Structure of a Quantized Vortex near the BCS-BEC Crossover in an Atomic Fermi Gas

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In order to clarify the structure of a singly quantized vortex in a superfluid fermion gas near the Feshbach resonance, we numerically solve the generalized Bogoliubov–de Gennes equation in the boson-fermion model. The superfluid gap, which contains contributions from both condensed fermion pairs and condensed bosons, is self-consistently determined, and the quasiparticle excitation levels bound in the vortex core are explicitly shown. We find that the boson condensate contributes to enhance the matter density depletion and the discreteness of localized quasiparticle spectrum inside the core. It is predicted that the matter density depletion and the discrete core levels are detectable in the vicinity of the BCS–Bose-Einstein condensation crossover point.

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The realization of superfluidity in the ultracold atomic Fermi gases [1,2] is a landmark achievement in the history of physics. The superfluid is achieved by utilizing a crossover to BCS superfluidity from Bose-Einstein condensation (BEC) of tightly bound molecules [3–6]. In these systems, the interaction causing BCS superfluidity or making a molecule from two fermion atoms originates from the Feshbach resonance [7] which is controllable by tuning an external magnetic field.

The clear-cut confirmation of the BCS-like pairing state in the superfluid Fermi gas has not yet been achieved. The observation of a quantized vortex will provide the most direct confirmation [8,9]. In this Letter, we theoretically investigate the single-vortex state of a superfluid Fermi gas near the Feshbach resonance on the basis of the generalized Bogoliubov–de Gennes (BdG) equation in the boson-fermion model [4–6].

Vortices in conventional BCS superconductors have so far been extensively studied [10–12]. The matter density of a vortex core is known to be little depleted in conventional weak-coupling BCS superconductors [13–16]. This is because the superfluid density is much less than the normal-fluid density; i.e., $E_F \gg \Delta$ (where E_F and Δ denote Fermi energy and superconducting gap, respectively). However, in strong-coupling superconductors the situation is different, because $\Delta \sim E_F$. As a result, the depletion of the superfluid density is expected to be large and detectable [9,13–15]. In fact, very recently vortices in a neutron star [17] and an atomic Fermi gas [9] have been discussed on the basis of this idea.

In the atomic Fermi gasses, the pairing character in the superfluid state changes from BCS-like to BEC-like when the threshold energy of the Feshbach resonance approaches the chemical potential [4–6]. Near this crossover region, the system goes into the strong-coupling limit. Then, since the superfluid is a mixture of BCS pairs and BEC molecules, the vortex structure is expected to be more complex than that in the single-component case [9]. Then one can raise the question of how the condensed bosons affect the vortex structure, that is, whether the depletion of the matter density inside the core is enhanced by the condensed bosons. To answer this, in this Letter we study the structure of a vortex in the boson-fermion model [4–6]. We numerically solve the generalized BdG equation in a fully self-consistent fashion [11] and obtain both fermionic and bosonic gap functions. Using the solutions, we clarify the density profile of both components in the single-vortex state as a function of the threshold energy from BCS to the crossover regime.

The fermionic quasiparticle excitations localized in a vortex core are well understood in the BCS weak-coupling case [10]. The lowest excitation energy and the spacing between the core levels are an order of $\Delta^2/E_F(\ll E_F)$ [10], which indicates that the quasiparticle spectrum is nearly gapless and continuous. Then the vortex core is regarded as the "normal core" [18]. However, such a picture does not make sense in the strong-coupling limit, since the separations between core levels become significant [13]. In this Letter, we calculate the energy spectrum of the fermionic quasiparticles in the single-vortex state and show that the separations between the core levels exceed $0.1E_F$ near the crossover point, which can be classified as an ultra clean vortex [19]. Such widely separated quasiparticle levels will be detectable by rf-tunneling spectroscopy [20]. The detection of the core levels also provides proof for the existence of quantized vortices.

In neutral fermionic superfluids the collective phase oscillation mode (the Goldstone mode) is known to appear below the fermion-pair excitation energy in the uniform state, i.e., twice the Meissner gap energy [21]. Since the vortex core levels are also situated below the Meissner gap, the core levels and the collective mode can couple with each other, which causes the damping of the core levels and the collective phase oscillation mode. This interaction is characteristic of the neutral fermionic superfluids. In this Letter, we briefly discuss this effect and show that the vortex solution in the present mean-field approximation is valid in the crossover regime.

The Hamiltonian of the boson-fermion model is given as [4–6]

$$H_{BF} = \int d\mathbf{r}\bigg[\psi^\dagger_\sigma(\mathbf{r})\Big(-\frac{1}{2m}\nabla^2 - \mu\Big)\psi_\sigma(\mathbf{r}) - U\psi^\dagger_\uparrow(\mathbf{r})\psi^\dagger_\downarrow(\mathbf{r})\psi_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r}) + \varphi^\dagger_B(\mathbf{r})\Big(-\frac{1}{4m}\nabla^2 + 2\nu - 2\mu\Big)\varphi_B(\mathbf{r}) + g[\varphi^\dagger_B(\mathbf{r})\psi_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r}) + \varphi_B(\mathbf{r})\psi^\dagger_\uparrow(\mathbf{r})\psi^\dagger_\downarrow(\mathbf{r})]\bigg], \tag{1}$$

where $\psi_\sigma(\mathbf{r})$ and $\varphi_B(\mathbf{r})$ are the field operators of the fermionic atoms with pseudospin $\sigma = \uparrow, \downarrow$ and the quasimolecular bosons, respectively, U is the BCS attractive interaction ($U > 0$), μ is the chemical potential, and 2ν is the threshold energy of the Feshbach resonance. The last term with the coupling constant g in Eq. (1) represents the process that a boson is created from two fermion atoms and vice versa due to the Feshbach resonance [4–6]. Note that the fermionic atoms are almost bound to two-atom molecules at $\nu = \mu$, whereas molecules and fermionic atoms can coexist when $\nu > \mu$ [4–6]. In the mean-field approximation, the gap functions are given by the vacuum expectation values as

$$\phi^B(\mathbf{r}) = g\langle\varphi_B(\mathbf{r})\rangle, \tag{2}$$

$$\Delta^F(\mathbf{r}) = U\langle\psi_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r})\rangle, \tag{3}$$

and the generalized BdG equation is derived from Eq. (1) as

$$\begin{pmatrix} H_\sigma & \Delta^F(\mathbf{r}) + \phi^B(\mathbf{r}) \\ \Delta^{F*}(\mathbf{r}) + \phi^{B*}(\mathbf{r}) & -H^*_\sigma \end{pmatrix}\begin{pmatrix} u_{n\sigma} \\ v_{n\sigma}\end{pmatrix} = E_n\begin{pmatrix} u_{n\sigma} \\ v_{n\sigma}\end{pmatrix}, \tag{4}$$

where $H_\sigma \equiv -\frac{1}{2m}\nabla^2 - \mu$. Similarly, an equation for $\phi^B(\mathbf{r})$ is obtained as

$$\bigg[-\frac{1}{4m}\nabla^2 + 2\nu - 2\mu\bigg]\phi^B(\mathbf{r}) = \frac{g^2}{U}\Delta^F(\mathbf{r}). \tag{5}$$

These equations are solved self-consistently together with the BCS gap equation [22],

$$\Delta^F(\mathbf{r}) = U\sum_n u_n(\mathbf{r})v^*_n(\mathbf{r}). \tag{6}$$

Let us study an isolated vortex in the 2D s-wave case [11] without a trapping potential. We note that the core states confined within the coherence length which is normally much less than the condensate diameter are not qualitatively affected by the trapping potential. By expressing the gap functions as $\Delta^F(\mathbf{r}) = \Delta^F(r)e^{-i\theta}$ and $\phi^B(\mathbf{r}) = \phi^B(r)e^{-i\theta}$ in the 2D cylindrical coordinates $\mathbf{r} = (r, \theta)$, the eigenfunctions $u_n(\mathbf{r})$ and $v_n(\mathbf{r})$ can be expanded as

$$\begin{aligned} u_{n,\eta}(\mathbf{r}) &= \sum_i c_{n,i}\phi_{i,\eta-1/2}(r)\exp[i(\eta - 1/2)\theta], \\ v_{n,\eta}(\mathbf{r}) &= \sum_i d_{n,i}\phi_{i,\eta+1/2}(r)\exp[i(\eta + 1/2)\theta], \end{aligned} \tag{7}$$

where $\phi_{i,m}(r) \equiv [\sqrt{2}/RJ_{m+1}(\alpha_{im})]J_m(\alpha_{im}r/R)$, $|\eta| = 1/2, 3/2, 5/2, \ldots$, and i is an positive integer ($\leq M$), depending on the value η [11]. Thus, the BdG equation can be solved as an eigenvalue problem for $2M(\eta) \times 2M(\eta)$ matrices [11]. The gap functions are expanded as $\phi^B(r) = \sum_i f_i\phi_{i1}(r)$ and $\Delta^F(r) = \sum_i g_i\phi_{i1}(r)$, and the expansion coefficients, f_i and g_i, are determined self-consistently from Eqs. (5) and (6) [15]. Furthermore, we have a constraint for the total number of particles, $N = N_F + 2N_B = \text{const}$, where $N_F = 2\int d\mathbf{r}\sum_i|v_i(\mathbf{r})|^2$ and $N_B = \int d\mathbf{r}\frac{|\phi^B(\mathbf{r})|^2}{g^2}$. The particle number N is kept constant during the iterative calculations in $\nu > \mu$. For a given value of ν, we determine μ from the constraint. Throughout this Letter, the energy and the radial distance are normalized by unit of E_F and $1/k_F$, respectively. The coupling constants are fixed to $U = 0.5$ and $g = 0.6$, and R is taken as $R = 40\frac{1}{k_F}$. We note that results of other cases are qualitatively the same as in the present one.

Let us now present self-consistent solutions for the single-vortex state. Figure 1(a) shows a solution for the gap functions $\Delta^F(r)$ and $\phi^B(r)$ in $\nu = 3.0$. The obtained chemical potential $\mu(= 0.9915)$ is slightly shifted from the value of the weak-coupling limit $\mu = E_F = 1$ in this self-consistent solution. The Bose condensate is not well developed, and then the BCS pairs mainly contribute to the superfluidity. Note that $\Delta^F \sim 0.1E_F$ outside the core, which indicates that the system belongs to the intermediate-coupling range. Therefore, the quantum oscillation, such as the Friedel oscillation, is also visible in the core region as in Refs. [13–16]. The gap functions depend strongly on the threshold energy when ν is close to μ, i.e., the resonant point. In Fig. 1(b), we plot the result at $\nu = 1.1$, where the chemical potential μ is reduced to a value of 0.621 [6]. As seen in this figure, ϕ^B grows and exceeds Δ^F. In this case, the spectrum of the localized quasiparticles strongly depends not only on $\Delta^F(r)$ but also on $\phi^B(r)$, since the off-diagonal component in Eq. (4) is given by their sum, $\Delta^F(r) + \phi^B(r)$, in the boson-fermion model [4–6]. It is found that the Friedel-oscillation-like behavior in the spatial dependence becomes obscure; that is, the spatial dependence of the gap functions becomes monotonic. Hence, one understands that the Bose condensate works to diminish the quantum character of the quasifermions. In Fig. 1(c), we plot the asymptotic values of the gap functions for $r \to \infty$, Δ^F_∞, and ϕ^B_∞ against ν. When ν approaches the resonant point ($\nu = 1$), ϕ^B_∞ exceeds Δ^F_∞ at a certain value of ν, and both Δ^F_∞ and ϕ^B_∞ are enhanced. These results reflect that the crossover takes place at the resonant point [6].

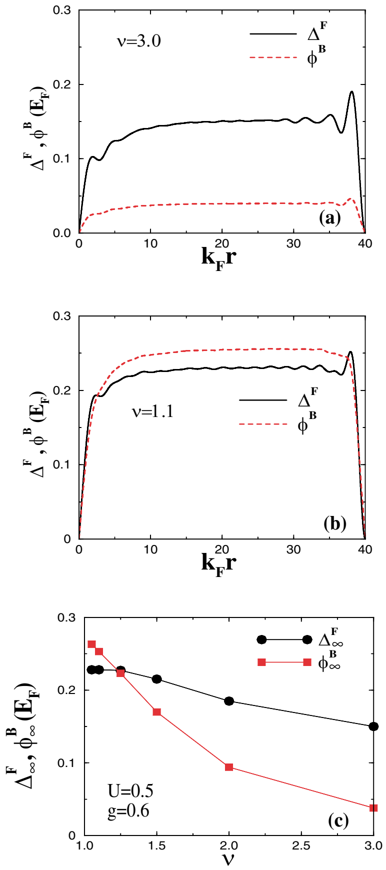

FIG. 1 (color online). The radial distribution of the superfluid gap functions, Δ^F and ϕ^B at (a) $\nu = 3.0$ and (b) $\nu = 1.1$. (c) The ν dependence of the asymptotic values of the gap functions, Δ_∞^F and ϕ_∞^B for $r \rightarrow \infty$.

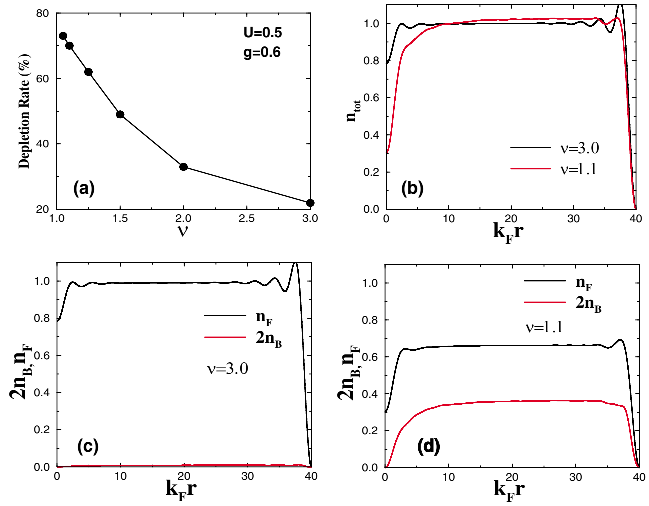

FIG. 2 (color online). (a) The ν dependence of the depletion rate (%). (b) The radial distribution of $n_{\rm tot}$ at $\nu = 3.0$ and at $\nu = 1.1$. The radial distributions for the components n_F and $2n_B$ are shown in (c) for $\nu = 3.0$ and in (d) for $\nu = 1.1$.

Let us next study the density profile near the vortex core. As was pointed out in Ref. [9], the observation of a vortex core is easier in a strong-coupling BCS superfluid since the depletion of matter density is relatively obvious. In this Letter, the depletion of the matter density in the vortex core is clarified as a function of ν. In order to quantify the depletion inside the core, we introduce the depletion rate defined as $[n_{\rm tot}^\infty - n_{\rm tot}(r = 0)]/n_{\rm tot}^\infty$, where $n_{\rm tot}(\mathbf{r}) = 2n_B(\mathbf{r}) + n_F(\mathbf{r})$. The bosonic and fermionic densities, $n_B(\mathbf{r})$ and $n_F(\mathbf{r})$, are calculated by the eigenfunctions of the generalized BdG equation as $n_B(\mathbf{r}) = |\phi^B(\mathbf{r})|^2/g^2$ and $n^F(\mathbf{r}) = 2\sum_i |v_i(\mathbf{r})|^2$. Figure 2(a) shows the ν dependence of the depletion rate in percentage terms (%). As seen in this figure, the depletion rate exceeds 70% when the threshold energy approaches the resonant point. The total matter density profile around the vortex in two cases are shown in Fig. 2(b). In $\nu = 3.0$, which is far from the resonant point, not very significant depletion is seen in the core region [16], while the depletion is clearly seen in $\nu = 1.1$. The total matter density can be split into two parts, i.e., the bosonic $n_B(r)$ and fermionic $n_F(r)$ ones. Figure 2(c) shows the result for $\nu = 3.0$. In this case, the density of the condensed bosons is negligibly small, and then the fermionic contribution is dominant for the core depletion. This result is consistent with that given in Refs. [9,13,15,16]. The results for $\nu = 1.1$ are presented in Fig. 2(d). The boson density is not small, which is sharply contrasted to the $\nu = 3$ case, and the depletion inside the core is found to be complete. We also notice that the depletion in the fermionic density is enhanced in the crossover region compared with that in $\nu = 3.0$. This is because the BCS coupling constant is effectively enhanced by the molecule formation and therefore the strong-coupling character becomes stronger [9]. From these results, one understands that a vortex is well visible in the density profile measurements in the vicinity of the resonant point, but it becomes obscure as the system departs from the resonant point [see Fig. 2(a)].

Now, let us focus on the excitation spectrum of the quasifermions in the single-vortex state [10]. In Fig. 3(a), the lowest two excitation energies, δE_1 and δE_2, are plotted as a function of ν (see also the right panel in which the relation between the core levels and the excitations is schematically shown). Note that δE_1 increases to a large value of about $0.15E_F$ for $\nu \rightarrow 1$. In Fig. 3(b), we plot the energy eigenvalues as a function of η in the case of $\nu = 3.0$. The low-lying branch of the spectrum represents the excitation levels bound in the vortex core. As seen in this figure, nearly continuous spectrum appears inside the Meissner gap. This quasiparticle spectrum is very similar to that in the weak-coupling BCS superconductors [11]. Near the BCS-BEC crossover, the core levels are drastically altered, especially in the low-energy region, as seen in Fig. 3(c). We notice, for example, that the lowest excitation energy becomes quite large, i.e., $\delta E_1 \sim 0.15E_F$, and also the distances between the other low-lying core levels are largely expanded. Then the spectrum cannot be considered as being continuous; that is, the vortex is not a normal core in this crossover regime.

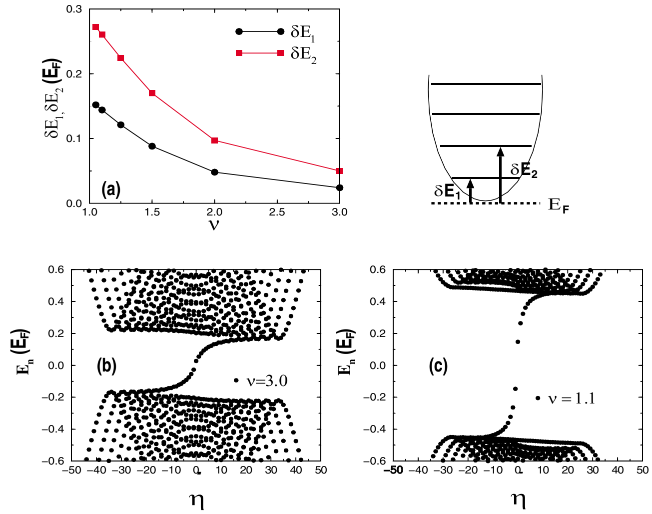

FIG. 3 (color online). (a) The ν dependence of the lowest two excitation energies, δE_1 and δE_2. The quasiparticle levels inside the core and the excitation energies, δE_1 and δE_2, are schematically shown in the right panel. The energy spectra of the quasifermions are plotted as a function of the angular quantum number η in (b) for $\nu = 3.0$ and in (c) for $\nu = 1.1$.

Finally, we briefly discuss the effect of collective excitations (the Goldstone mode) on the vortex core states. This effect arises from the self-energy correction to the fermion core levels. Note that the self-energy correction can be described in terms of the virtual processes that induce the transitions between core levels by emitting or absorbing the collective excitation modes, which causes both the shift and the broadening of the core levels. In the present system, the normal modes of the collective excitations can be labeled in terms of the integer 2D angular quantum numbers $n = 0, \pm 1, \pm 2, \ldots$, reflecting the cylindrical symmetry, and the collective mode with the quantum number n can cause transitions between the core states satisfying the selection rule for angular momentum, $\eta - \eta' = \pm n$. We can show on the basis of a random-phase-approximation–like calculation that the correction becomes large only if the energy of the collective mode satisfying the selection rule is close to one of the core level spacings. But the collective mode energy does not generally coincide with the level spacings, which indicates that the correction to the mean-field core levels is not significant, especially in the BEC-BCS crossover region, because the core levels are well separated in the crossover region. Thus, the present core levels are well justified.

In summary, we have studied microscopically the single-vortex state in the boson-fermion model for the first time. We clarified the variation of the vortex structure and the core excitation spectrum as a function of the threshold energy of the Feshbach resonance. Near the BCS-BEC crossover region, the strong-coupling BCS superfluid state, which includes the large number of condensed molecules, is realized. Then the quasiparticle levels bound in the vortex core are well separated, and the number of core levels is drastically reduced in this region. This strong-coupling effect forming the condensed molecules may be observed as the strong depletion of the matter density and the widely discrete density of states. These features are in marked contrast to those of the vortex core in conventional BCS superfluids.

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