

Effective Desynchronization by Nonlinear Delayed Feedback

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We show that nonlinear delayed feedback opens up novel means for the control of synchronization. In particular, we propose a demand-controlled method for powerful desynchronization, which does not require any time-consuming calibration. Our technique distinguishes itself by its robustness against variations of system parameters, even in strongly coupled ensembles of oscillators. We suggest our method for mild and effective deep brain stimulation in neurological diseases characterized by pathological cerebral synchronization.

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Synchronization is a natural property of interacting oscillators or functional units, intensively studied in many physical [1–3], engineering [4,5], chemical [6,7], biological [8], and medical [9,10] systems. However, synchronization is not always desirable. In fact, several neurological diseases, such as essential tremor and tremor in Parkinson's disease, are caused by synchronized populations of oscillatory neurons [11,12]. The standard therapy for such patients at an advanced stage is permanent electrical deep brain stimulation (DBS) at high frequencies [13]. However, in some patients DBS may not help or may cause side effects, or its therapeutic effect may decline over time [14]. Thus, there is a significant clinical need for mild stimulation techniques which restore desynchronized (i.e., normal [12]) dynamics in networks of oscillatory neurons [10].

To control the dynamics of chaotic oscillators Pyragas [15] suggested to use delayed feedback. Nowadays, the impact of delay on the collective dynamics of coupled oscillators is intensively studied; see, e.g., [5,16–19]. In particular, delay-induced synchronization and multistability of many phase-locked states [16] as well as of stable synchronous and desynchronous states [18] has been reported. Recently, Rosenblum and Pikovsky [19] proposed to use linear delayed feedback to control synchrony in ensembles of coupled oscillators. In the parameter plane of feedback amplification and delay they found bounded regions of desynchronization, complemented by wide areas of stimulus-enhanced synchronization; see also [17,18]. However, the risk of unintentional boosting of synchronization limits the linear method's applicability to DBS, especially due to large variations (of 30% and more) of the frequency of the neuronal target populations [20].

In this Letter we study the impact of nonlinear delayed feedback, i.e., linear delayed feedback nonlinearly combined with the instantaneous signal, on synchronization. This yields a completely different result: robust desynchronization. For several reasons this is important: (i) We show that linear delayed feedback and nonlinear delayed feedback have completely different effects and mecha-

nisms of action. Our nonlinear delayed feedback cannot reinforce synchronization, which is extremely important for medical applications. Because of its nonlinear nature, this type of feedback acts on saturation mechanisms by effectively suppressing the amplitude of collective oscillations. (ii) Our findings may contribute to a better understanding of the function of nonlinear feedback loops in the central nervous system [9]. (iii) We develop a reliably desynchronizing stimulation technique which requires no time-consuming calibration, which works on demand, with a vanishing amount of stimulation, even in the case of strong coupling, and is robust against parameter variations. Our novel technique specifically counteracts the undesired coupling and restores the oscillators' natural frequencies. Hence, our method may be particularly suitable for both technical and medical applications requiring robust desynchronization by mild stimulation.

The method is demonstrated on ensembles of coupled limit-cycle and phase oscillators. We consider the following system of N globally coupled limit-cycle oscillators:

$$\dot{Z}_j(t) = (a_j + i\omega_j - |Z_j|^2)Z_j(t) + C\bar{Z}(t), \quad (1)$$

where $Z_j = x_j + iy_j$ are the complex variables of the individual oscillators and $\bar{Z}(t) := N^{-1} \sum_{j=1}^N Z_j(t)$ is the mean field of the ensemble. Without coupling ($C = 0$) all oscillators independently rotate on limit cycles with individual radii $\sqrt{a_j}$ and individual natural frequencies ω_j . The variations of the mean field $\bar{Z}(t)$ are of order $1/\sqrt{N}$ [2,21]. If the coupling strength increases, all oscillators synchronize, i.e., start to rotate with the same frequency, where all mutual phase differences decay (mod 2π) [21]. The synchronized dynamics is then characterized by large variations of the mean field [6].

We administer an external stimulus $S_j(t)$ to the synchronized ensemble (1) by adding the signal $S_j(t)$ to the right-hand side of Eq. (1). In our first example, the stimulation signal $S_j(t)$ is considered to take the form

$$S_j(t) := K\bar{Z}(t)Z_j(t)Z_j^*(t - \tau), \quad (2)$$

where K is a stimulus amplification parameter, τ is a time delay, and the asterisk denotes complex conjugacy. Results of numerical simulations of system (1) stimulated with signal (2) are shown in Fig. 1(a). Stimulation with $K = 3$ results in a suppression of the synchronization, leading to small variations of the mean field. Moreover, an increase of the number of oscillators in the ensemble further reduces the variations: For $N = 900$ (red curve) the amplitude of $\bar{X}(t)$ in the stimulated regime is about 3 times smaller than that for $N = 100$ (blue curve). Thus, the oscillators again become uncorrelated as in the uncoupled regime for $K = C = 0$. In addition, the stimulation does not destroy the natural oscillatory activity of the individual oscillators [Fig. 1(c)].

The desynchronizing effect of stimulation with signal (2) depends on the stimulation parameters K and τ as shown in Fig. 2(a), where the averaged order parameter $\langle R(t) \rangle = \langle |N^{-1} \sum_{j=1}^N Z_j(t) / |Z_j(t)|| \rangle$ is plotted versus K and τ . $\langle \cdot \rangle$ denotes averaging over time. Let T denote the mean period of ensemble (1) without stimulation ($T = 5$ in Figs. 1 and 2). Then one can distinguish two different regions in the (K, τ) -parameter plane: (i) In the broad stripelike parameter regions around $\tau = T(2n + 1)/2$, $n = 0, 1, \dots$, the order parameter practically vanishes. These regions correspond to an ideal, complete desynchronization in system (1) and (2). (ii) In the complementary stripe-like regions around $\tau = Tn$, $n = 0, 1, \dots$, the synchronization is also strongly suppressed, but the order parameter $\langle R(t) \rangle$ attains slightly larger values. The differences of $\langle R(t) \rangle$ between two parameter regions (i) and (ii) become less pronounced for larger delay values τ , where ideal desynchronization is achieved irrespectively of τ .

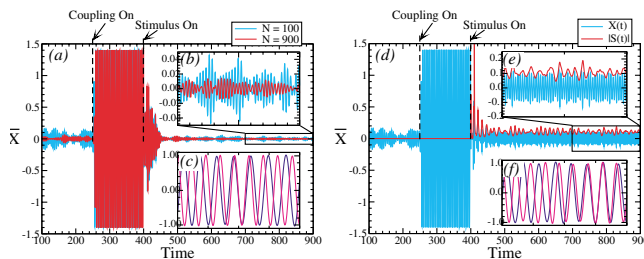


FIG. 1 (color). Time course of the $\bar{X}(t)$ component of the mean field of system (1) stimulated with signal (2) in (a) and signal (3) in (d). Coupling (C) and stimulation (K) are switched on at different times: $C = K = 0$ for $t < 250$, $C = 1$ and $K = 0$ for $t \in (250, 400)$, and $C = 1$ and (a) $K = 3$ and (d) $K = 150$ for $t > 400$. In (a) the mean field is depicted for $N = 100$ (blue curve) and $N = 900$ (red curve). In (d) the mean field (blue curve) and the corresponding stimulation force $|S(t)|$ (red curve) are shown for $N = 100$. In subplots (b) and (e) the graphs are enlarged for $t \in (700, 900)$, and in (c) and (f) two trajectories x_j are depicted for $t \in (700, 740)$. Delay $\tau = 7.5$ in (a) and $\tau = 5$ in (d). The mean period of ensemble (1) $T = 5$ (mean natural frequency $\bar{\omega} = 2\pi/T = 1.2566\dots$), the frequency distribution is Gaussian with deviation $\sigma = 0.1$; $a_j = 1.0$.

Moreover, changing the polarity of the stimulation signal ($K < 0$) reverses the situation: For $K < 0$ ideal desynchronization can easily be obtained for delay values τ in regions (ii), whereas the oscillators are slightly less desynchronized in regions (i). Hence, for small τ connected with a subideal level of desynchronization, the latter can reliably and easily be improved by simply reversing the polarity of the stimulation.

Since the stimulation signal (2) directly depends on the mean field $\bar{X}(t)$, the stimulation force, i.e., the amplitude of the stimulation signal $|S_j(t)|$, vanishes as soon as a desynchronized state of system (1) and (2) is achieved. This realizes a noninvasive (in terms of control theory) and demand-controlled method for desynchronization in a natural way. We call the stimulation protocol above with the stimulation signal (2) *individual stimulation*. From the standpoint of applications this protocol requires on-site measurement and stimulation of each individual oscillator. In some applications individual stimulation might not be feasible. To overcome this limitation, one can use only a finite number of stimulation and recording sites. Below we present a simulation protocol which we call *global stimulation*. It operates with only one stimulation site and one registration site. The stimulus affects all oscillators instantaneously with the same stimulation signal $S_j(t) = S(t)$, $j = 1, 2, \dots, N$:

$$S(t) := K \bar{X}^2(t) \bar{X}^*(t - \tau). \quad (3)$$

The results of numerical simulations of system (1) and (3) are presented in Fig. 1(d). In the stimulated regime ($t > 400$), the variations of the mean field (blue curve) are reduced by a factor of approximately 14, if compared to the synchronized regime [$t \in (250, 400)$]. This indicates a high level of desynchronization. Moreover, the stimulation force $|S(t)|$ (red curve) is of the same order as the amplitude of the mean field. Note that the delay τ in Fig. 1(d) is equal to the mean period T of ensemble (1). Such a choice of τ appears to be unfavorable for the individual stimulation with positive stimulus amplification K [Fig. 2(a)]. In contrast, in the case of global stimulation the choice of the delay is of minor importance for the desynchronization outcome [Fig. 2(b)]. For any $\tau > 0$ the averaged order parameter $\langle R(t) \rangle$ monotonously decays with increasing K .

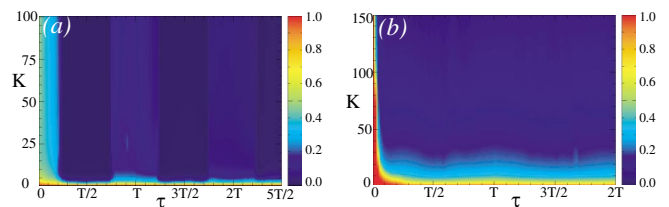


FIG. 2 (color). Averaged order parameter $\langle R(t) \rangle$ of the stimulated ensemble (1) with $N = 100$ versus parameters K and τ with stimulation signal (2) in (a), and signal (3) in (b). The other parameters are the same as in Fig. 1.

In this way, any level of desynchronization, i.e., any desirable small values of $\langle R(t) \rangle$, can be achieved by simply increasing K .

We investigate the phase dynamics of ensemble (1) globally stimulated with signal (3) in more detail. To this end, we consider a system of globally coupled phase oscillators corresponding to system (1) stimulated by the phase representation of signal (3):

$$\dot{\psi}_j(t) = \omega_j + CR(t) \sin[\Theta(t) - \psi_j(t)] + KR^2(t)R(t - \tau) \times \sin[2\Theta(t) - \Theta(t - \tau) - \psi_j(t)], \quad (4)$$

where $\psi_j(t)$ are the phases of the limit-cycle oscillators (1). $R(t)$ and $\Theta(t)$ are the order parameter and the mean phase, respectively, defined by $R(t)e^{i\Theta(t)} = N^{-1} \sum_{j=1}^N e^{i\psi_j(t)}$ [6].

To explore the desynchronization mechanism of the global stimulation, we follow the evolution of the individual averaged frequencies $\bar{\omega}_j = \langle \dot{\psi}_j(t) \rangle$ of oscillators (4) as the stimulus amplification K increases. As before, the distribution of the natural frequencies $\{\omega_j\}$ is Gaussian with mean $\bar{\omega} = 2\pi/5 \approx 1.26$ (mean period $T = 5$) and deviation $\sigma = 0.1$. In a synchronized regime without

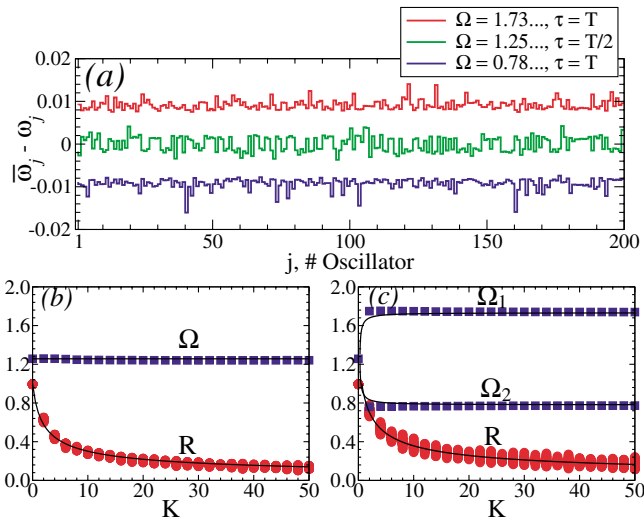


FIG. 3 (color). Desynchronization mechanism of global stimulation and multistability of desynchronized states. (a) Differences between the individual averaged frequencies $\bar{\omega}_j$ and the natural frequencies ω_j for $K = 150$ and for $\tau = T/2$ (green curve) and $\tau = T$ (blue and red curves, obtained for two different initial conditions). Ω is the mean frequency of a desynchronized state. In (b) and (c) the order parameter R (red symbols) and the mean frequency Ω (blue symbols) obtained by simulation of $N = 100$ phase oscillators (4) are shown. The values of $R(t)$ are plotted for time intervals of 40 mean periods T after skipping a transient of 2000 periods. The solid curves are the theoretical prediction of R and Ω from Eq. (6). In (c) the mean frequencies Ω_1 and Ω_2 (blue symbols) realized for two different initial conditions of system (4) correspond to red and blue curves in (a), respectively. Parameters $C = 1.0$, $T = 5.0$, (b) $\tau = 2.5$, and (c) $\tau = 5.0$.

stimulation, all oscillators rotate with the same averaged frequency $\bar{\omega}_j = \bar{\omega}$, $j = \overline{1, N}$. When the stimulation is on and the stimulus amplification K increases, the individual averaged frequencies approach the natural frequencies. This is illustrated in Fig. 3(a), where the differences $\bar{\omega}_j - \omega_j$ are shown for $K = 150$ and for $\tau = T/2$ (green curve) and $\tau = T$ (blue and red curves). In the latter case $\tau = T$, two different initial conditions of system (4) are used to illustrate the phenomenon of the stimulation-induced multistability of desynchronized states. The multistability manifests itself in different mean frequencies Ω , i.e., averaged frequencies of the mean phase $\Theta(t)$ of the desynchronized states: $\Omega \approx 0.78$ (blue curve) and $\Omega \approx 1.73$ (red curve) in Fig. 3(a). On the other hand, for τ close to $T/2, 3T/2, 5T/2$, etc., the stimulation preserves the mean frequency of the ensemble, as illustrated in Fig. 3(a) for $\tau = T/2$: $\Omega = \bar{\omega} \approx 1.26$ (green curve). The small values of the differences $\bar{\omega}_j - \omega_j$ indicate that the oscillators get back their original frequencies for sufficiently large values of parameter K . In other words, the stimulation causes a robust desynchronization by restoring the natural frequencies of the oscillators.

We model the dynamics of the mean field $W(t) = R(t)e^{i\Theta(t)}$ of the strongly synchronized and stimulated ensemble (4) by a single limit-cycle oscillator in the form

$$\dot{W}(t) = \frac{C}{2}[1 - |W(t)|^2]W(t) + i\bar{\omega}W(t) + \frac{K}{2}W^2(t)W^*(t - \tau), \quad (5)$$

where $\bar{\omega}$ is the mean natural frequency of the ensemble. In a strongly synchronized regime, for large enough $C > 0$ and $K = 0$, the dynamics of the mean field of ensemble (4) for large N can be approximated by a uniform rotation on the unit circle with the frequency $\Omega = \bar{\omega}$ emerging in a Hopf bifurcation [6]. Global stimulation (3) affects all oscillators in the same way and is modeled by the same stimulation term in Eq. (5).

From Eq. (5) one can see that the cubic term of the stimulation signal is the nonlinearity of smallest order, which does not destroy the uniform Hopf oscillation of the mean field, but only suppresses its amplitude. Indeed, the trivial solution $W(t) \equiv 0$ of Eq. (5) is unstable for $C > 0$, with positive eigenvalue $C/2$. The other solutions of Eq. (5) have the form $R(t) = R \neq 0$, $\Theta(t) = \Omega t + \text{const}$, where the constant amplitude R and the mean frequency Ω can be derived from

$$\Omega = \bar{\omega} + \frac{KC \sin(\Omega\tau)}{2C - 2K \cos(\Omega\tau)}, \quad R^2 = \frac{C}{C - K \cos(\Omega\tau)}. \quad (6)$$

Equation (6) has infinitely many solutions Ω for $|K| \geq C$ [22]. However, with increasing $|K|$ any selected mean frequency Ω saturates, and thus, the order parameter $R(t)$ of ensemble (4) and the stimulation force $|S|$ decay with

increasing stimulus amplification K according to

$$R \sim |K|^{-1/2}, \quad |S| \sim |K|^{-1/2}. \quad (7)$$

An optimal solution of Eq. (6) is given by $\Omega = \bar{\omega}$ and $R^2 = C/(C + |K|)$. According to Eq. (6) such a solution can be realized for $\tau = T/2, 3T/2, 5T/2, \dots$, if $K > -C$, or for $\tau = T, 2T, 3T, \dots$, if $K < C$. These values of the delay τ are optimal in the sense that the order parameter R attains its minimum given by the optimal solution with the smallest proportionality coefficient in Eq. (7). With the use of the linear stability theory [23], we found that the solutions $W(t) = Re^{i\Theta(t)}$ of Eq. (5) with amplitude and frequency of the optimal solution are stable.

In Figs. 3(b) and 3(c) we illustrate the good agreement between the dynamics of the mean field of ensemble (4) and the theoretical model Eq. (5). In Fig. 3(b) the delay $\tau = 2.5 = T/2$, at which the optimal solution of Eq. (5) is stable for $K > 0$ (solid curves). This corresponds to the case, where the natural frequencies of oscillators (4) are completely restored [Fig. 3(a), green curve]. In Fig. 3(c) the delay $\tau = 5.0 = T$, at which the optimal solution is not admissible for $K \geq C$ [22]. This solution loses its stability with increasing K at $K_{\text{pf}} = 2C/(C\tau + 2)$ [$K_{\text{pf}} \approx 0.286$ in Fig. 3(c)] via a pitchfork bifurcation giving birth to two new stable frequencies $\Omega_{1,2}$ shown in Fig. 3(c). Depending on the initial conditions the dynamics of the stimulated and desynchronized system (4) can realize different mean frequencies Ω as illustrated in Fig. 3(c). This corresponds to the case, where the natural frequencies of oscillators (4) are not completely restored but are slightly shifted compared to the uncoupled case [Figs. 3(a), red and blue curves].

In conclusion, nonlinear delayed feedback opens up new avenues of approach to the issue of effective control of synchronization. We propose a powerful method for desynchronization of even strongly synchronized ensembles of coupled oscillators. Because of its inherently demand-controlled design, our method restores the natural frequencies with a minimal amount of stimulation. The individual and the global stimulation protocols introduced here constitute borderline cases of a variety of intermediate stimulation protocols, which use a finite number of recording and stimulation sites, where the number of sites can flexibly be chosen as desired for the respective application. The method works equally well if a small latency time is introduced in the stimulation signal. Our nonlinear delayed feedback method allows us to parry large variations of system parameters, such as the mean frequency in DBS target populations. Its effectiveness and robustness make our method superior to previously designed desynchronization methods using either repeated administration of pulse sequences [10] or linear delayed feedback [19]. We propose our method for the therapy of neurological diseases with pathological synchronization, e.g., tremor in Parkinson's disease or essential tremor.

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