

$D_{s_0}^+(2317)-D_0(2308)$ Mass Difference as Evidence for Tetraquarks

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We argue that the anomalously small mass difference between the $D_{s_0}^+(2317)$ and the recently observed $D_0(2308)$ (Belle) state is the first “smoking gun” experimental evidence of their tetraquark $[cq(\bar{q}\bar{q})]$ structure. The recently reported $D_{sJ}^+(2632)$ (SELEX) state completes the low-lying nonexotic tetraquark spectrum, as predicted by ’t Hooft’s instanton-induced effective quark interaction.

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Introduction.—The recent discovery of positive parity mesons with open charm and strangeness $D_{sJ}^+(2317)$ [1] and $D_{sJ}^+(2460)$ [2,3], about 200 MeV below their predicted masses (under the assumption of $c\bar{s}$ state in the quark model), has elicited a great deal of interest and puzzlement or excitement: Several authors, Ref. [4], suggested that these two states form a $J^P = (0^+, 1^+)$ “parity doublet” of the heavy-light ($c\bar{s}$) quark chiral symmetry. Several other groups, Refs. [1,5–8], on the other hand, suggested that these states are scalar ($J^P = 0^+$) open charm “tetraquark” ($q^2\bar{q}^2$) states. The Belle collaboration, Ref. [3], has in the meantime established the $D_s^+(2460)$ ’s spin parity as $J^P = 1^+$, thus apparently confirming the heavy quark symmetry’s prediction and refuting the tetraquark interpretation. More recently, however, (1) the SELEX collaboration [9] reported a discovery of the charmed strange state $D_{sJ}^+(2632)$, and (2) the Belle collaboration [10] reported a discovery of the nonstrange states $D_{J=0}^+(2308)$, which ought to be a partner of the $D_{s_0}^+(2317)$ in an $SU(3)$ $\bar{3}$ -plet.

These states present new problems for any model based on their assumed $c\bar{q}$ structure: (1) the $D_{sJ}^+(2632)$ is too low to be a radial excitation of the $D_{s_0}^+(2317)$ [11] (moreover, by the same token one would expect similar radial excitations of pseudoscalar and vector mesons around 315 MeV above their respective ground states, where there have been found none), and (2) the $D_{s_0}^+(2317) - D_0^+(2308)$ mass difference is far too small to account for the $s-u/d$ quark mass difference of 150 ± 30 MeV.

We shall show in this Letter that only the tetraquark model naturally accounts for both of these problems: (1) as for the proliferation of states, the $D_{sJ}^+(2632)$ is the second-lowest $D_{s_0}^+$ tetraquark (Ref. [7] suggested that this is a member of the $\bar{15}$ -plet, but we shall show that it is a mixture of the $\bar{15}$ -plet and the $\bar{3}_S$ -plet with a larger contribution of the latter), and (2) the small mass difference between the nonstrange $D_0^+(2308)$ and the strange $D_{s_0}^+(2317)$, states which ought to form an $SU(3)$ $\bar{3}$ -plet, is quite naturally explained in the tetraquark model: This model predicts equal masses of all three members of the $\bar{3}_A$ -plet, due to their antisymmetric flavor $SU(3)$ wave functions, as can be seen in Table II. of Ref. [8]. This is a general property of the $\bar{3}_A$ -plet in any tetraquark model regardless of the quark interaction; the ordering of $SU(3)$

multiplets in energy or mass depends on the dynamics, however. It is the property of the ’t Hooft interaction, however, that the $\bar{3}_A$ -plet is the lowest one.

Mass splitting in the $\bar{3}_A$ -plet.—The following flavor $SU(3)$ multiplets: $\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \bar{\mathbf{3}}_A \oplus \bar{\mathbf{3}}_S \oplus \mathbf{6} \oplus \bar{\mathbf{15}}$ appear in the charm $C = 1$ tetraquark system. States with the quantum numbers of D_s^+ and $D_0^{0,+}$ appear in two distinct triplets, as well as in the $\bar{15}$ -plet, Fig. 1, in this Clebsch-Gordan series. The three (bare) states with equal quantum numbers mix due to $SU(3)$ symmetry breaking effects, such as the quark mass differences, and thus produce three (physical) mass eigenstates. The 3×3 mass matrix for the D_s states is nondiagonal in general, but there is no mixing between the $\bar{3}_A$, and the other two multiplets ($\bar{3}_S, \bar{15}$), which do mix among themselves. It is a straightforward exercise to convince oneself that all three members of the $\bar{3}_A$ -plet,

$$\begin{aligned}
 |D^0 \in \bar{3}_A\rangle &= \frac{1}{2} |c(s\bar{u}\bar{s} - \bar{s}\bar{u}) - d(\bar{d}\bar{u} - \bar{u}\bar{d})\rangle, \\
 |D^+ \in \bar{3}_A\rangle &= \frac{1}{2} |c(s\bar{d}\bar{s} - \bar{s}\bar{d}) - u(\bar{d}\bar{u} - \bar{u}\bar{d})\rangle, \\
 |D_s^+ \in \bar{3}_A\rangle &= \frac{1}{2} |c(u\bar{u}\bar{s} - \bar{s}\bar{u}) - d(\bar{d}\bar{s} - \bar{s}\bar{d})\rangle,
 \end{aligned}
 \tag{1}$$

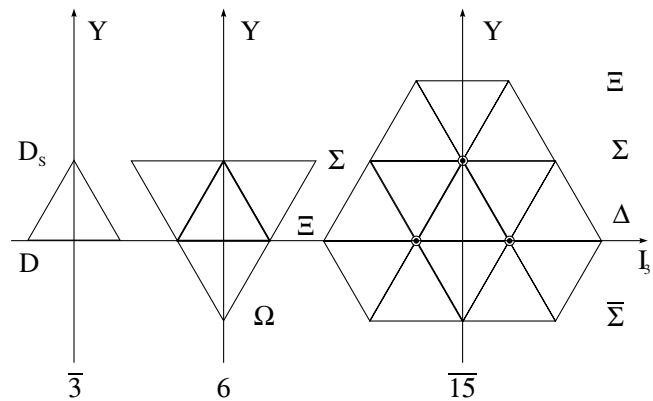


FIG. 1. Weight diagrams of $SU(3)$ irreducible representations appearing in the charmed tetraquark Clebsch-Gordan series. Note the double occupancy (double circles) of the “inner” triangle within the $\bar{15}$ -plet.

have the same mass $M_{\bar{3}_A}$, which in the noninteracting quark model may be expressed in terms of quark masses as $M_{\bar{3}_A} = m_c + m_u + m_d + m_s$. This is in good agreement with experimental observations [3,10] (the small mass difference of 8 MeV is entirely within the substantially larger systematic and statistical uncertainties). This tetraquark mass in the 't Hooft interaction model is discussed below.

Fitting the $D_s^+(2317)$ and $D_{sJ}^+(2632)$ masses.—The second problem we have to solve is fitting the masses of the $D_s^+(2317)$ and $D_{sJ}^+(2632)$ states. The following values of the free parameters in this model, that we may have to change now, were taken in Ref. [8]: (a) the quark masses: u/d quark masses were taken as $m_{u/d} = 313/318$ MeV, and the strange quark mass $m_s = m_q + 150 \pm 30$ MeV; (b) the 't Hooft interaction coupling constant $K = 390$ GeV $^{-5}$; (c) the quark condensate $\langle \bar{q}q \rangle_0 = -(225 \pm 25 \text{ MeV})^3$; (d) the confinement oscillator frequency $\omega = 500$ MeV.

First we shall show that the easiest way of fitting the $D_{sJ}^+(2632)$ mass is by raising all four quark masses. Then

we shall fit the $D_s^+(2317)$ mass, which is sensitive to the three-body interaction and consequently can be easily modified to fit the experiment by slightly changing the quark (mean) oscillator frequency ω and/or the u, d quark mass $m_{u,d}$. Moreover, in Ref. [8] we approximated the quadratic A dependence of the three-body term with a linear one. Restoring the quadratic A dependence substantially facilitates the fit.

$D_{sJ}^+(2632)$.—As stated above, in Ref. [7] it was suggested that the $D_{sJ}^+(2632)$ is a member of the $\bar{\mathbf{15}}$ -plet, based on its decay properties, but we shall show that it is a mixture of the $\bar{\mathbf{15}}$ -plet and the $\bar{\mathbf{3}}_S$ -plet and that the decay properties are not spoiled by this fact.

Mass and state mixing.—Flavor state mixing determines the flavor content of the physical tetraquarks, which in turn determines their decay properties. Flavor SU(3) symmetry breaking by quark mass differences, leads to splitting within flavor multiplets and to mixing of members of different multiplets. The 't Hooft interaction, although SU(3) symmetric, also adds to the mixing.

The 2×2 mass matrix for the D_s ($\bar{\mathbf{3}}_S, \bar{\mathbf{15}}$) states is nondiagonal in general

$$M_{D_s} = \begin{pmatrix} [m_c + \frac{3}{2}(\bar{m} + m_s) + \delta E(\bar{\mathbf{3}}_S)] & \frac{3}{2}(\bar{m} - m_s) \\ \frac{3}{2}(\bar{m} - m_s) & [m_c + \frac{3}{2}(\bar{m} + m_s) + \delta E(\bar{\mathbf{15}})] \end{pmatrix} \quad (2)$$

where $\bar{m} = \frac{1}{3}(m_u + m_d + m_s)$ (there is no mixing with the $\bar{\mathbf{3}}_A$).

$$\begin{aligned} |D^0 \subset \bar{\mathbf{3}}_S\rangle &= \frac{1}{\sqrt{8}} |c(-s(\bar{u}\bar{s} + \bar{s}\bar{u}) + 2u\bar{u}\bar{u} - d(\bar{d}\bar{u} + \bar{u}\bar{d}))\rangle, \\ |D^+ \subset \bar{\mathbf{3}}_S\rangle &= \frac{1}{\sqrt{8}} |c(-s(\bar{d}\bar{s} + \bar{s}\bar{d}) + 2d\bar{d}\bar{d} - u(\bar{d}\bar{u} + \bar{u}\bar{d}))\rangle, \\ |D_s^+ \subset \bar{\mathbf{3}}_S\rangle &= \frac{1}{\sqrt{8}} |c(-2s\bar{s}\bar{s} + u(\bar{u}\bar{s} + \bar{s}\bar{u}) - d(\bar{d}\bar{s} + \bar{s}\bar{d}))\rangle. \end{aligned} \quad (3)$$

$$\begin{aligned} |D^0 \subset \bar{\mathbf{15}}\rangle &= \frac{1}{\sqrt{24}} |c(3s(\bar{u}\bar{s} + \bar{s}\bar{u}) + 2u\bar{u}\bar{u} - d(\bar{d}\bar{u} + \bar{u}\bar{d}))\rangle, \\ |D^+ \subset \bar{\mathbf{15}}\rangle &= \frac{1}{\sqrt{24}} |c(3s(\bar{d}\bar{s} + \bar{s}\bar{d}) - 2d\bar{d}\bar{d} + u(\bar{d}\bar{u} + \bar{u}\bar{d}))\rangle, \\ |D_s^+ \subset \bar{\mathbf{15}}\rangle &= \frac{1}{\sqrt{8}} |c(2s\bar{s}\bar{s} + u(\bar{u}\bar{s} + \bar{s}\bar{u}) - d(\bar{d}\bar{s} + \bar{s}\bar{d}))\rangle. \end{aligned} \quad (4)$$

Diagonalization is accomplished by way of mixing the $\bar{\mathbf{3}}_S$ and the $\bar{\mathbf{15}}$ states

$$|D_{s+}\rangle = \cos\Theta_s |D_s(\bar{\mathbf{15}})\rangle + \sin\Theta_s |D_s(\bar{\mathbf{3}}_S)\rangle, \quad |D_{s-}\rangle = -\sin\Theta_s |D_s(\bar{\mathbf{15}})\rangle + \cos\Theta_s |D_s(\bar{\mathbf{3}}_S)\rangle, \quad (5)$$

and the (strange) mixing angle Θ_s is determined by

$$\tan 2\Theta_s = \frac{3(\bar{m} - m_s)}{\delta E(\bar{\mathbf{3}}_S) - \delta E(\bar{\mathbf{15}})}. \quad (6)$$

Similar mass matrices and mixing angles can be written for

the nonstrange tetraquark members of the two triplets and the $\bar{\mathbf{15}}$ -plet.

Without 't Hooft interaction, $A = 0$ in the equations above, we find that the diagonal mass matrix or spectrum consists of two degenerate lighter tetraquark triplets with opposite exchange symmetries at $m_c + m_u + m_d + m_s$, and a third one at $m_c + 3m_s$, i.e., the heavier and the two

lighter states are separated by exactly two strange- u,d quark mass differences $2m_s - (m_u + m_d) = 300 \pm 60$ MeV. This is “ideal mixing”, and corresponds to the strange tetraquark mixing angle of $\Theta_s = -45^\circ$.

Matrix elements.—In Ref. [8] we have calculated the single-charm ($C = 1$) tetraquark ($cq\bar{q}^2$) mass spectrum with the 't Hooft interaction and found that three SU(3)-flavor multiplets ($\bar{\mathbf{3}}_A, \mathbf{6}, \bar{\mathbf{15}}$) are significantly lowered, while the $\bar{\mathbf{3}}_S$ -plet is lifted, as compared with their unperturbed masses, as a consequence of the 't Hooft interaction (both two-body and three-body).

The ratio of strengths of the two-body and three-body contributions varies with the change in the ratio of the quark condensate $\langle\bar{q}q\rangle_0$ and $I = \langle\Psi|\delta(\mathbf{r}_1 - \mathbf{r}_2)|\Psi\rangle = (\frac{m_q\omega}{2\pi})^{3/2}$, due to the relation

$$\langle\Psi|\delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_1 - \mathbf{r}_3)|\Psi\rangle = \left(\frac{m_q\omega}{\sqrt{3}\pi}\right)^3 = \left(\frac{2}{\sqrt{3}}\right)^3 I^2 \approx 1.54I^2. \quad (7)$$

Therefore we may introduce a new constant c to vary with the ratio of the two-body and three-body contributions, which allows one to “dial” the mass splitting between the first and the second-lowest states.

We shall not repeat the evaluation of the 't Hooft interaction's matrix elements in first order perturbation theory, shown in Ref. [8], but use the results shown therein. Thus we have the following energy/mass shifts for scalar tetraquarks in the color $[\bar{\mathbf{3}}_{12}\mathbf{3}_{34}]$ and in the lowest nonrelativistic approximation while ignoring the $m_s - m_{u/d}$ mass difference

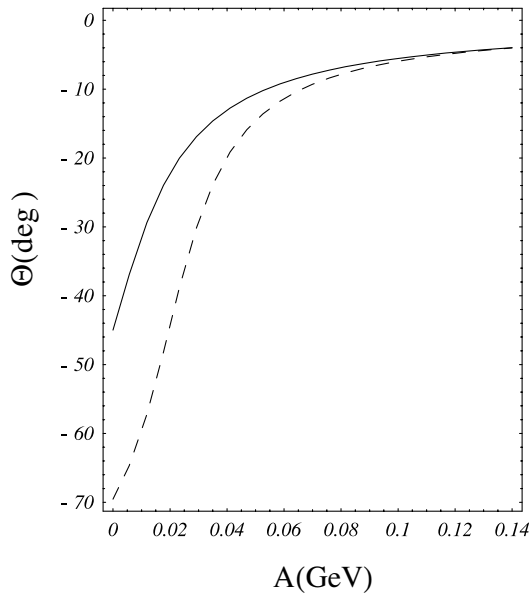


FIG. 2. The mixing angles in the strange (solid lines) and nonstrange (dashed line) sectors as functions of the effective KMT coupling A .

$$\delta E(\bar{\mathbf{3}}_A) = 8KI[\langle\bar{q}q\rangle_0 - 3.08cI] \approx -\left(2 + 1.7c\frac{A}{70}\right)A < 0 \quad (8)$$

$$\delta E(\bar{\mathbf{3}}_S) = -16K\langle\bar{q}q\rangle_0 I \equiv 4A > 0 \quad (9)$$

$$\delta E(\bar{\mathbf{15}}) = 12K\langle\bar{q}q\rangle_0 I \equiv -3A < 0 \quad (10)$$

$$\delta E(\mathbf{6}) = 8KI[2\langle\bar{q}q\rangle_0 + 3.08cI] \approx -\left(4 - 1.7c\frac{A}{70}\right)A < 0, \quad (11)$$

where the numerical coefficient 1.7 in front of the three-body term was evaluated using $-(225 \text{ MeV})^3$ as the value of the quark condensate $\langle\bar{q}q\rangle_0$, and the u/d quark mass $m_q = 313$ MeV and $\omega = 500$ MeV, which in turn led to $A = 70$ MeV with $K = 390 \text{ GeV}^{-5}$.

In Fig. 1 of Ref. [8] we showed the splitting of these two (diagonalized) states (at $c = 1$) that grows with the effective 't Hooft coupling strength $A = -4K\langle\bar{q}q\rangle_0 I > 0$. (There we neglected the quadratic A dependence of the three-body term.) As all of the $\bar{\mathbf{3}}_S - \bar{\mathbf{15}}$ mixing matrix elements are independent of the three-body term, and thus independent of the new variable c , the diagonalized masses $D_{s\pm}$ do not depend on c .

Thus the only way of raising the D_{s-} mass from 2505 to 2632 is by raising all constituent quark masses by 32 MeV. That brings the u/d constituent quark mass m_q to 345 MeV, up from 313 MeV, and leaves the fit value of A at 108 MeV. This increase in the quark mass raises I by about 15%; however, its effect is as if c were 1.15. The previously neglected quadratic A dependence of the three-body term adds another 54%: $108/70 \approx 1.54$.

$D_s^+(2317)$.—With the above corrections it turns out that all one needs to fit the $D_s^+(2317)$ is $c = 1.01$. The remaining increase in c by 1% can come about either from reducing the condensate value $\langle\bar{q}q\rangle_0 = -(225 \pm 25 \text{ MeV})^3$, which is well within the error bars, or from an increase of the (harmonic) oscillator frequency in this state, which may come about due to color-dependent three-body force, or from both.

$\bar{\mathbf{3}}_S - \bar{\mathbf{15}}$ mixing and $D_s(2632)$ decays.—In Ref. [7] it was argued that the assumption of $D_s(2632)$ belonging to a

TABLE I. Quark contents of the $cq\bar{q}^2$ tetraquark $\bar{\mathbf{3}}$ -plet and $\bar{\mathbf{15}}$ -plet states that mix, and their predicted masses (MeV). The states denoted by $(\bar{\mathbf{3}}_S - \bar{\mathbf{15}})_\pm$ are the heavier and lighter admixtures, respectively.

	\bar{D}^0 $T(cs\bar{u}\bar{s})$	\bar{D}^+ $T(cs\bar{s}\bar{d})$	\bar{D}_s^+ $T(cq\bar{s}\bar{q})_{I=0}$
$\bar{\mathbf{3}}_A$	2317	2317	2317
$(\bar{\mathbf{3}}_S - \bar{\mathbf{15}})_-$	2566	2561	2632
$(\bar{\mathbf{3}}_S - \bar{\mathbf{15}})_+$	3229	3224	3437

TABLE II. Quark contents of the $cq\bar{q}^2$ tetraquark **6**-plet and their predicted masses (MeV).

$T(c\bar{d}\bar{u}\bar{d})_{I=1/2}$	$T(cu\bar{u}\bar{d})_{I=1/2}$	$T(cs\bar{u}\bar{d})_{I=0}$	$T(c\bar{d}\bar{u}\bar{s})_{I=1}$	$T(cq\bar{s}\bar{q})_{I=1}$	$T(cu\bar{d}\bar{s})_{I=1}$
2724	2724	2724	2724	2724	2724

$\bar{\mathbf{15}}$ -plet fits very well with its decay properties. The mixing with the $\bar{\mathbf{3}}_S$ -plet was completely ignored, however. The $\bar{\mathbf{3}}_S$ -plet analogon of Ref. [7]’s Eq. (19) reads

$$\left| \frac{\lambda(D_s^+(\bar{\mathbf{3}}_S) \rightarrow D^0 K^+)}{\lambda(D_s^+(\bar{\mathbf{3}}_S) \rightarrow D_s \eta_8)} \right| = \frac{1}{2}, \quad (12)$$

which is not far from the $\bar{\mathbf{15}}$ -plet value of $1/\sqrt{6}$, and is also consistent with the experimental branching ratio. Of course, only the proper admixture ought to be compared with experiment.

The mixing, with ’t Hooft interaction, is far from being “ideal”: at the fitted value of $A = 108$ MeV (see above) we find $\Theta_s = -5.1^\circ$ vs an ideal mixing angle of -45° (at $A = 0$). This is similar to stating that the $D_s^+(2632)$ is very close to being a pure $D_s^+(\bar{\mathbf{3}}_S)$. Similarly, in the nonstrange sector, the mixing angle is $\Theta = -5.4^\circ$ vs an ideal mixing angle of -69.5° (at $A = 0$) in this channel. In Fig. 2 we show the mixing angles as functions of the effective ’t Hooft coupling A . Therefore the conclusion of Ref. [7] is unchanged: the decay pattern of $D_s^+(2632)$, at the present level of experimental precision, agrees with experiment if it is a $\bar{\mathbf{3}}_S$ -plet, or if it is a $\bar{\mathbf{15}}$ -plet.

Prediction of other tetraquark masses.—Our results show that the ’t Hooft interaction may be the cause of $\bar{D}_s^+(2317)$ ’s anomalously low mass. For the same value of ’t Hooft coupling A , another heavier scalar \bar{D}_s^+ tetraquark state ought to exist: A manifest candidate for this state is the $D_s^+(2632)$ now, and its decay properties seem to confirm this assumption.

Note that one may expect an even larger proliferation of states: many new exotic tetraquark states ought to exist. Here we update the mass predictions of Ref. [8], with the higher quark masses and the value of A fixed at 108 MeV and $c = 1.7$ in the aforementioned manner. Thus we find the open charm ($C = 1$) tetraquark spectrum tabulated in Tables I, II, and III.

Conclusions.—We have looked into the question of identifying the recently observed states $D_0(2308)$, $D_s^+(2317)$ and $D_{sJ}^+(2632)$ with open charm and strangeness

TABLE III. Quark contents of the $cq\bar{q}^2$ tetraquark $\bar{\mathbf{15}}$ -plet and their predicted masses (MeV).

$T(c\bar{d}\bar{s})$	$T(cu\bar{s})$	$T(cq\bar{q})$	$T(c\bar{d}\bar{u})$	$T(cq\bar{s})_{I=1}$	$T(cu\bar{d})$
2657	2652	2383 ± 10	2520	2520	2520
$T(cq\bar{u})$	$T(cq\bar{d})$	$T(cq\bar{s})$	$T(cs\bar{u})$	$T(cs\bar{q})_{I=1}$	$T(c\bar{s}\bar{u})$
			2520	2520	2520

tetraquarks in the presence of ’t Hooft interaction and found that:

(1) The lowest open charm and strangeness scalar cryptoexotic tetraquarks $D_0(2308)$, $D_s^+(2317)$ belong to the $SU(3)$ $\bar{\mathbf{3}}_A$ -plet, a fact which naturally explains their small mass difference. The ’t Hooft interaction moves the strange state below both of its strong decay thresholds, while leaving the nonstrange one far above the $D(1870) + \pi(140)$ “fall apart” threshold, which accounts for its large decay width. Thus it appears that the tetraquark structure is the only natural explanation of these states at present, so our findings lend credence to the tetraquark interpretation of $D_0(2308)$ and $D_{s0}^+(2317)$.

(2) $D_{sJ}^+(2632)$ is the lighter admixture of the $\bar{\mathbf{3}}_S$ -plet and $\bar{\mathbf{15}}$ -plet. The lighter admixture’s mass is lowered by the ’t Hooft interaction, whereas the heavier one’s is elevated. $D_{sJ}^+(2632)$ consists predominantly of $\bar{\mathbf{3}}_S$ -plet, with only a small fraction of $\bar{\mathbf{15}}$ -plet, yet it is consistent with the observed decay properties.

We are not the only ones to have revived the notion of tetraquarks and suggest that the $D_s^+(2317)$ is one: that has been done in Refs. [1,5,6,12], and by others. Similarly, there have been several suggestions of tetraquark structure for the $D_s^+(2632)$ [6,7,12,13]. Most of these authors did not discuss the $\bar{\mathbf{3}}_S$ - $\bar{\mathbf{15}}$ -plet mixing, however, nor did they attempt a dynamical explanation of the tetraquarks’ anomalously low masses.

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