Deuteron Transfer in $N = Z$ Nuclei

P. Van Isacker,[1] D. D. Warner,[2] and A. Frank[3]
[1]*Grand Accélérateur National d'Ions Lourds, B.P. 55027, F-14076 Caen Cedex 5, France*
[2]*CCLRC Daresbury Laboratory, Daresbury, Warrington WA4 4AD, United Kingdom*
[3]*Instituto de Ciencias Nucleares, UNAM, Apdo. Postal 70-543, 04510 México, D.F. Mexico*
(Received 14 September 2004; published 29 April 2005)

Predictions are obtained for $T = 0$ and $T = 1$ deuteron-transfer intensities between self-conjugate $N = Z$ nuclei on the basis of a simplified interacting boson model which considers bosons without orbital angular momentum but with full spin-isospin structure. These transfer predictions can be correlated with nuclear binding energies in specific regions of the mass table.

DOI: 10.1103/PhysRevLett.94.162502 PACS numbers: 21.60.Fw, 21.10.Dr, 25.70.Hi

In the study of atomic nuclei, it was recognized at an early stage that the attractive interaction between the fermionic constituents of the nucleus would lead to pairing analogous to that found between electrons in the superfluid phase of condensed matter [1]. In nuclei near stability with sufficient valence nucleons to form a condensate, the effects of pairing between like nucleons, neutrons or protons are observable, but it is only recently that the opportunity has arisen to study the exotic, proton-rich species with masses in the range of 60–100 where deuteronlike, neutron-proton pairing may be manifest. This is predicted to occur on the specific locus defined by equal numbers of neutrons and protons ($N = Z$), where the valence nucleons move in orbits with identical quantum numbers. Neutron-proton pairing can be either isoscalar $T = 0$ (spin-triplet) or isovector $T = 1$ (spin-singlet) and this leads to a generalized structure of the condensate, typical of superfluidity in a two-component system. The most obvious experimental signature for the existence and character of this generalized condensate is an enhancement in the probability for the transfer of a pair (either $T = 0$ or $T = 1$) into or out of it and it is therefore the purpose of this Letter to obtain first estimates for the cross sections for deuteron transfer between medium-heavy $N = Z$ nuclei. These estimates can be tested by contrasting the reaction $(d, {}^4\mathrm{He})$ where only $T = 0$ deuteron transfer is possible with reactions like $(p, {}^3\mathrm{He})$ which allow both $T = 0$ as well as $T = 1$ deuteron(like) transfer. The relevant experiments will require the new generation of radioactive beam facilities.

The interacting boson model (IBM) [2] provides a description of nuclei in terms of correlated nucleon-pair excitations which are treated as bosons. As such, it offers a natural framework to discuss the issue of two-nucleon transfer. Two-neutron and two-proton transfer have been analyzed in the early days of the model (see, e.g., Refs. [3,4]) using the neutron-proton version of the model, IBM-2 [5], which includes neutron-neutron (nn) and proton-proton (pp) bosons. A description of deuteron transfer requires a more complicated version of IBM which involves bosons corresponding to np pairs. Such extensions have been considered in the past, and of particular relevance is the so-called IBM-4 [6] since it contains np pairs with isospin $T = 0$ and $T = 1$. The full IBM-4 is a rich spectroscopic model [7] with bosons with orbital angular momentum $L = 0$ (s boson) or $L = 2$ (d boson), with intrinsic spin $S = 0$ or $S = 1$, and with isospin $T = 0$ (if $S = 1$) or $T = 1$ (if $S = 0$). This particular choice of bosons is justified on the basis of the nuclear shell model [8].

To avoid the complexity of the full IBM-4, it is instructive to confine the analysis to $L = 0$ bosons. This simplification preserves the complete spin-isospin structure of the model—crucial for the study of deuteron-transfer properties—and can be put to use in the analysis of the competition between isoscalar and isovector pairing in self-conjugate nuclei [9]. The dynamical algebra (in the sense of Ref. [10]) of the $L = 0$ IBM-4 is U(6), obtained from two vector bosons. One is vector in isospin while scalar in spin and the other boson is vector in spin while scalar in isospin. Based on a comparison with the full IBM-4 and its interpretation in terms of the shell model, one can justify the use of a simplified IBM-4 in all $N = Z$ nuclei and also in even-even $N \neq Z$ but not in odd-odd $N \neq Z$ nuclei. In the latter case the favored U(6) representation of the full IBM-4 is nonsymmetric [6] and is not contained in the simplified $L = 0$ IBM-4.

Two different symmetry classifications occur in the $L = 0$ IBM-4:

$$\mathrm{U}(6) \supset \left\{ \begin{matrix} \mathrm{SU}(4) \\ \mathrm{U}_T(3) \otimes \mathrm{U}_S(3) \end{matrix} \right\} \supset \mathrm{SO}_T(3) \otimes \mathrm{SO}_S(3). \quad (1)$$

The total number of bosons $[N_\mathrm{b}]$ labels U(6) while $\mathrm{SO}_T(3)$ and $\mathrm{SO}_S(3)$ are associated with the total isospin T and the total spin S of the bosons. Mathematical details on the two limits in (1) can be found in Ref. [11] where also the correspondence is studied between the U(6) model and its fermionic analogue, the SO(8) model of $T = 0$ and $T = 1$ pairing with neutrons and protons [12]. A simple Hamiltonian that describes the transition from one limit of (1) to the other is of the form

$$H_0 = aC_2[\mathrm{SU}(4)] + bC_1[\mathrm{U}_S(3)], \quad (2)$$

where $C_n[G]$ denotes a linear or quadratic ($n = 1, 2$) Casimir operator of the algebra G. The first term in (2) is associated with SU(4) and implies equal single-boson energies and boson-boson interactions in the two isospin-spin channels $(T, S) = (0, 1)$ and $(1, 0)$. The second term breaks this equivalence between the two channels. With respect to a previous study [9], the linear instead of the quadratic Casimir operator of $U_S(3)$ is considered in (2). Any of the two (or a combination of them) leads to comparable results but the linear operator is taken here since it can be shown [13] to originate from the one-body spin-orbit term of the shell model (see below).

The transition from SU(4) to $U_T(3) \otimes U_S(3)$ is governed by the single parameter b/a and intermediate results can be obtained after diagonalization [9].

Deuteron transfer is described in this model by the operators $b^\dagger_{01}$ ($b^\dagger_{10}$) for $T = 0$ ($T = 1$) transfer, where $b^\dagger_{01}$ ($b^\dagger_{10}$) creates a boson with $T = 0$ and $S = 1$ ($T = 1$ and $S = 0$), both with orbital angular momentum $L = 0$. To establish the connection with measured cross sections, we note that the amplitude for two-nucleon transfer in the reaction $\alpha \equiv A + a \to \beta \equiv B + b$ is given by [14]

$$\mathcal{T}_{\alpha\to\beta} = \lambda \sum_N G_N(L, S, J) K_{NLM_L}(\vec{k}_\alpha, \vec{k}_\beta), \qquad (3)$$

where λ depends on the isospin and spin labels of the initial and final states, and $K_{NLM_L}(\vec{k}_\alpha, \vec{k}_\beta)$ is a kinematical factor, obtained in a distorted wave Born approximation and depending on the relative initial and final momenta $\vec{k}_\alpha$ and $\vec{k}_\beta$ in the center-of-mass frame. The nuclear structure dependence is contained in the factor $G_N(L, S, J)$ which, in the specific case of orbital angular momentum $L = 0$ transfer, reduces to

$$G_N(L = 0, S = J) = \sum_{nl} \langle 00N0; 0|nlnl; 0\rangle \beta^{TS}_{nl}, \qquad (4)$$

where the sum is subject to the constraint $N = 2(2n + l)$. Two terms enter into this expression. The first is the Talmi-Moshinsky bracket [15,16] necessary to transform from individual to relative and center-of-mass coordinates of neutron and proton, which can be done exactly in case of single-particle wave functions of the harmonic oscillator. The second term is the parentage amplitude

$$\beta^{TS}_{nl} = \langle \Phi_B \parallel (a^\dagger_{nl1/21/2} \times a^\dagger_{nl1/21/2})^{(0TS)} \parallel \Phi_A\rangle, \qquad (5)$$

where Φ_A and Φ_B are the wave functions of the target and product nuclei A and B. The operator in (5) corresponds to a pair of nucleons in an nl orbit coupled to orbital angular momentum $L = 0$, isospin T, and spin S.

The shell-model matrix element (5) can be related to a boson matrix element. The boson is associated to a *correlated* fermion pair which implies the correspondence

$$b^\dagger_{TS} \Longleftrightarrow \sum_{nl} \alpha_{nl} (a^\dagger_{nl1/21/2} \times a^\dagger_{nl1/21/2})^{(0TS)} \equiv S^{TS}_+, \qquad (6)$$

with α_{nl} certain coefficients. For example, in the limit of degenerate single-particle energies, they are $\alpha_{nl} = \sqrt{\Omega_l}$ with $\Omega_l \equiv 2l + 1$. As such, the boson matrix element corresponds to the fermion matrix element of the specific linear combination given in (6). To find the matrix elements of the *individual nl* pairs, needed in (4), we use the following scaling property:

$$\beta^{TS}_{nl} = \frac{\sqrt{\Omega_l}}{\sum_{n'l'} \alpha_{n'l'} \sqrt{\Omega_{l'}}} \langle \Phi_B \parallel S^{TS}_+ \parallel \Phi_A\rangle. \qquad (7)$$

This can be shown to be exactly valid for the transfer of a correlated pair with $\alpha_{nl} = \sqrt{\Omega_l}$ between the ground states Φ_A and Φ_B in the SU(4) limit of the SO(8) model. The property (7) then follows from a generalization to arbitrary coefficients α_{nl}.

The boson transfer operator must be corrected with a Pauli factor which is obtained by requiring identical matrix elements in boson and fermion spaces [3]. For the present purpose, we associate the U(6) $\supset$ SU(4) limit of the $L = 0$ IBM-4 with the SO(8) $\supset$ SU(4) limit of the neutron-proton pairing model. In both cases the matrix elements can be obtained analytically with use of known U(6) $\supset$ SU(4) and SO(8) $\supset$ SU(4) isoscalar factors [17,18]. From this analysis one concludes that the boson transfer operator which, in the SU(4) limit exactly reproduces the fermionic ground-to-ground transition between $N = Z$ nuclei, is

$$\begin{aligned} \text{even-even} \to \text{odd-odd:} &\sqrt{\frac{1}{2}(2\Omega - \hat{N}_\text{b} + 1)} b^\dagger_{TS}, \\ \text{odd-odd} \to \text{even-even:} &\sqrt{\frac{1}{2}(2\Omega - \hat{N}_\text{b} + 6)} b^\dagger_{TS}, \end{aligned} \qquad (8)$$

valid for $N_\text{b} \leq 2\Omega$, where $\Omega \equiv \sum_l \Omega_l$ is the shell degeneracy.

In this derivation, care has been taken to relate the matrix element of $b^\dagger_{TS}$ to the shell-model amplitude which enters the deuteron-transfer amplitude (3). This requires the application of the Talmi-Moshinsky transformation, the use of the scaling property (7), and the inclusion of the spin-isospin Pauli factors. Since the transfer intensity between the states $|[N_\text{b}]\phi_A T_A S_A\rangle$ and $|[N_\text{b} + 1]\phi_B T_B S_B\rangle$ (where ϕ_A and ϕ_B are additional labels) is proportional to the square of the matrix element of $b^\dagger_{TS}$, we shall study the quantity

$$C^2_T \equiv \langle [N_\text{b} + 1]\phi_B T_B S_B \parallel b^\dagger_{TS} \parallel [N_\text{b}]\phi_A T_A S_A\rangle^2, \qquad (9)$$

where, for convenience, the matrix element is reduced in isospin and spin.

The properties of the Hamiltonian (2) have been studied previously [9]. The ground state of even-even nuclei (N_b even) has $T = 0$ while the ground state of an odd-odd nucleus has $T = 0$ for $b/a < 0$ and $T = 1$ for $b/a > 0$; $b/a = 0$ yields degeneracy between the lowest $T = 0$ and

$T = 1$ states and corresponds to the SU(4) limit. Thus by varying the ratio b/a one can study the qualitative features of deuteron transfer with changing $T = 0$ *versus* $T = 1$ pairing correlations. The result for $N = Z$ nuclei is shown in Fig. 1.

Deuteron transfer at the $N = Z$ line has unique properties since, starting from an even-even $N = Z$ nucleus with $T = 0$ ground state, one finds two states excited in the low-energy region of the odd-odd nucleus corresponding to $T = 0$ and $T = 1$ transfer, respectively. Not surprisingly, the two states are equally excited in the SU(4) limit ($b/a = 0$) while otherwise the sign of b/a determines which of the two transfer intensities is strongest. It is evident from Fig. 1 that the transfer intensities change rapidly around the SU(4) limit (which corresponds to a phase-transitional point in the limit $N_b \to \infty$) but saturate quickly at large values of $|b/a|$. Simple predictions for the transfer intensities in the three limits $b/a = 0$, $b/a \ll -1$, and $b/a \gg +1$ are shown in Table I.

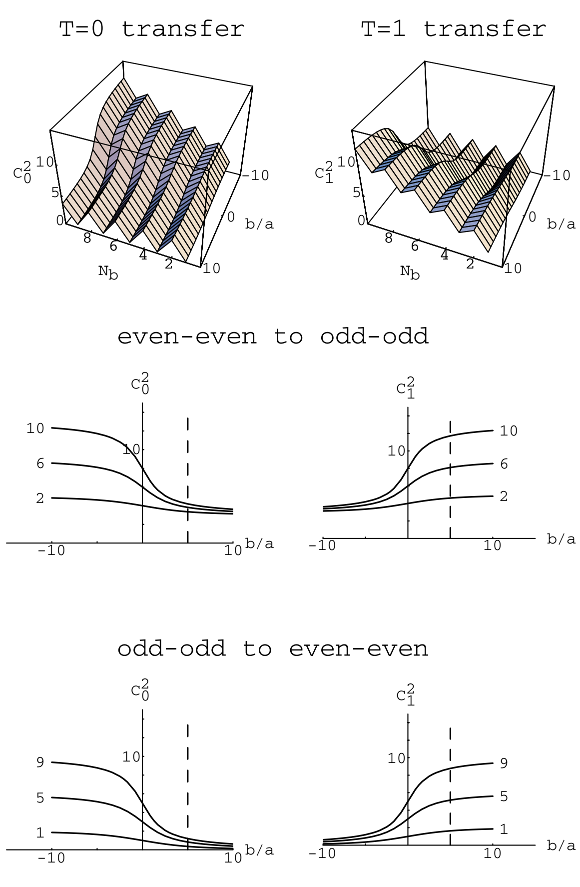

FIG. 1 (color online). The $T = 0$ and $T = 1$ deuteron-transfer intensities C_T^2 between $N = Z$ nuclei with boson numbers N_b and $N_b + 1$ as a function of b/a between the lowest $T = 0$ and $T = 1$ eigenstates of the Hamiltonian (2). In the upper part the entire surface is shown for $1 \leq N_b \leq 10$ and $-10 \leq b/a \leq 10$. In the middle and lower parts the even-even to odd-odd and odd-odd to even-even intensities are displayed for specific values of N_b. In odd-odd nuclei the $T = 0$ and $T = 1$ states are close in energy and the figure shows the isospin-allowed intensity to or from both, i.e., to/from $T = 0$ for C_0^2 and to/from $T = 1$ for C_1^2. The dashed line indicates the value of b/a obtained from nuclear masses in the first half of the 28–50 shell.

What are appropriate values of b/a in actual nuclei? Qualitative arguments can give an idea of the sign. The physical origin of SU(4) symmetry is the attractive short-range interaction between nucleons which favors spatially symmetric states. Because of the overall antisymmetry of the nuclear wave function this leads to ground states with least symmetry in isospin-spin SU(4) and hence to $a > 0$. On the other hand, the operator $C_1[\mathrm{U}_S(3)]$ arises from a mapping of the spin-orbit term $v_{\mathrm{so}}\sum_i \bar{l}_i \cdot \bar{s}_i$ of the shell model into the IBM-4 space. In fact, it can be shown [13] that a pairing Hamiltonian with a (one-body) spin-orbit term maps into a combination of $C_2[\mathrm{SU}(4)]$, $C_1[\mathrm{U}_T(3)]$, and $C_1[\mathrm{U}_S(3)]$ with additional N_b-dependent terms. Since $C_1[\mathrm{U}_T(3)] + C_1[\mathrm{U}_S(3)] = N_b$, one of the linear operators can be eliminated, and, if $C_1[\mathrm{U}_S(3)]$ is kept, its strength is approximately given by $4l(l+1)v_{\mathrm{so}}^2/3g\Omega$ where $-g$ is the strength of the pairing interaction ($g > 0$), Ω is the shell degeneracy, and l is the orbital angular momentum of the two fermions that make up a boson. On the basis of this argument one thus expects $b > 0$.

A more quantitative estimate can be given based on nuclear masses. The $L = 0$ IBM-4 can be used to calculate binding energies of $N = Z$ nuclei [19]. The proposed Hamiltonian contains, in addition to (2) and linear and quadratic terms in N_b, a $T(T+1)$ term which is known to be of importance for the nuclear symmetry energy:

$$H = H_0 + c_1 C_1[\mathrm{U}(6)] + c_2 C_2[\mathrm{U}(6)] + d C_2[\mathrm{SO}_T(3)]. \tag{10}$$

Since the additional three terms are diagonal in both bases (1), they leave the wave functions unaltered which still only depend on the ratio b/a. The results for binding energies in nuclei in the first half of the 28–50 shell—a mass region currently of particular interest for deuteron-transfer experiments—are shown in Fig. 2.

TABLE I. Predicted deuteron-transfer intensities C_T^2 between even-even (EE) and odd-odd (OO) $N = Z$ nuclei in the SU(4) ($b/a = 0$) and $\mathrm{U}_T(3) \otimes \mathrm{U}_S(3)$ ($|b/a| \gg 1$) limits.

Limit	Reaction	$C_{T=0}^2$	$C_{T=1}^2$
$b/a = 0$	$\mathrm{EE} \to \mathrm{OO}_{T=0}$	$\frac{1}{2}(N_b + 6)$	0
	$\mathrm{EE} \to \mathrm{OO}_{T=1}$	0	$\frac{1}{2}(N_b + 6)$
	$\mathrm{OO}_{T=0} \to \mathrm{EE}$	$\frac{1}{2}(N_b + 1)$	0
	$\mathrm{OO}_{T=1} \to \mathrm{EE}$	0	$\frac{1}{2}(N_b + 1)$
$b/a \ll -1$	$\mathrm{EE} \to \mathrm{OO}_{T=0}$	$N_b + 3$	0
	$\mathrm{EE} \to \mathrm{OO}_{T=1}$	0	3
	$\mathrm{OO}_{T=0} \to \mathrm{EE}$	$N_b + 1$	0
$b/a \gg +1$	$\mathrm{EE} \to \mathrm{OO}_{T=0}$	3	0
	$\mathrm{EE} \to \mathrm{OO}_{T=1}$	0	$N_b + 3$
	$\mathrm{OO}_{T=1} \to \mathrm{EE}$	0	$N_b + 1$

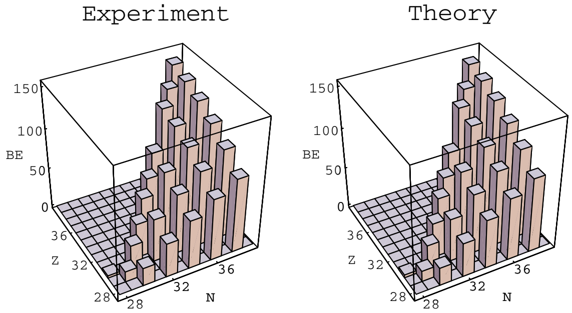

FIG. 2 (color online). Binding energies (in MeV) of even-even (all N, Z) and odd-odd (only $N = Z$) nuclei in the first half of the 28–50 shell. All binding energies are relative to ^{56}Ni. The data are taken from Ref. [23]. The calculated values are obtained from (10) with $a = 0.238$, $b = 1.261$, $c_1 = -23.466$, $c_2 = -0.083$, and $d = 0.765$, in MeV.

Note that the study of Ref. [19] has now been extended to include even-even $N \neq Z$ nuclei. The root-mean-square deviation is 0.306 MeV (0.254 MeV if experimental errors are included via the method of maximizing the likelihood function). The fit to the nuclear masses yields the value $b/a \approx 5$, confirming our earlier qualitative arguments that the ratio should be positive. Even if there is considerable uncertainty in the value of this ratio, the fact that the deuteron-transfer intensities quickly saturate for large b/a leads to a clear prediction of this analysis: the favored deuteron-transfer mode in this mass region has $T = 1$ rather than $T = 0$ character. Some appreciable strength of the latter can only be expected in the transfer from an even-even to an (excited) $T = 0$ state of an odd-odd nucleus.

The $N = Z$ line in the 28–50 neutron and proton shells represents the ideal region in which to search experimentally for the competition between $T = 0$ and $T = 1$ pairing; the valence space is sufficiently large to allow the development of collective features and the lowest states of each isospin lie close to each other in energy. Moreover, it was pointed out some time ago [20], following studies of lighter nuclei in a Hartree-Fock-Bogoliubov framework, that a dominance of $T = 0$ over the $T = 1$ mode can only be expected to occur in $N = Z$ nuclei; the addition of only two neutrons is sufficient to reestablish the normal, like-nucleon pairing encountered throughout the rest of the nuclear chart. Thus the limitation, pointed out earlier, that the $L = 0$ IBM-4 is not applicable to odd-odd $N \neq Z$ nuclei does not pose a serious problem in the current study.

In considering future experiments to determine the relative contributions of the different pairing modes on the $N = Z$ line, it is important to recall that the $N = Z$ nuclei are located increasingly far from stability as mass increases so that the study of deuteron transfer mandates the use of radioactive beams and inverse kinematics, either at classical transfer energies or through knockout reactions at the higher energies available at fragmentation facilities. The possibility to extract two-particle spectroscopic factors from the latter method has recently been demonstrated for the first time for two-proton transfer [21,22]. The extraction of spectroscopic factors describing deuteron transfer poses particular additional problems in either approach and will require an enhancement in both the beam intensity and experimental sensitivity currently available in this type of study to achieve meaningful results. Nevertheless, the new generation of exotic beam accelerators currently proposed or under construction promises just such a degree of enhancement and the study presented here provides a first prediction of what may be observed in this new class of experiments.

This research was supported in part by CONACyT (Mexico).

[1] A. Bohr, B. R. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958).
[2] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, England, 1987).
[3] A. Arima and F. Iachello, Phys. Rev. C **16**, 2085 (1977).
[4] J. A. Cizewski, in *Interacting Bose-Fermi Systems in Nuclei*, edited by F. Iachello (Plenum, New York, 1981).
[5] A. Arima, T. Otsuka, F. Iachello, and I. Talmi, Phys. Lett. B **66**, 205 (1977).
[6] J. P. Elliott and J. A. Evans, Phys. Lett. B **101**, 216 (1981).
[7] P. Halse, J. P. Elliott, and J. A. Evans, Nucl. Phys. A **417**, 301 (1984).
[8] O. Juillet, P. Van Isacker, and D. D. Warner, Phys. Rev. C **63**, 054312 (2001).
[9] P. Van Isacker and D. D. Warner, Phys. Rev. Lett. **78**, 3266 (1997).
[10] A. Frank and P. Van Isacker, *Algebraic Methods in Molecular and Nuclear Structure* (Wiley-Interscience, New York, 1994).
[11] P. Van Isacker, J. Dukelsky, S. Pittel, and O. Juillet, J. Phys. G **24**, 1261 (1998).
[12] B. H. Flowers and S. Szpikowski, Proc. Phys. Soc. **84**, 193 (1964); **84**, 673 (1964).
[13] O. Juillet and S. Josse, Eur. Phys. J. A **8**, 291 (2000).
[14] N. K. Glendenning, *Direct Nuclear Reactions* (Academic, New York, 1983).
[15] I. Talmi, Helv. Phys. Acta **25**, 185 (1952).
[16] M. Moshinsky, Nucl. Phys. **13**, 104 (1959).
[17] S. C. Pang, Nucl. Phys. A **128**, 497 (1969).
[18] F. Iachello and P. Van Isacker, *The Interacting Boson-Fermion Model* (Cambridge University Press, Cambridge, England, 1991).
[19] E. Baldini-Neto, C. L. Lima, and P. Van Isacker, Phys. Rev. C **65**, 064303 (2002).
[20] A. L. Goodman, Adv. Nucl. Phys. **11**, 263 (1979).
[21] D. Bazin *et al.*, Phys. Rev. Lett. **91**, 012501 (2003).
[22] D. D. Warner, Nature (London) **425**, 570 (2003).
[23] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. A **729**, 337 (2003).