Novel Mechanism of Anomalous Electron Heat Conductivity and Thermal Crashes during Alfvénic Activity in the Wendelstein 7-AS Stellarator

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Enhanced plasma heat conductivity in the presence of kinetic Alfvén waves (KAW) is predicted theoretically. The enhancement is shown to be strongest when the electron collision frequency exceeds the particle transit frequency in the wave field. Alfvén waves (both KAW and ideal MHD Alfvén eigenmodes generating the KAW) are studied in a shot of the Wendelstein 7-AS stellarator. On the basis of these results, strong thermal crashes observed during bursting Alfvénic activity in the mentioned shot are explained.

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Alfvén instabilities driven by fast ions were observed in many experiments on tokamaks and stellarators [1–3]. In tokamaks, they can strongly affect the fast ion confinement but not the bulk plasma (only weak local thermal crashes have been observed [4]). In contrast to this, experiments on the Wendelstein 7-AS (W7-AS) stellarator have shown that Alfvénic activity can strongly deteriorate the global energy confinement time: strong thermal crashes (the temperature dropped by up to 30%) were reported in Ref. [2]; strong drops of the plasma energy content during Alfvén instabilities were observed also in a more recent series of experiments in 2002. This motivated the fulfillment of the present work, where a theory of the influence of a monochromatic Alfvén wave on the electron heat flux across the magnetic field is suggested, and it is shown that the revealed mechanism of enhancement of heat conductivity by Alfvén waves can be responsible for strong thermal crashes observed in W7-AS.

The basic idea of this mechanism is that the finite electric conductivity of the plasma leads to a wave-induced enlargement of the particle deflections from the magnetic flux surfaces, whereas the Coulomb collisions provide a random walk of the particles with enlarged steps. The waves and the particles interact through the resonance $\omega = k_\parallel v_\parallel$ (ω is the wave frequency, $k_\parallel$ and $v_\parallel$ are the wave vector and the particle velocity along the magnetic field, respectively); because of this, the particle orbits are divided into trapped and untrapped ones. Therefore, by similarity with the neoclassical theory of the particle transport in tokamaks (see, e.g., the overview in Ref. [5]), we can introduce three regimes of the anomalous transport: collisional, plateau, and of weak collisions. The collisional regime takes place when the collision frequency $\nu_e \equiv \nu_{ee} + \nu_{ei}$ (ν_{ee} and ν_{ei} are the elecron-electron and electron-ion collision frequencies, respectively) exceeds the transit frequency of the thermal electrons $\omega_t = k_\parallel v_e$ ($v_e = \sqrt{T_e/M_e}$, T_e the electron temperature, M_e the electron mass). The regime of weak collisions takes place when the bounce frequency of the particles trapped in the wave field $\omega_b \equiv k_\parallel (\delta v)_{\rm res}$ exceeds the effective collision frequency $\nu_{\rm ef} \equiv \nu_e v_e^2/(\delta v)^2_{\rm res}$, where $(\delta v)_{\rm res}$ is the resonance width, $(\delta v)_{\rm res} \ll v_e$. The plateau regime occurs for $\omega_t (\delta v)^3_{\rm res}/v_e^3 < \nu_e < \omega_t$. The mechanism suggested is effective provided that the wave electric field along equilibrium magnetic field, $\tilde{E}_\parallel$, is considerable (thus, it does not work in the ideal MHD approximation). A possible source of this field is the transformation of ideal MHD Alfvén waves destabilized by fast ions into kinetic Alfvén waves (KAW).

Let us consider first the electron transport in the plateau regime. We take the perturbed quantities in the form $\tilde{X} = \hat{X}(r)\exp(-i\omega t + im\vartheta - in\varphi)$ and proceed from the following equations of the particle drift motion:

$$\dot{r} = v_\parallel \frac{\tilde{B}_r}{B} + c\frac{\tilde{E}_\theta}{B} = -\frac{\Omega}{k_\parallel}\frac{\tilde{B}_r}{B} + \frac{ck_\vartheta \tilde{E}_\parallel}{k_\parallel B}, \tag{1a}$$

$$\dot{\vartheta} = \frac{v_\parallel \iota}{R} + \frac{v_E}{r}, \qquad \dot{\varphi} = \frac{v_\parallel}{R}, \qquad \dot{\mathcal{E}} = e v_\parallel \tilde{E}_\parallel, \tag{1b}$$

where r, ϑ, and φ are the radial, poloidal, and toroidal flux coordinates, respectively, ι is the rotational transform, m and n are the wave numbers, $\Omega = \omega - k_\vartheta v_E - k_\parallel v_\parallel$, $k_\parallel = (m\iota - n)/R$, $k_\vartheta = m/r$, R is the major radius of the torus, $v_E = c[\mathbf{E}_0 \times \mathbf{B}_0]_\vartheta / B^2 = -cE_0/B$, $\mathbf{E}$ and $\mathbf{B}$ are the electric field and the magnetic field, respectively, $E_0 \equiv E_{0r}$, the subscript "0" labels unperturbed quantities, and $E_\parallel = \mathbf{E}\cdot\mathbf{B}/B$. Equations (1) take into account that $\tilde{B}$ is negligible for the Alfvén waves. Using Eqs. (1), we can write the drift kinetic equation for the distribution function of the circulating electrons as follows:

$$\frac{d}{dt}\left(\tilde{f} + \frac{\tilde{B}_r}{ik_\parallel B_0}\frac{\partial f_0}{\partial r}\right) - C(\tilde{f}) = -\frac{ck_\vartheta}{k_\parallel}\frac{\tilde{E}_\parallel}{B_0} \times \left[\frac{\partial f_0}{\partial r} + k_\parallel v_\parallel \frac{eB_0}{ck_\vartheta}\frac{\partial f_0}{\partial \mathcal{E}}\right], \tag{2}$$

where $d/dt = \partial/\partial t + (v_\parallel \iota/R + v_E/r)\partial/\partial\vartheta + (v_\parallel/R)\partial/\partial\varphi$, $C(\tilde f)$ is the linearized collision operator, and f_0 is the Maxwellian distribution. Note that, as is well known, the role of the Coulomb collisions in the plateau regime is reduced to resolving the singularity arising in the considered case because of the resonance $\Omega = 0$. Therefore, a simple way to obtain a solution of Eq. (2) is to assume that ω has a small positive imaginary part and omit $C(\tilde f)$ (a similar approach is often used for the calculation of the Landau damping and the neoclassical transport in the plateau regime). Then we obtain the following for the resonance part of $\tilde f$:

$$\tilde f_{\rm res} = -\pi c\delta(\Omega)\frac{k_\vartheta}{k_\parallel}\frac{\tilde E_\parallel}{B_0}\left[\frac{\partial f_0}{\partial r} - \frac{eB_0}{T}\frac{k_\parallel}{k_\vartheta}\frac{v_{\rm res}}{c}f_0\right]. \quad (3)$$

Knowing $\tilde f_{\rm res}$ and taking into account that $\mathrm{Re}(\overline{\tilde E_\parallel^2}) = (1/2)|\hat E_\parallel|^2$ (the overline means the phase averaging), we can calculate the electron flux across the magnetic field, $\Gamma_e = \int d^3v \tilde f_{\rm res}\dot r$, with $\dot r$ given by Eq. (1a):

$$\Gamma_e = -\chi_e^{\rm plat} n_e\left[\frac{n_e'}{n_e} + \frac{1}{2}\frac{T_e'}{T_e}\left(\frac{v_{\rm res}^2}{v_e^2} - 1\right) - \frac{e_e B_0}{cT_e}\frac{k_\parallel}{k_\vartheta}v_{\rm res}\right], \quad (4)$$

where $v_{\rm res} = (\omega - k_\vartheta v_E)/k_\parallel$ is the resonance velocity, primes mean radial derivatives, χ_e is the diffusivity coefficient given by

$$\chi_e^{\rm plat} = \frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{c^2k_\vartheta^2}{v_e k_\parallel^3}\frac{|\hat E_\parallel|^2}{B_0^2}\exp\left(-\frac{v_{\rm res}^2}{2v_e^2}\right), \quad (5)$$

the superscript "plat" labels quantities in the plateau regime.

The ion flux is described by equations similar to Eqs. (4) and (5). It is negligible when $v_{\rm res} \gg v_i$ ($v_i = \sqrt{T_i/M_i}$, T_i and M_i are the ion thermal velocity and mass, respectively). Therefore, the ambipolarity condition is $\Gamma_e = \Gamma_{\rm beam}$, where $\Gamma_{\rm beam}$ is the flux of the beam ions. Moreover, because $n_{\rm beam} \ll n_e$, where $n_{\rm beam}$ is the beam density, and $\Gamma_{\rm beam} \ll \chi_e n_e'$, we have $\Gamma_e \approx 0$, which gives (for $v_{\rm res} \ll v_e$)

$$v_{\rm res} = \frac{\omega - k_\vartheta v_E}{k_\parallel} \approx \frac{k_\vartheta}{k_\parallel}\frac{cT_e}{e_eB_0}\left(\frac{n_e'}{n_e} - \frac{1}{2}\frac{T_e'}{T_e}\right). \quad (6)$$

Equation (6) implies that sheared plasma rotation will be formed as a result of the considered diffusion:

$$\omega_A^0 - m(\omega_E - \omega_E^0) = \frac{k_\vartheta cT_e}{e_eB_0}\left(\frac{n_e'}{n_e} - \frac{1}{2}\frac{T_e'}{T_e}\right), \quad (7)$$

where $\omega_A^0 = \omega_A(r = r_0)$, $\omega_A(r) = k_\parallel v_A$, $\omega_E(r) = v_E/r$ is the frequency of the poloidal plasma rotation, $\omega_E^0 = \omega_E(r_0)$, r_0 is a point where $\omega = \omega_A^0$, i.e., r_0 is a point such that in the frame rotating with the frequency ω_E^0 the mode frequency is equal to that of the Alfvén continuum (AC) or close to an extremum of AC. When the characteristic inhomogeneity lengths are of the order of the plasma radius, then $\omega_A \gg \omega_{*e}$, where ω_{*e} is the electron diamagnetic frequency, and the right-hand side of Eq. (7) is relatively small. Therefore, Eq. (7) requires that the rotation terms compensate approximately for ω_A, which is possible away from the point r_0 for a moderate plasma rotation frequency provided that $k_\theta \gg k_\parallel$ and the shear of the plasma rotation is sufficiently large. In the vicinity of $r = r_0$, the gradients of plasma parameters must increase as a result of the diffusion.

With Eq. (6) satisfied, we find the following electron thermal flux:

$$Q_e = \int d^3v \frac{M_e v^2}{2}\dot r f_e = -\chi_e^{\rm plat} n_e T_e'. \quad (8)$$

Now we consider the collisional regime. Integrating Eq. (1), we find the particle radial displacement, ξ_r, as follows:

$$\xi_r = \frac{ick_\vartheta}{k_\parallel(\omega - k_\vartheta v_E - k_\parallel v_\parallel)}\frac{\tilde E_\parallel}{B_0} - \frac{i\tilde B_r}{k_\parallel B_0}. \quad (9)$$

The first term in Eq. (9) depends on the velocity of the individual particle, whereas the second one describes the oscillations of the particles together with the plasma. Therefore, only the first term describes the deflection of the particles from the perturbed flux surfaces. Keeping only this term, taking $v_\parallel = v_e$, and assuming that $|\omega - k_\vartheta v_E| \ll |k_\parallel|v_e$, we obtain the characteristic radial displacement of a particle during its random walk, Δ, as $\Delta \approx ck_\vartheta\hat E_\parallel/(v_e k_\parallel^2 B_0)$. Then, evaluating the diffusivity coefficient in the collisional regime as $\Delta^2\nu_e/2$, we arrive at the following estimate:

$$\chi_e^{\rm col} \approx \frac{\nu_e}{2}\frac{c^2k_\vartheta^2}{v_e^2k_\parallel^4}\frac{|\hat E_\parallel|^2}{B_0^2}. \quad (10)$$

Comparing Eq. (10) and Eq. (5), we conclude that, as one could expect, $\chi_e^{\rm col} = \chi_e^{\rm plat}$ for $\nu_e = \omega_t$ (up to a factor of $\sqrt{\pi/2}$), i.e., the diffusivity coefficients are equal at the collision frequency separating the collision region from plateau region.

Finally, we consider the regime of weak collisions. It follows from Eq. (9) that the maximum deflection of the particles trapped in the resonance with the wave is $\Delta_{\rm res} \approx \Delta v_e/(\delta v)_{\rm res}$. Because $(\delta v)_{\rm res} \ll v_e$, the effective collision frequency is $\nu_e v_e^2/(\delta v)_{\rm res}^2$. The fraction of the resonant particles is $(\delta v)_{\rm res}/v_e$. Taking these facts into account, we can estimate the electron diffusivity coefficient in the weak collisions regime as $\chi_e^{\rm w/c} \approx 0.5\nu_e\Delta^2 v_e^3/(\delta v)_{\rm res}^3$.

The magnitude of $(\delta v)_{\rm res}$ for Alfvén waves is determined as in the case of a perturbation with $\tilde E_\parallel = -ik_\parallel\tilde\Phi$, where $\tilde\Phi$ the scalar potential: $(\delta v)_{\rm res} = 2\sqrt{eE_\parallel/(M_e k_\parallel)}$. This expression for $(\delta v)_{\rm res}$ was obtained from the following invariant of the particle motion:

$$\frac{M}{2}(v_\parallel - v_{\rm res})^2 + U = \mathrm{const}, \quad (11)$$

where $U = e\tilde{\Phi} + e\tilde{A}_{\parallel}\omega/(ck_{\parallel})$, $\tilde{A}_{\parallel}$ is the longitudinal vector potential (so that $\tilde{E}_{\parallel} = -\nabla_{\parallel}U/e$). This invariant results from the symmetry of the particle Lagrangian provided that detuning from the resonance occurs mainly due to variation of $v_{\parallel}$.

As the perturbed magnetic field rather than the electric field is usually measured in experiment, we express $\tilde{E}_{\parallel}$ in terms of $\tilde{B}_r$. With this purpose, we use the equations $\tilde{E}_{\parallel} = 4\pi i\tilde{j}_{\parallel}/(\omega\epsilon_{\parallel})$, $\tilde{\mathbf{j}} = (c/4\pi)\nabla\times\tilde{\mathbf{B}}$, and $\nabla\cdot\mathbf{B} = 0$, where $\epsilon_{\parallel} = \omega_{pe}^2/(k_{\parallel}^2 v_e^2)$ is the plasma dielectric permeability along the magnetic field for $\omega \ll k_{\parallel}v_e$, $\tilde{\mathbf{j}}$ is the perturbed current, and ω_{pe} is the plasma frequency. Then we can write the coefficients of the electron heat conductivity as follows:

$$\chi_e^{\mathrm{col}} = \chi_e^{\mathrm{plat}}\frac{\nu_e}{k_{\parallel}v_e}, \qquad \chi_e^{\mathrm{plat}} = \frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{k_{\parallel}v_{Ae}^4}{\nu_e\omega^2}\left(\frac{\hat{B}_r}{B_0}\right)^2 k_{\perp}^4\rho_s^4, \tag{12a}$$

$$\chi_e^{\mathrm{w/c}} = \frac{1}{16}\sqrt{\frac{\pi}{2}}\frac{\nu_e v_e v_{Ae}}{\omega^{1/2}\omega_{Be}^{3/2}}\left(\frac{k_{\vartheta}}{k_{\parallel}}\right)^{3/2}\sqrt{\frac{\hat{B}_r}{B_0}}k_{\perp}\rho_s, \tag{12b}$$

where $v_{Ae} = B/\sqrt{4\pi n_e M_p}$, $\rho_s = v_s/\omega_{Bp}$, $v_s = \sqrt{T_e/M_p}$, ω_B is the gyrofrequency, the subscripts p label proton quantities, and $k_{\perp}$ is the transverse wave number.

Now we proceed to considering an experiment on W7-AS. The instability in the W7-AS shot No. 34723 (see Fig. 1) had a bursting character and was characterized by strong frequency chirping down, from about 70 to 45 kHz. Thermal crashes occurred at the final stage of the bursts when the instability was strongest. The duration of the instability bursts was about 2.5 ms, and the repetition period of the bursts was 8–10 ms. The dominant poloidal mode numbers during the instability were $m_1 = 3$ and $m_2 = 5$. The plasma density was high, $n_e \sim 10^{20}$ m^{-3}, whereas the temperature was low, $T(0) = 290$ eV, and the effective charge number was $z_{\mathrm{eff}} \sim 2$. Because of this, the collision frequency was high ($\nu_e \sim 2\times10^6$ s^{-1}) and exceeded the electron transit frequency ($\omega_t \sim 7\times10^5$ s^{-1} for $k_{\parallel} = 10^{-3}$ cm^{-1}, $T = 260$ eV). Therefore, the heat conductivity was in the collisional regime. The equilibrium magnetic field was $B_0 = 1.2$ T, and the minor radius of the plasma, $a = 15$ cm.

Although the harmonics mentioned are coupled through the ellipticity coupling number, $\mu \equiv m_2 - m_1 = 2$, the ellipticity-induced Alfvén eigenmode (EAE) instability was dismissed as an explanation since the cylindrical branches of AC with $m_1 = 3$ and $m_2 = 5$ intersect at $\iota = 0.5$, whereas $\iota(r)$ was predicted to be less than 0.5 from equilibrium calculations. An eigenmode analysis carried out with the code BOA [6] revealed eigenmodes with a toroidal mode number $n = 2$. An eigenmode with the frequency near the maximum of the AC branch $m/n = 5/2$ is shown in Fig. 2. The mentioned branch, as well as the whole AC in the region below 120 kHz, was calculated with the AC code COBRA [6], see Fig. 3. Both Figs. 2 and 3 are obtained for a plasma with a slightly nonmonotonic rotational transform. It follows from our analysis that the observed mode is not a gap mode; therefore, it can be identified as an unusual global Alfvén eigenmode (GAE) with a frequency lying above the AC.

It is clear that the plasma confinement can be significantly affected by the anomalous transport only when the waves exist in a considerable part of the plasma cross section. This is not the case for the core localized mode shown in Fig. 2. Nevertheless, it affects a considerable part of the plasma due to conversion into a KAW, which, in addition, has significant $\tilde{E}_{\parallel}$. The KAW arises because the frequency of the GAE considered lies above the corre-

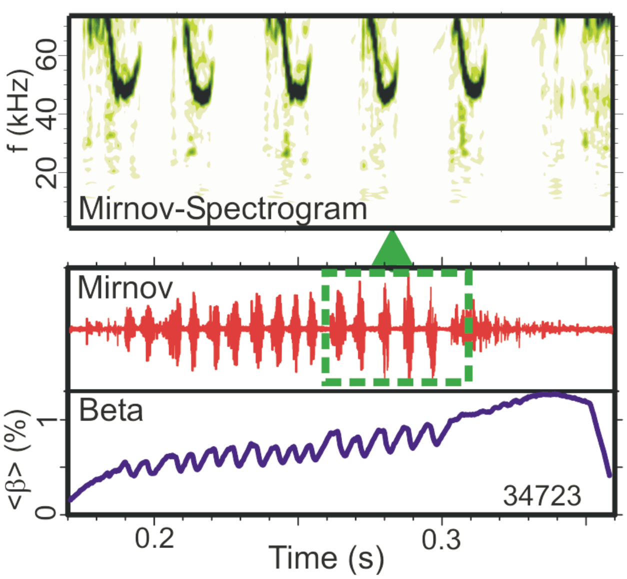

FIG. 1 (color online). Bursts of Alfvén instabilities in W7-AS shot No. 34723.

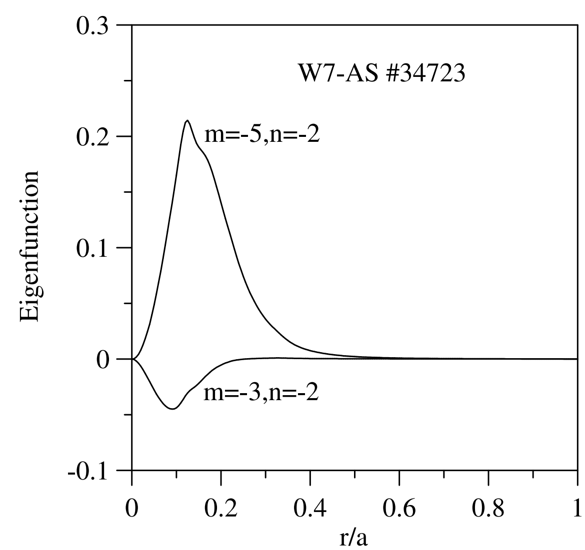

FIG. 2. Scalar potential of an Alfvén eigenmode calculated with the ideal MHD code BOA [6] for W7-AS shot No. 34723.

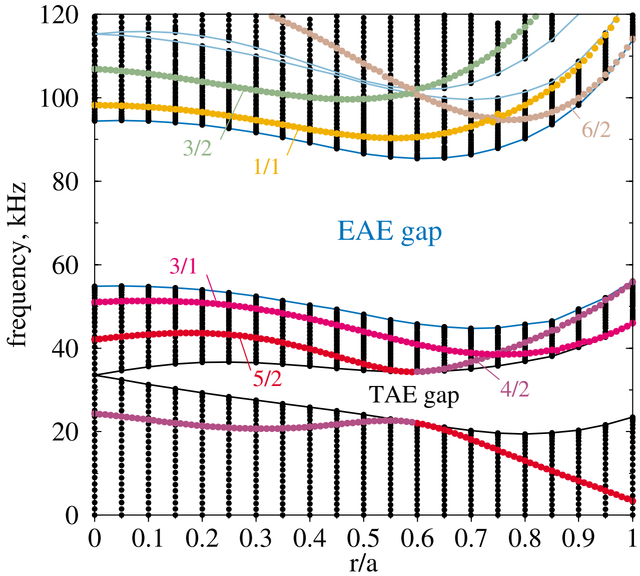

FIG. 3 (color). The Alfvén continuum calculated by the code COBRA for W7-AS shot No. 34723. Black vertical lines, AC at several radii; thick curves, selected AC branches labeled by the mode numbers, m/n; thin curves, boundaries of main gaps.

sponding AC branch, $\omega_A(r)$ (in contrast to the conventional GAE). Our analysis (which will be published elsewhere) shows that in this case a KAW is generated due to "tunnel" interaction with the continuum, like in the case of TAE (toroidicity-induced Alfvén eigenmodes) [7]. The ratio of the amplitude of the radiated KAW to the GAE amplitude can be estimated as $\exp[-(\delta\omega/\omega)(L/\rho_i)]$, where $\delta\omega = \omega - \omega_{A\max}$, $\omega_{A\max} = \max\omega_A(r)$, and $L = \omega_A/[(\omega_A^2)'']^{1/2}$. For the GAE shown in Fig. 2, $\delta\omega/\omega \sim 10^{-2}$, and $L \sim 10$ cm. As $\rho_i = 0.25$ cm, we conclude that the KAW amplitude is comparable to the GAE amplitude. The absorption length of the KAW is

$$l_{\rm abs} = 2\sqrt{\frac{2}{\pi}}\frac{v_e}{v_A}\frac{\omega}{\omega_A}\left(\frac{1}{k_\perp^2\rho_i^2} - \frac{3}{4}\right)k_r^{-1}, \tag{13}$$

where

$$k_\perp^2\rho_i^2 = \frac{\omega^2 - \omega_A^2(r)}{\omega_A^2(r) + \frac{3}{4}\omega^2}. \tag{14}$$

One can show from Eq. (14) that $k_\perp \approx 0$ at the point where the KAW is generated, and $k_{\perp\max}^2\rho_i^2 \approx 0.3$. This leads to $l_{\rm abs} \sim 10$ cm. Because of this, the destabilization of the ideal GAE localized in the central region affects a considerable part of the plasma. Analysis of the dispersion relation of the KAW shows that the coupling of Fourier harmonics of the wave results in transformations of KAW branches near rational-ι points where $|k_\parallel|$ of the branches coincide, in our case from an $m/n = 5/2$ into an $m/n = 4/2$ branch at $r/a \sim 0.5$; the latter propagates to the point $r/a \sim 0.7$, where $\omega = \omega_A$ (see Fig. 3). At this point the wave is reflected because the KAW is evanescent when its frequency is below the corresponding AC branch. Thus, we conclude that the instability affects about two-thirds of the plasma radius, which roughly coincides with the experimentally observed inversion radius of the thermal crashes.

Let us evaluate the amplitude of the wave required to account for the observed thermal crashes. Taking $\tau_E = a^2/(\mu_0^2\chi_e) = 3$ ms (where $\mu_0 = 2.4$) during the thermal crash [$\tau_E = a^2/(\mu_0^2\chi_e) \sim 10$ ms before the considered crash], $k_\perp^2\rho_s^2 \sim 1/3$, $k_\parallel \sim 10^{-3}$ cm^{-1}, we obtain from Eq. (12a) that $\tilde{B}_r/B \lesssim 10^{-3}$. This magnitude of $\tilde{B}_r/B$ is quite reasonable from the point of view of available experimental data: the Mirnov-measured amplitude of $\tilde{B}_\theta/B$ outside the plasma was as large as 10^{-4}, whereas soft x-ray measurements indicated that the instability was localized in the core, which implies that its amplitude inside the plasma well exceeded the level of 10^{-4}.

Although the electron heat flux was strongly enhanced, the electron confinement time was weakly affected in the experiment because of both a small number of fast ions and a very flat electron density profile.

In summary, it is revealed that the presence of Alfvén waves, especially KAW, can strongly enhance the electron heat flux. The heat flux is largest in the "anomalous collisional" regime [when $k_\parallel v_e < \nu_e$]. As our analysis shows, it was this regime that took place during the Alfvénic activity in the W7-AS shot No. 34723. The instability observed in W7-AS is identified as an unconventional GAE mode (with a frequency above the AC) accompanied by the generation of KAW. The calculated GAE eigenmodes are localized at the plasma center; the generation of the KAW considerably extends the region affected by the instability. The mechanism of the anomalous heat transfer considered explains the oscillations of the plasma energy content without noticeable density variations in W7-AS shot No. 34723. On the other hand, this mechanism is typically less effective in tokamak plasmas, where regimes of weak collisions and plateau take place. This explains why large thermal crashes were not observed in tokamaks.

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