamma_i < \gamma_{\rm ad}$ (in practice we may take $\gamma_{\rm ad} \sim \Omega/10$). Fast BFs are described by $\xi_f(t)$ or better modeled by a set of quantum impurities as in Ref. [6]. In this case $\xi_f(t) \to \hat{\xi}_f$ and we have a quantum environment able to produce also spontaneous decay. The reduced density matrix of the qubit can be written as

$$\rho(t) = \int \mathcal{D}\xi(t)P[\xi(t)]\rho_f[t|\xi(t)],$$

where $\rho_f[t|\xi(t)]$ is the qubit density matrix resulting from the elimination of the fast environment, under the "drive" $\xi(t)$, and can be found within the weak coupling theory [24]. This is very simple if we treat slow noise in the SPA,

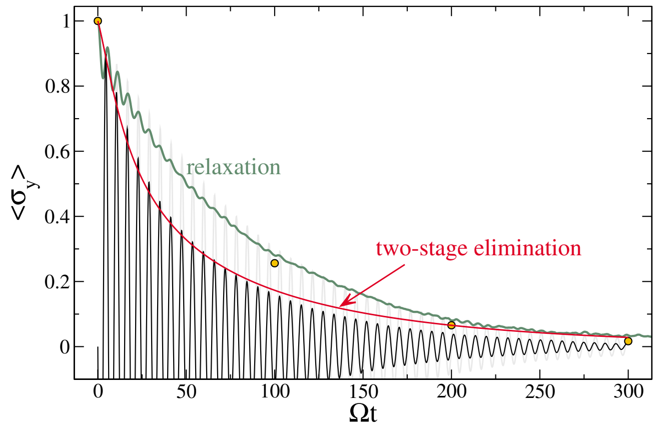

FIG. 3 (color online). Results of simulations with an adiabatic plus fast BF-$1/f$ environment (same parameters of Fig. 1 except for $\gamma_M = 10^{11}$ rad/s). Relaxation (thick solid green line) is given by the weak coupling result (dots). The initial suppression of the oscillation amplitude is partially removed by a feedback protocol (shaded curves) and is well described by the two-stage elimination SPA theory (solid red line), Eq. (4).

where $\xi(t) = \xi_0$. We are left with averages over $P(\xi_0)$ of the entries of $\rho_f[t|\xi_0]$. For instance, the decay of the coherences at $\theta = \pi/2$ is given by

$$e^{-1/4S_f(\Omega)t - 1/2\ln|1+[i\Omega+S_f(0)-1/2S_f(\Omega)]\sigma_\xi^2 t/\Omega^2|}, \qquad (4)$$

where $S_f(\omega)$ refers to the set of fast BFs, whereas σ_ξ^2 refers to the set of slow BFs. Equation (4) agrees very well with simulations (Fig. 3).

We notice that the validity of Eq. (4) is not limited to the $\sim 1/\gamma$ distribution of switching rates giving rise to $1/f$ noise. According to this description, relaxation and inhomogeneous broadening are due to separate sets of BFs. Therefore, no special relation is expected to hold between T_1 and T_2. The mixed term in Eq. (4), due to the interplay between slow and fast BFs, does not qualitatively change this conclusion. Finally, Eq. (4) is rather independent from the nature of the noise sources and the form of the spectrum and it is applicable in other situations, e.g., when slow impurity noise combines with fast electromagnetic noise. Equation (4) becomes exact if ξ_f determines white noise, a scenario recently proposed to fit decoherence in phase-charge qubits.

We come now to effects of the discrete nature of noise. Results presented so far rely on the SPA and on the weak coupling theory; therefore, they apply to situations where discrete and Gaussian noise are indistinguishable. Striking differences appear when only decoherence during time evolution [6,16] matters, or if the distribution of environment couplings v_i is wide [8], the realistic scenario for the solid state. We now study the interplay of $1/f$ noise with RTN produced by one BF which is more strongly coupled with the qubit. The model for the BF is minimal: it is an incoherent slow fluctuator, having $\gamma_0 \ll 1/t \ll \Omega$ but

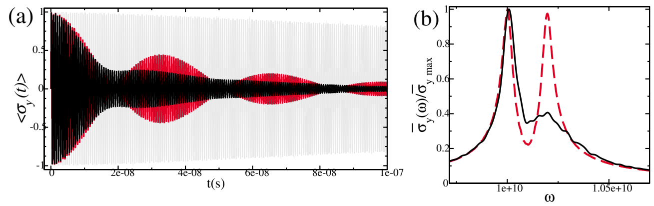

FIG. 4 (color online). (a) $\langle\sigma_y\rangle$ at $\theta = \pi/2$, $\Omega = 10^{10}$ Hz. The effect of weak adiabatic $1/f$ noise (light gray line) ($\gamma[10^5, 10^9]$ Hz, uniform $v = 0.002\ \Omega$, $n_d = 250$) is strongly enhanced by adding a single slow ($\gamma/\Omega = 0.005$) more strongly coupled ($v_0/\Omega = 0.2$) BF (black line), which alone would give rise to beats (red line). (b) When the BF is present the Fourier transform of the signal may show a split-peak structure. Even if peaks are symmetric for the single BF alone (dashed line), $1/f$ noise broadens them in a different way (solid line).

$v_0 \lesssim \Omega$. Even if the BF is not resonant with the qubit [25] it strongly affects the output signal. If $g_0 > 1$ [7], it determines beats in the coherent oscillations and split peaks in spectroscopy, which are signatures of a discrete environment. The additional BF makes bistable the working point of the qubit and amplifies defocusing due to $1/f$ noise. Even if the device is initially optimally polarized, during t_m the BF may switch it to a different working point. The line shape of the signal will show two peaks, split by $\sim\Omega' - \Omega$ and differently broadened by the $1/f$ noise in background. The corresponding time evolution will show damped beats, this phenomenology being entirely due to the non-Gaussian nature of the environment. For illustrative purposes we show results of a simulation at the optimal point, where $1/f$ noise is adiabatic and weaker than the typical noise level in charge qubits (see Fig. 4). This picture applies to smaller devices. The fact that even a single impurity on a $1/f$ background causes a substantial suppression of the signal poses the problem of reliability of charge based devices. An analytic two-stage elimination combining the SPA with the solutions for the dynamics of a qubit coupled to an impurity [6,7] can be developed, and will be presented elsewhere.

Recently, effects of the resonant coupling of the qubit with a quantum two-level system, simulating defects in the tunnel oxide, have been proposed to explain features of the dynamics of phase Josephson qubits [25]. We have shown that these effects are present even if the impurity behaves as a slow stochastic fluctuator. Our model describes a very common situation in the solid state [8], and it is a minimal model for charge noise in charge and charge-phase qubits. Finally, the interplay between slow noise and fast noise with generic spectrum is likely to be important in general and can be studied with Eq. (4). The main open questions are the accurate characterization beyond phenomenology of the physics of the noise sources and the design of specific strategies to defeat them and to improve reliability of devices.

We acknowledge discussion with D. Esteve, R. Fazio, G. Ithier, Y. Nakamura, G. Schoen, and A. Shnirman. We acknowledge support from projects EU-SQUBIT2 (IST-2001-390083) and MIUR-FIRB (RBAU01A9PM).

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