## Tilt-Induced Localization and Delocalization in the Second Landau Level

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We have investigated the behavior of electronic phases of the second Landau level under tilted magnetic fields. The fractional quantum Hall liquids at  $\nu = 2 + 1/5$  and 2 + 4/5 and the solid phases at  $\nu = 2.30$ , 2.44, 2.57, and 2.70 are quickly destroyed with tilt. This behavior can be interpreted as a tilt driven localization of the 2 + 1/5 and 2 + 4/5 fractional quantum Hall liquids and a delocalization through the melting of solid phases in the top Landau level, respectively. The evolution towards the classical Hall gas of the solid phases is suggestive of antiferromagnetic ordering.

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Two-dimensional electron systems (2DESs) subjected to perpendicular magnetic fields B exhibit a myriad of ground states. Perhaps the most well known of these are the incompressible states called the integer (IQHLs) and fractional quantum Hall liquids (FQHLs) [1]. The low frequency transport signature of these IQHLs and FQHLs is the quantized Hall resistance  $R_{xy}$  accompanied by a vanishing diagonal resistance  $R_{xx}$ . While the IQHLs are the consequence of purely single particle physics, the FQHLs forming at certain fractional values of the Landau level (LL) filling factor  $\nu$  can be explained only by considering interparticle interactions [2,3]. The various series of FQHLs are successfully accounted for by the composite fermion theory [3].

Strong interparticle interactions give rise to a second class of many particle ground states: that of compressible solids. With the availability of samples with continuously improving quality, a number of solid phases have been found [4-7]. The first examples are the high field insulating and reentrant insulating phases of the lowest LL at the highest B fields [4], phases that have been associated with the Wigner solid (WS) [8]. A recent work in the highest quality samples available found that there are two types of WS phases in this regime [5]. Microwave resonances close to  $\nu = 1, 2, 3$ , and 4 have also been interpreted as being due to the WS [6]. Other examples of solid phases are the recently discovered electronic stripe and bubble phases in high Landau levels also referred to as charge density waves (CDWs) [7]. While the transport signature of stripes is anisotropic, that of the bubble phases is isotropic, and it is described by the reentrant integer quantum Hall (RIQH) effect [7]. The RIQH effect is manifest in a  $R_{xy}$  quantized to an integer multiple of the quantum Hall resistance combined with a vanishing  $R_{xx}$  but which, unlike the integer quantum Hall effect, is centered at a filling factor that is different from the integer value to which the  $R_{xy}$  plateau is quantized. This behavior is a consequence of a disorder pinned solid phase forming in the top LL when multiple LLs are occupied. While substantial progress has been made, the nature of the CDW phases has not yet been fully understood.

The second Landau level is very special being at the borderline of the two very different regimes [9,10]. On one hand, the lowest LL is dominated by FQHLs, a series of phases that is terminated on the low filling side by the high field WS. On the other hand, stripe and bubble phases prevail in the third LL and beyond. Thus in the second LL the FQHL phases of the lowest LL and the CDW phases of high LLs are expected to compete leading to an intricate behavior. Indeed, the  $\nu = 3 + 1/5, 3 + 4/5, 2 + 1/5, 2 + 1/5, 2 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3 + 1/5, 3$ 1/3, 2 + 2/3, 2 + 4/5, 2 + 2/5, and possibly 2 + 2/7FQHLs as well as eight RIQH states have been reported in the second LL [9,10]. Besides the alternating FQHL and RIQH states, there are special fingerprint FQHLs at even denominator filling that are present in the second LL only. These states at  $\nu = 5/2$ , 7/2 [9–11], and possibly at 2 + 3/8 [10] are believed to arise from a BCS-like pairing of composite fermions [12]. The evolution with tilted magnetic fields of the  $\nu = 5/2$  and 7/2 states toward anisotropic states that are very similar to stripes of half-filled higher LLs [13] and the presence of the RIQH states between the FQHLs are regarded as evidence of the delicate balance between the phases of the second LL. While it has been suggested [9,10] and theoretically independently obtained [14] that the RIQH states of the second LL are isotropic collective insulators also called bubble phases, an independent experimental verification is still lacking.

In this Letter, we have investigated the influence of a magnetic field parallel to the confinement plane of the 2DES on the various electronic phases in the second LL in the  $2 < \nu < 3$  range using the tilted field technique. We find that the recently discovered RIQH states are rapidly destroyed with tilt. Such a behavior is not consistent with single particle localization in the top LL; therefore it constitutes experimental evidence that the RIQH states in the second LL are collectively pinned insulators. The rapid evolution with tilt of  $R_{xy}$  of the RIQH states from the values of the nearby integer plateaus towards the classical Hall value can be interpreted as melting of this collective phase into a classical Hall gas. Furthermore, since tilting changes the ratio of the Zeeman and cyclotron energies, the data suggest that spin interaction plays an important role in the formation of these collective phases. We surmise that the RIOH phases are not fully spin polarized but have substantial antiferromagnetic order. These phases could be the first examples of antiferromagnetically ordered solids in a single layer 2DES in the quantum Hall regime. In addition, the well-developed  $\nu = 2 + 1/5$  and 2 + 4/5FQHLs are found to be driven insulating, while the 2 +1/3 and 2 + 2/3 states survive to the largest tilt angles we can reach.

The low frequency magnetoresistance measurements were performed on a  $\delta$ -doped 30 nm wide GaAs/ AlGaAs quantum well at an excitation current of 1 nA. The 2DES has been prepared with a brief illumination with a red light emitting diode at low temperatures and has a density of  $3.0 \times 10^{11}$  cm<sup>-2</sup> and an exceptionally high mobility of  $2.7 \times 10^7$  cm<sup>2</sup>/V s. The challenging task of cooling to millidegrees Kelvin temperatures and *in situ* tilting in this low temperature environment is achieved in a special hydraulically driven rotator [15] equipped with sintered silver heat exchangers immersed in a <sup>3</sup>He bath [11].

The diagonal and off-diagonal resistances as a function of the magnetic field component perpendicular to the 2DES  $B_{\perp}$  at a set of representative tilt angles  $\theta$  are shown in Fig. 1. The bath temperature is 9 mK. The traces in purely perpendicular field or at  $\theta = 0^{\circ}$  are located in the middle of Fig. 1. These traces are similar to those of samples of comparable parameters [9,10]. At  $\theta = 0^{\circ}$  we observe the FQHLs at  $\nu = 2 + 1/5, 2 + 1/3, 2 + 2/3$ , and 2 + 4/5 as well as the well-developed  $\nu = 5/2$  state. In the vicinity of  $\nu \simeq 2.30$ , 2.44, 2.57, and 2.70, the RIQH states develop since the vanishing  $R_{xx}$  is accompanied by an  $R_{xy}$ that jumps to the nearest integer value. In between the FQHL and RIQH states,  $R_{xy}$  follows the classical Hall line. We note that in a recent experiment the  $\nu =$ 2 + 1/5 FOHL is well developed at 40 mK, but it started to evolve toward a localized phase at 16 mK and the RIQH phase around  $\nu = 2.44$  is interrupted by the developing 2 + 2/7 FOHL [10].

As shown in Fig. 1, with increasing tilt the 2 + 1/3 FQHL stays robust while the 2 + 1/5 FQHL quickly vanishes. Once destroyed, the 2 + 1/5 FQHL does not reemerge with further tilting. This behavior is summarized in Fig. 2, where we have plotted the tilt dependence of  $R_{xy}$  at filling factors at which FQHLs develop. None of these FQHLs display the cusp in  $R_{xx}$  that is associated with spin transitions [16,17]. Our data therefore suggest that

the 2 + 1/3 and 2 + 1/5 FQHLs are spin polarized, just as their  $\nu = 1/3$  and 1/5 counterparts are in the lowest LL [2]. It is interesting to note that similar tilts could lead to a spin-unpolarized state for the  $\nu = 2 + 2/3$  FQHL since the  $\nu = 2/3$  FQHL in the lowest LL has convincingly been shown to be spin unpolarized [17,18]. However, up to the largest tilt angle  $\theta = 54.4^{\circ}$  of our experiment we do not observe any sign of a spin transition. We explain this behavior by either a *B* field that is too small to polarize this state or, more likely, a fully spin-polarized  $\nu = 2 + 2/3$  FQHL. At negligible spin mixing, a fully spinpolarized 2 + 2/3 state can be derived from the  $\nu = 2 + 1/3$  spin-polarized FQHL by particle-hole symmetry within the top LL.

We have seen that the  $\nu = 2 + 1/5$  and 2 + 4/5 FQHLs turn insulating with tilt. A similar localization transition has been recently observed in the lowest LL in a low density 2D hole sample in which tilt localizes the terminal FQHL with  $\nu = 1/3$  [19]. One route to localization in the



FIG. 1 (color online). Dependence of  $R_{xx}$  and  $R_{xy}$  on  $B_{\perp}$  at various tilt angles  $\theta$  measured at 9 mK. Filling factors are shown in the top scale. Horizontal lines highlight the plateaus of  $R_{xy}$  of the FQHLs at  $\theta = 0^{\circ}$ .



FIG. 2 (color online). Tilt dependence of  $R_{xy}$  at  $\nu = 2 + 1/5$ , 2 + 1/3, 2 + 1/2, 2 + 2/3, and 2 + 4/5 measured at 9 mK. While  $R_{xy}$  for the 2 + 1/3 and 2 + 2/3 FQHLs is unchanged, for the 2 + 1/5 and 2 + 4/5 FQHLs it evolves from  $h/\nu e^2$  toward the nearest integer quantum Hall value.

top LL with effective filling 1/5 is the enhanced surface roughness scattering. This scattering mechanism is due to the single particle wave function being squeezed in tilted field against the interface of the confining potential. We think, however, that such a scenario is unlikely. At the approximately 15° tilt at which these FQHLs are destroyed the center of the single particle wave function in a 30 nm quantum well does not shift substantially. A second route that renders the FQHL localized and that most likely explains our data is given by the evolution with tilt of the ground state from the FQHL toward a pinned collective insulator. Such a transition occurs if an electronic solid becomes energetically favored at large tilt angles.

We focus next on the evolution of the RIQH states with tilt. At small tilt angles the RIQH phase at  $\nu = 2.70$  and at constant T gradually weakens with tilt, a response briefly mentioned in Ref. [9]. As the tilt increases we find that  $R_{xy}$ evolves from  $h/3e^2$ , the value of the nearby  $\nu = 3$  IQHL plateau, and reaches a value very close to the classical Hall value beyond  $\theta \simeq 10^{\circ}$  tilt. This behavior, together with the results for the other RIQH states, is summarized in Fig. 3(a). The observed behavior is interpreted as a delocalization transition under tilt. Since for localized single particles there is no known mechanism for delocalization driven by the application of a magnetic field parallel to the 2DES, the reentrant behavior cannot be due to single particle localization. The tilt-induced destruction of the RIQH behavior therefore demonstrates a collectively pinned solid at  $\theta = 0^{\circ}$ .

With increasing tilt, the  $\nu = 5/2$  state weakens and beyond  $\theta \simeq 44^{\circ}$  cannot be discerned any more. It has been proposed that this is due to a symmetry breaking mechanism induced by the parallel component of the *B* field [13]. As a result, the isotropic  $\nu = 5/2$  state at zero tilt becomes an anisotropic phase similar to the stripe



FIG. 3 (color online). Angle dependence of  $R_{xy}$  at  $\nu = 2.30$ , 2.44, 2.57, and 2.70 at 9 mK [panel (a)], and *T* dependence at  $\nu = 2.57$  and at zero tilt [panel (b)]. Lines are guides to the eye.

phases in higher LL. We have investigated if a similar symmetry breaking occurs for the RIQH states by running the current both parallel and perpendicular to the direction of the parallel component of the *B* field. In doing so, there is little change in the data (not shown). We conclude that, similar to the  $\theta = 0$  case, in tilted fields there is no anisotropic behavior except in the vicinity of the  $\nu = 5/2$ .

The rapid transformation with tilt of the RIQH states into the classical Hall gas can be interpreted as a tiltinduced melting of the bubble phase. The evolution at  $\nu =$ 2.30, 2.44, 2.57, and 2.70 of  $R_{xy}$  from the nearby integer plateau toward the classical Hall value cannot be explained by a transition from the bubble phase to a phase of singly localized particles. This is consistent with the earlier observation that single particle localization due the enhanced interface roughness scattering is most likely not substantial for the FQHLs. Thus, the destruction of the RIQH states with tilt is most likely not disorder driven. A second, intriguing possibility that can explain the behavior with tilt is that the bubble phase is only partially spin polarized. We consider such a scenario because a fully spin-polarized solid is not expected to be affected by tilting. Melting of the partially polarized bubble phase can occur if its energy becomes higher than that of the classical Hall gas as the tilt angle increases. The abrupt T dependence of  $R_{xy}$ , shown in Fig. 3(b) for  $\nu = 2.57$ , is not inconsistent with such a scenario. A partially spin-polarized electronic solid most likely has antiferromagnetic ordering.

The exchange interaction modeled by the Heisenberg Hamiltonian for the spin 1/2 is an essential ingredient in understanding magnetic properties of quantum solids. Magnetism in the B = 0 WS was found to be determined by the competition of different ring exchange processes [20]. Even and odd circular permutations lead to ferromagnetic and antiferromagnetic coupling, respectively, and the type of lattice can influence the dominant term [20]. A known experimental realization of a 2D quantum solid of

spin 1/2 particles is the solidified second layer <sup>3</sup>He prepared on a graphite substrate [21]. Since the single particle wave functions of electrons in the top LL have significant overlap, in our system there is a considerable exchange. We speculate that the exchange has a dominant antiferromagnetic term in the bubble phases of the second LL.

Finally, we note that, at tilt angles beyond 30°, as the IQH plateaus extend over the 2 + 1/5 and 2 + 4/5 fillings, any sign of reentrance has disappeared and  $\nu = 2.30$  and 2.70 become the demarcation line between the 2 and 2 +1/3 and the 3 and 2 + 2/3 plateaus, respectively. This results, as shown in Fig. 1, in pronounced peaks in  $R_{xx}$ and a steep  $R_{xy}$  as function of  $B_{\perp}$  close to  $\nu = 2.70$  and 2.30. We note that, due to this steep  $R_{xy}$ , its value at  $\nu =$ 2.30 and 2.70 has a substantial error propagating from small errors of the tilt angle. At the highest tilt angle  $\theta >$ 48° we observe an asymmetry between the plateaus of the 2 + 1/3 and 2 + 2/3 FQHLs that we do not understand. Using  $B_{\perp}$  as abscissa, the plateau length of the 2 + 1/3FQHL shrinks and that of the 2 + 2/3 FQHL grows. In fact, at  $\theta = 54.4^{\circ}$ , as shown in Fig. 1, the plateau of the 2 + 2/3 FQHL will extend beyond  $B_{\perp}$  corresponding to  $\nu = 2.57.$ 

To summarize, we found that an interesting tilt and filling factor dependent interplay of localization and delocalization shapes the dc transport in the second LL. These transitions are a result of the delicate balance of various phases of the second LL. We think that an important consequence is that the 2 + 1/5 FQHL cedes its place as a ground state to a collective solid, while the collectively pinned solids associated with reentrance of the integer quantum Hall plateaus melt into a classical Hall gas. The RIQH states at zero tilt are electronic solid phases with possible antiferromagnetic order.

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*Note added in proof.*—After the submission of this work, the microwave response of the RIQH states was found to be consistent with the formation of electronic solids [22].

- See *Perspectives in Quantum Hall Effects*, edited by S. Das Sarma and A. Pinczuk (John Wiley & Sons, New York, 1997).
- [2] R.B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
- [3] J.K. Jain, Phys. Rev. Lett. 63, 199 (1989).
- [4] R. L. Willett *et al.*, Phys. Rev. B 38, R7881 (1988); H. W. Jiang *et al.*, Phys. Rev. Lett. 65, 633 (1990); D. C. Glattli

*et al.*, Surf. Sci. **229**, 344 (1990); F. I. B. Williams *et al.*, Phys. Rev. Lett. **66**, 3285 (1991); M. A. Paalanen *et al.*, Phys. Rev. B **45**, 11 342 (1992); L. W. Engel *et al.*, Solid State Commun. **104**, 167 (1997).

- [5] Y. P. Chen, R. M. Lewis, L. W. Engel, D. C. Tsui, P. D. Ye, Z. H. Wang, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 93, 206805 (2004).
- [6] Y. Chen *et al.*, Phys. Rev. Lett. **91**, 016801 (2003); R. M. Lewis *et al.*, Physica (Amsterdam) **22E**, 104 (2004).
- [7] M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 82, 394 (1999); R. R. Du et al., Solid State Commun. 109, 389 (1999); R. M. Lewis et al., Phys. Rev. Lett. 89, 136804 (2002).
- [8] E. Wigner, Phys. Rev. 46, 1002 (1934).
- [9] J.P. Eisenstein, K.B. Cooper, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 88, 076801 (2002).
- [10] J.S. Xia et al., Phys. Rev. Lett. 93, 176809 (2004).
- [11] W. Pan et al., Phys. Rev. Lett. 83, 3530 (1999).
- [12] M. Greiter, X. G. Wen, and F. Wilczek, Phys. Rev. Lett. 66, 3205 (1991); G. Moore and N. Read, Nucl. Phys. B360, 362 (1991); K. Park, V. Melik-Alaverdian, N.E. Bonesteel, and J.K. Jain, Phys. Rev. B 58, R10167 (1998); R. H. Morf, Phys. Rev. Lett. 80, 1505 (1998); N.E. Bonesteel, Phys. Rev. Lett. 82, 984 (1999); E. Fradkin, C. Nayak, and K. Schoutens, Nucl. Phys. B546, 711 (1999); E. H. Rezayi and F. D. M. Haldane, Phys. Rev. Lett. 84, 4685 (2000); V.W. Scarola, K. Park, and J. K. Jain, Nature (London) 406, 863 (2000).
- [13] J.P. Eisenstein *et al.*, Phys. Rev. Lett. **61**, 997 (1988);
   W. Pan *et al.*, Solid State Commun. **119**, 641 (2001).
- [14] M. O. Goerbig, P. Lederer, and C. M. Smith, Phys. Rev. B 68, 241302(R) (2003).
- [15] J.S. Xia, E.D. Adams, N.S. Sullivan, W. Pan, H.L. Stormer, and D.C. Tsui, Int. J. Mod. Phys. B 16, 2986 (2002).
- [16] R.G. Clark *et al.*, Phys. Rev. Lett. **62**, 1536 (1989); J.P. Eisenstein, H.L. Stormer, L. Pfeiffer, and K.W. West, Phys. Rev. Lett. **62**, 1540 (1989); R.R. Du, A.S. Yeh, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **75**, 3926 (1995).
- [17] J. P. Eisenstein, H. L. Stormer, L. N. Pfeiffer, and K. W. West, Phys. Rev. B 41, R7910 (1990); L. W. Engel, S. W. Hwang, T. Sajoto, D. C. Tsui, and M. Shayegan, Phys. Rev. B 45, 3418 (1992); I. V. Kukushkin, K. v. Klitzing, and K. Eberl, Phys. Rev. Lett. 82, 3665 (1999).
- [18] N. Freytag et al., Phys. Rev. Lett. 87, 136801 (2001).
- [19] W. Pan, G. A. Csáthy, D. C. Tsui, L. N. Pfeiffer, and K. W. West, Phys. Rev. B 71, 035302 (2005).
- [20] M. Roger, Phys. Rev. B 30, 6432 (1984); X. Zhu and S. G. Louie, Phys. Rev. B 52, 5863 (1995); V. V. Flambaum, I. V. Ponomarev, and O. P. Sushkov, Phys. Rev. B 59, 4163 (1999); B. Bernu, Ladir Cândido, and D. M. Ceperley, Phys. Rev. Lett. 86, 870 (2001).
- [21] H. Godfrin, R.E. Rapp, and H.J. Lauter, Physica (Amsterdam) 169B, 177 (1991); C.P. Lusher, J. Saunders, and B.P. Cowan, Europhys. Lett. 14, 809 (1991);
  M. Siqueira, J. Nyéki, B. Cowan, and J. Saunders, Phys. Rev. Lett. 78, 2600 (1997).
- [22] R.M. Lewis et al., Phys. Rev. B 71, 081301 (2005).