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Resonant Activation in a Nonadiabatically Driven Optical Lattice

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We demonstrate the phenomenon of resonant activation in a nonadiabatically driven dissipative optical lattice with broken time symmetry. The resonant activation results in a resonance as a function of the driving frequency in the current of atoms through the periodic potential. We demonstrate that the resonance is produced by the interplay between deterministic driving and fluctuations, and we also show that by changing the frequency of the driving it is possible to control the direction of the diffusion.

The problem of the escape of a Brownian particle out of a potential well, first characterized by Kramers [1], plays a central role in the description of many processes in physics, chemistry, and biology [2,3]. Kramers' law predicts an escape rate of the form $\exp(-U/kT)$, where U is the depth of the well, T the temperature, and k the Boltzmann constant. In the case of a nonadiabatically driven Brownian particle, the aforementioned scenario may change significantly, and much work has been devoted to the study of this nonequilibrium phenomenon [4–11]. It has been shown that in the presence of nonadiabatic driving the lifetime of the particle in the potential well can be significantly reduced, a phenomenon named resonant activation.

Resonant activation was first observed in a current-biased Josephson tunnel junction [4], and more recently for a Brownian particle optically trapped in a double well potential [10]. Resonant activation has also been theoretically studied for Brownian particles in periodic potentials, a configuration relevant for the realization of Brownian motors [12]. Also in this case the nonadiabatic driving may result in a significant enhancement of the activation rate. Furthermore, it has been predicted that, whenever the spatiotemporal symmetry of the system is broken, the resonant activation gives rise to *resonant* rectification of fluctuations [6,8,13]. Indeed, very recently Brownian motors have been realized with nonadiabatically driven Brownian particles [14,15]. However, in those works the resonant nature of the rectification mechanism and the underlying resonant activation process have not been investigated. In particular, the possibility of controlling the amplitude and the direction of the diffusion by changing the frequency of the driving field, as predicted by theoretical work [6,8,11], has not yet been demonstrated.

In this Letter we demonstrate experimentally the phenomenon of resonant activation in a Brownian motor by using cold atoms in a driven dissipative optical lattice as a model system. Indeed, we observe the appearance of a resonance while monitoring the current of atoms through the lattice as a function of the driving frequency. We show that this resonance is due to the interplay between fluctuations and deterministic driving, and we demonstrate that by

varying the driving frequency it is possible to reverse the current direction.

In our experiment we use cold atoms in a dissipative optical lattice [16], in which the atom-light interaction determines both the periodic potential for the atoms and the dissipation mechanism which leads to a friction force—the so-called Sisyphus cooling—and to fluctuations in the atomic dynamics. This system offers the significant advantage of being easily tunable over a wide range of parameters: the potential depth, the fluctuation's level, and the parameters (frequency and amplitude) of the driving force can be varied and carefully controlled over a large interval of values. This is an essential feature for the investigation presented here.

Before presenting the experimental results, we analyze our system through numerical simulations. For the sake of clarity, our theoretical analysis is limited to the simplest one-dimensional configuration in which Sisyphus cooling takes place: a $J_g = 1/2 \rightarrow J_e = 3/2$ atomic transition and two counterpropagating laser fields with orthogonal linear polarizations—the so-called lin $\perp$ lin configuration. The light interference pattern results in two optical potentials $U_\pm$ for the atoms, one for each ground state $|\pm\rangle$, in phase opposition: $U_\pm = U_0[-2 \pm \cos 2kz]/2$, where z is the atomic position along the axis Oz of light propagation, k the laser field wave vector, and U_0 the depth of the optical potential. The stochastic process of optical pumping transfers the atom from one ground state to the other one, changing in this way the optical potential experienced by the atom. This stochastic process results in a friction force and produces fluctuations in the atomic dynamics [16]. The departure rates $\gamma_{\pm \rightarrow \mp}(z)$ from the $|\pm\rangle$ states can be written in terms of the photon scattering rate Γ' as $\gamma_{\pm \rightarrow \mp} = \Gamma'(1 \pm \cos 2kz)/9$ [16]. It appears therefore that the amplitude of the fluctuations can be quantitatively characterized by the photon scattering rate Γ', which is an experimentally accessible parameter.

To study the phenomenon of resonant activation, we need to drive the atoms with a zero-mean oscillating force. We consider here an ac force consisting of two harmonics, $A_1 \cos(\omega t)$ and $A_2 \cos(2\omega t - \pi/2)$, so that the resonant activation should lead, following the breaking of the time

symmetry of the system, to a resonant generation of a current, as predicted by theoretical work [6,8,17]. In the numerical work it is obviously possible to "apply" directly an homogeneous ac force to the atoms, by simply including the appropriate terms in the equation of motion. On the contrary, in the experiment this is not possible, and forces can be applied only by phase modulating the lattice beams. For consistency, we follow the same approach in the theoretical analysis, and we consider a phase modulation $\alpha(t)$ of one of the lattice beams of the form

$$\alpha(t) = \alpha_0[A_1 \cos(\omega t) + \tfrac{1}{4}A_2 \cos(2\omega t - \pi/2)]. \quad (1)$$

In this way in the accelerated frame in which the optical potential is stationary the atoms experience an inertial force

$$F(t) = \frac{m\omega^2\alpha_0}{2k}[A_1 \cos(\omega t) + A_2 \cos(2\omega t - \pi/2)], \quad (2)$$

where m is the atomic mass and k the laser field wave vector.

To study the atomic dynamics in the presence of the nonadiabatic driving, we follow the same procedure developed to investigate laser cooling processes in (undriven) optical lattices. The Fokker-Plank–type equation for the undriven system and the Monte Carlo technique to derive the atomic trajectories have been discussed in detail in Refs. [16]. The generalization of that method for the driven system of interest here is straightforward, and consists of the inclusion of a time-dependent shift $\alpha(t)$ [see Eq. (1)] in the relative phase between the two laser fields generating the optical lattice.

Through Monte Carlo simulations, we calculated the mean atomic velocity v as a function of the frequency ω of the driving, for different amplitudes of the ac force. The results of our calculations are shown in Fig. 1. For each data set, in order to keep constant the amplitude $F_0 = m\omega^2\alpha_0/2k$ of the ac force [see Eq. (2)] we varied the amplitude α_0 of the phase modulation according to $\alpha_0 = \bar{\alpha}/\omega^2$, with $\bar{\alpha}$ constant for a given F_0. This is the same procedure used in the experiment.

Figure 1 shows clearly the appearance of a resonance in the current amplitude as a function of the driving frequency. Indeed, for weak driving the current is negligible. At a larger amplitude of the driving, the current differs significantly from zero and shows a well defined resonance. The resonance is observed in the regime of nonadiabatic driving, i.e., for driving frequencies of the order of or exceeding the vibrational frequency. The numerical simulations also show that by changing the driving frequency it is possible to reverse the current direction, as predicted by the general theory [6,8,11]. We note that we carried out numerical simulations for two different ratios of the force harmonics' amplitude: $A_1/A_2 = 1/4$ (Fig. 1) and $A_1/A_2 = 1$ (not shown). The two sets of calculations evidenced the same qualitative behavior.

To determine the nature of the resonance, we focus our attention on the range of frequencies of the ac fields
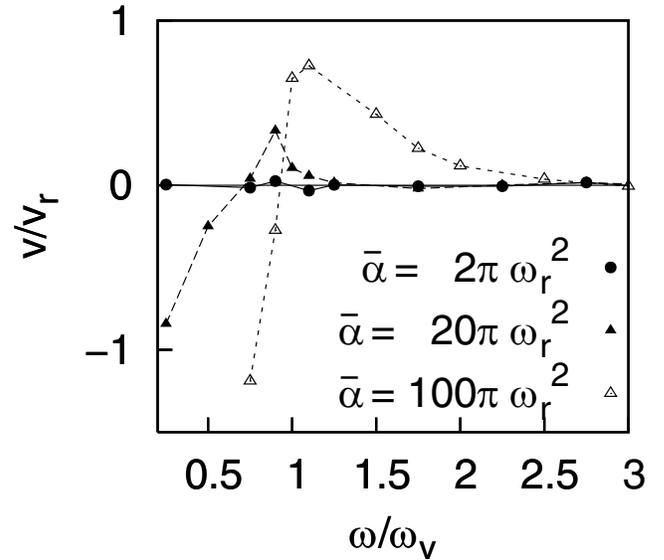

FIG. 1. Results of Monte Carlo simulations for a sample of $n = 10^4$ atoms in a 1D lin ⊥ lin optical lattice. The mean atomic velocity v, rescaled by the atomic recoil velocity $v_r = \hbar k/m$, is shown as a function of the driving frequency ω, for different amplitudes of the ac force. The driving frequency ω is expressed in terms of the vibrational frequency ω_v of the atoms at the bottom of the potential wells. Parameters of the calculation are $\Gamma' = 10\omega_r$ and $U_0 = 100E_r$, with E_r and ω_r the recoil energy and frequency, respectively. The amplitude of the two harmonics of the force are $A_1 = 1$ and $A_2 = 4$.

corresponding to nonadiabatic driving. This regime is illustrated in Fig. 2, where the resonance in the atomic current as a function of the driving frequency is shown for a given amplitude of the ac field and at different values of the scattering rate. From Fig. 2 it appears that the amplitude of the resonance shows a nonmonotonic dependence on the scattering rate Γ': at small Γ' the resonance amplitude increases with Γ', then reaches a maximum and finally at larger values of Γ' decreases. The constructive role played by the noise at low levels of Γ' shows that the resonance observed in the numerical simulations is determined by the interplay between the applied ac forces and the random fluctuations, which results in the rectification of the latter ones. This is at variance with the behavior observed at frequencies somewhat below ω_v where the magnitude of the (negative) current is decreased by an increase of the scattering rate, a behavior characteristic of deterministic rectification of forces. The conclusion of our numerical analysis is therefore that the resonant activation phenomenon should be observable in a dissipative optical lattice, and should result in a resonance in the atomic current as a function of the driving frequency.

The experiment is a direct implementation of what is described in the theoretical analysis. Instead of using a 1D optical lattice, as in the numerical work, we use a 3D lattice, which offers the significant advantage of confining the atoms in the three directions. This reduces the losses of atoms from the lattice during the experiment.
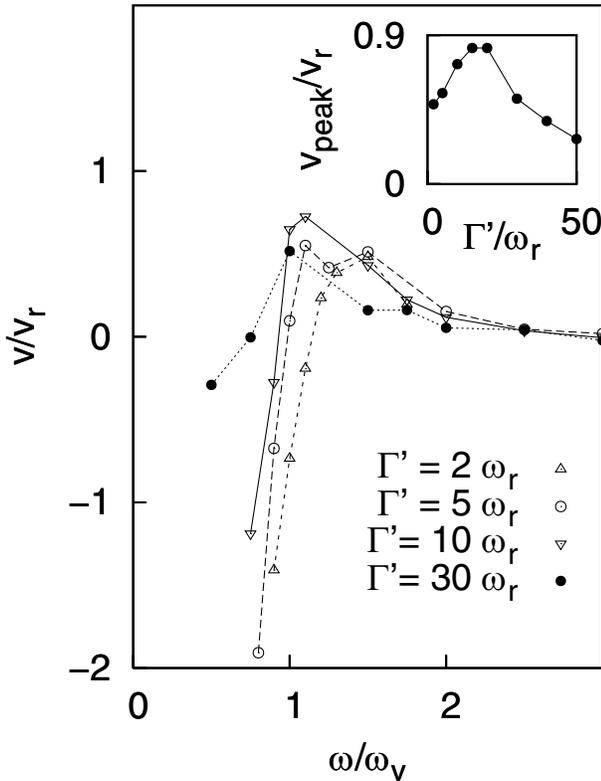
FIG. 2.   Results of Monte Carlo simulations for a sample of $n = 10^4$ atoms in a 1D lin $\perp$ lin optical lattice. The mean atomic velocity v is shown as a function of the driving frequency ω for different values of the scattering rate Γ'. In the inset, the mean atomic velocity v_{peak} at the maximum of the resonance is reported as a function of the scattering rate. The depth of the optical potential is $U_0 = 100E_r$; the ac force amplitude corresponds to $\bar{\alpha} = 100\pi\omega_r^2$. The amplitudes of the two harmonics of the force are $A_1 = 1$ and $A_2 = 4$. The lines are guides for the eye.

The experimental setup is the same as the one used in Ref. [15], and consists of four linearly polarized laser beams arranged in the so-called umbrellalike configuration [18]. One laser beam propagates in the z direction. This is the beam that is phase modulated. The three other laser beams propagate in the opposite direction, and are arranged along the edges of a triangular pyramid having the z direction as the axis, with the azimuthal angle between each pair of beams equal to $2\pi/3$. For this lattice beam configuration, the interference of the laser fields produces a periodic and spatially symmetric optical potential, with the potential minima associated with pure circular (σ^+ or σ^-) polarization of the light [18]. For an atom with a $F_g = F \rightarrow F_e = F + 1$ transition, the optical potential consists precisely of $2F + 1$ potentials, one for each ground state sublevel of the atom.

Cesium atoms are cooled and trapped in a magneto-optical trap (MOT). At a given instant the MOT is turned off and the four lattice beams are turned on. The atoms are left in this undriven optical lattice for 2 ms. This is sufficient for the atoms to thermalize and reach equilibrium.

Then the phase modulation $\alpha(t)$ [see Eq. (1)] is slowly turned on. The dynamics of the atoms is studied with a charged-coupled device camera. After a short transient, the center of mass of the atomic cloud is observed to be set into uniform motion along the z axis, and a center-of-mass velocity v is correspondingly derived.

Results for the average atomic velocity as a function of the driving frequency are shown in Fig. 3 for different values of the amplitude of the driving. We clearly observe the buildup of a resonance when the amplitude of the driving is progressively increased. The resonance appears in the regime of nonadiabatic driving ($2\omega \gtrsim \omega_v$), and a current reversal is observed on the low-frequency side of the resonance, in agreement with our simulations and with the general theory [6,8,11].

We examine now the dependence of the resonance on the scattering rate, i.e., the fluctuations' amplitude, with the aim to demonstrate that the observed resonance is, indeed, produced by the interplay of deterministic driving and fluctuations. Figure 4 shows our results for the average atomic velocity as a function of the driving frequency, at different values of the scattering rate. It clearly appears that the resonance amplitude shows a nonmonotonic dependence on the scattering rate: the resonance initially increases at increasing values of Γ', then reaches a maximum and starts decreasing at large values of the scattering
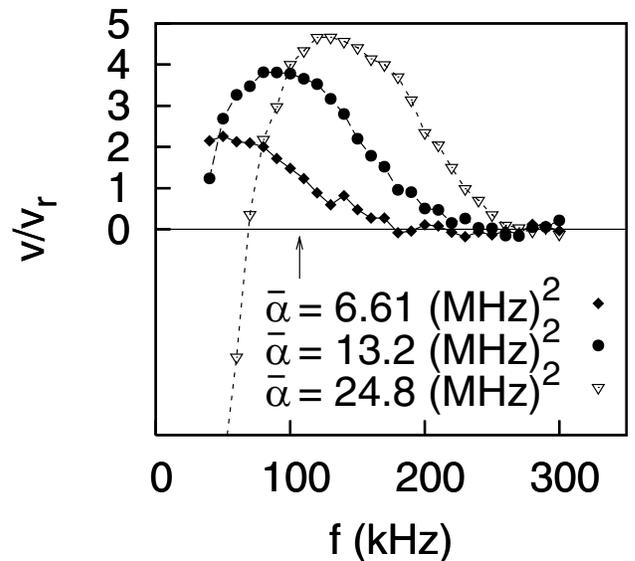

FIG. 3.   Experimental results for the average atomic velocity as a function of the driving frequency, for different amplitudes of the driving force. The optical potential is the same for all measurements and corresponds to a vibrational frequency $\omega_v = 2\pi \times 214$ kHz. The driving frequency satisfying the condition $2\omega = \omega_v$ is indicated by an arrow. The detuning Δ of the lattice from atomic resonance is $\Delta = 11.1\Gamma$, where Γ is the excited state linewidth. The values for the velocity are expressed in terms of the recoil velocity v_r, equal to 3.52 mm/s for the Cs D_2 line. The two harmonics of the force have equal amplitude: $A_1 = A_2 = 1$. The lines are guides for the eye.
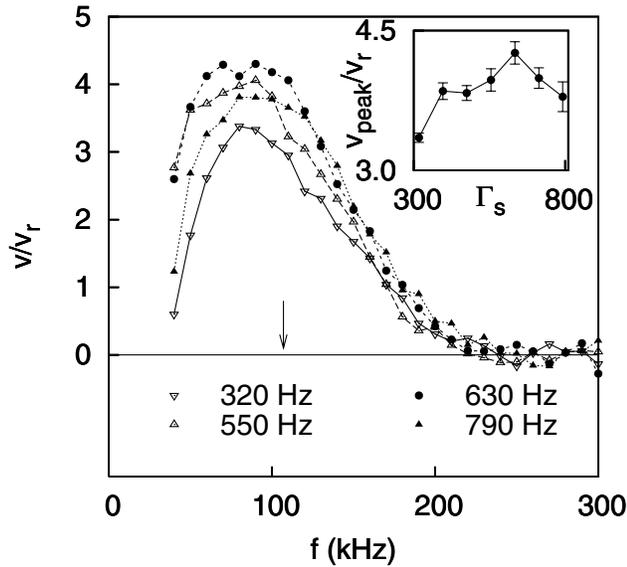
FIG. 4. Experimental results for the average atomic velocity as a function of the driving frequency, at different scattering rates. In the inset the resonance amplitude, i.e., the peak mean atomic velocity, is reported as a function of the scattering rate. The optical potential and the driving force amplitude are the same for all data sets, and they are characterized by $\omega_v = 2\pi \times 214$ kHz and $\bar{\alpha} = 13.3$ (MHz)^2. The driving frequency satisfying the condition $2\omega = \omega_v$ is indicated by an arrow. Different data sets correspond to different detunings Δ, i.e., to different scattering rates as the optical potential is kept constant. The data are labeled by the quantity $\Gamma_s = [\omega_v/(2\pi)]^2/\Delta$ which is proportional to the optical scattering rate. The two harmonics of the force have equal amplitudes: $A_1 = A_2 = 1$. The lines are guides for the eye.

rate. This is in agreement with our numerical results and clearly demonstrates that the resonance is determined by the interplay between deterministic driving and fluctuations and, due to the broken time symmetry, results in the rectification of the latter ones.

In conclusion, in this work we demonstrated the phenomenon of resonant activation in a nonadiabatically driven dissipative optical lattice with broken time symmetry. Because of the broken symmetry of the system, the resonant activation results in a resonance in the current of atoms through the periodic potential. We demonstrated that the resonance is produced by the interplay between deterministic driving and fluctuations, and we also showed that by changing the frequency of the driving it is possible to control the direction of the diffusion, as predicted by theoretical models. We notice that the rectification of fluctuations with nonadiabatically driven Brownian particles was already observed in previous work [14,15], but the resonant nature of the rectification mechanism was not demonstrated. Our work therefore also establishes experimentally the connection between resonant activation and

resonant rectification of fluctuations, and confirms the theoretical predictions. The present experimental realization, in which both deterministic and fluctuating forces originate from light fields, also shows the generality of the phenomenon of resonant activation, which is not restricted to systems in which the fluctuations are of thermal origin, as is usually considered in theoretical work.

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