Observation of the Anomalous Increase of the Longitudinal Energy Spread in a Space-Charge-Dominated Electron Beam

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We report a new experimental study of the growth of longitudinal energy spread in a space-charge-dominated electron beam, with a beam energy of several keV and beam current of approximately 100 mA. At relatively low beam densities, we measure growing energy spreads with distance along the transport channel, which are in remarkably good agreement with the theory of energy relaxation via Coulomb collisions. At higher beam densities, however, anomalous energy spreads exceeding the predictions of the relaxation theory are observed, which, we believe, could be caused by collective longitudinal-transverse instabilities observed in computer simulation studies. The onset of these instabilities occurs after several plasma periods according to calculations.

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Most beams of interest for advanced particle accelerator applications such as high-energy colliders, heavy-ion inertial fusion drivers, spallation neutron sources, and free electron lasers start in the space-charge-dominated regime, by which we mean that the space-charge term exceeds the emittance (pressure) term in the envelope equation. The physics of longitudinal beam energy spread in high-quality intense beams is very important, because a large beam energy spread degrades the beam quality. One of the mechanisms causing the growth of energy spread is temperature relaxation via Coulomb collisions. This occurs in a system with temperature anisotropy, which naturally happens in a beam accelerated in the longitudinal direction. The acceleration process decreases (cools) the longitudinal temperature of the beam dramatically by several orders of magnitude ($T_{\parallel} \ll T_{\perp}$) while the transverse temperature is kept roughly the same as in the cathode [1]. Coulomb collisions and other effects try to equilibrate this anisotropic state by transferring energy from the transverse direction into the longitudinal direction, thus causing an increase of the longitudinal beam temperature and energy spread. Several theoretical studies have been carried out on the longitudinal-transverse (LT) energy equipartitioning process due to the small angle Coulomb collisions [1–5]. The theory by Ichimaru and Rosenbluth for a nonrelativistic plasma with initially unequal longitudinal and transverse temperatures [2] was adopted for the theoretical studies in Ref. [1]. Knauer [3] predicted that the energy spread in the Boersch effect scaled with the beam current I, the distance L, and the beam radius a as $(IL/a)^{1/2}$, which is confirmed by our work. In their 1983 article, Rose and Spehr [4] present a thorough review and detailed references of the Boersch effect [6] and the various models to explain it as a collisional process, while Jansen's more recent book presents a comprehensive update of the field prior to 1993 [5]. Relaxation due to the Coulomb collisions is usually a slow process so that the theoretical distance to reach full equilibrium is much longer than the length of a linear accelerator, for instance. Equilibrium can be reached, however, in the many turns of high-energy proton rings, where the effect is known as intrabeam scattering (see Ref. [1], section 6.4.2 for a review). The relaxation process that we have investigated in our 2 m long transport experiment shows a significant longitudinal temperature increase in a very short distance. Recent simulations [7,8] also show that collective instabilities may expedite the energy equipartitioning process.

In addition to the relaxation between longitudinal and transverse directions, there is also a longitudinal-longitudinal (LL) relaxation effect inside the electron gun [9], which is due to the nonadiabatic acceleration in the gun. This LL relaxation process produces longitudinal energy spread, which is comparable to the LT effect in a distance of several plasma periods, and it is important to include it in the energy spread calculations. Taking into account both effects (LL and LT), the energy spread in a coasting beam after acceleration can be expressed as

$$\Delta\tilde{E}_{\parallel f} = \left(\frac{1}{\pi\varepsilon_0} q n^{1/3} q V_0 + 2qV_0 k_B T_{\parallel}\right)^{1/2}. \quad (1a)$$

Here, $\Delta\tilde{E}_{\parallel f}$ is the rms energy spread, qV_0 is the beam energy and $T_{\parallel}$ is an increasing function of time or distance of beam propagation, n is the beam density, and q is the electron charge. The first term in the brackets corresponds to the LL effect, which is a new feature in our experiments, and the second term to the LT effect. The longitudinal temperature $k_B T_{\parallel}$ in the second term is calculated by numerically solving the differential equation (6.149) in Ref. [1]. This equation is based on a collisional Fokker-Planck theory for a nonrelativistic electron beam, with the assumption that the magnetic field has no effect on the relaxation towards equilibrium (Larmor radius $r_L \gg$ Debye shielding length λ_D). After the beam has been transported for several plasma periods, the LT effect becomes dominant and the contribution from the LL effect can be ignored. In this approximation, the rms energy spread is given by

$$\Delta\tilde{E}_{\parallel f} = (2qV_0 k_B T_\parallel)^{1/2}, \tag{1b}$$

and one can derive from Eq. (6.154) of Ref. [1] an effective relaxation time $\tau_{\rm eff} = 1.34\tau_0$ given by

$$\frac{1}{\tau_{\rm eff}} = \frac{\pi^{3/2} n r_0^2 c}{1.34(k_B T_{\perp 0}/mc^2)^{3/2}} \ln\Lambda. \tag{2}$$

Here, n is the particle density, r_0 is the classical particle radius, defined as $r_0 = q^2/4\pi\varepsilon_0 mc^2 = qc/I_0$, where I_0 is the characteristic current, $\ln\Lambda$ is the Coulomb logarithm, $k_B T_{\perp 0}$ is the initial transverse temperature and related to the cathode temperature $k_B T_c$ by $k_B T_{\perp 0} = k_B T_c (r_c/a)^2$, where r_c is the cathode radius and a is the beam radius. In the region where the beam traveling time is much shorter than the relaxation time to reach full equilibrium, we can derive an approximate scaling law relating the energy spread and other beam parameters as

$$\Delta\tilde{E}_{\parallel f} = \left(\frac{2\ln\Lambda}{1.34} \frac{(mc^2)^{5/2}\pi^{1/2} r_0}{(k_B T_c)^{3/2} r_c} \frac{r_0 I L}{qca}\right)^{1/2}, \tag{3a}$$

$$\ln\frac{\Delta\tilde{E}_{\parallel f}}{\Delta E_{\parallel 0}} = \frac{1}{2}\ln\left(\frac{IL}{I_0 a}\right), \tag{3b}$$

where we used $r_0/(qc) = 1/I_0$. $\Delta\tilde{E}_{\parallel f}$ is the rms energy spread, $\Delta E_{\parallel 0}$ is a normalization constant defined as $\Delta E_{\parallel 0} = (\frac{2\ln\Lambda}{1.34} \frac{(mc^2)^{5/2}\pi^{1/2} r_0}{(k_B T_c)^{3/2} r_c})^{1/2}$, I is the beam current, a is the effective beam radius in the long solenoid, and L is the traveling distance. From this scaling law, the logarithm of the energy spread due to the LT relaxation is linearly proportional to the logarithm of $(I/I_0)(L/a)$, with a slope of 0.5, which agrees with Knauer's scaling mentioned above.

Hyatt and Beck [10,11] reported experimental measurements of the anisotropic temperature relaxation in a stationary, magnetically confined electron plasma, where they found good agreement between the experimental results and the small-momentum-transfer collision theory. In this Letter, we report a new experimental study of the energy relaxation in low energy, space-charge-dominated electron beams, the results of which would have direct applications to the intense beams for various advanced accelerator applications. The good agreement between the experimental results and the theoretical model has not been achieved in any other experiment, to the best of our knowledge (see, for instance, Knauer's comparison in Fig. 6 of his article [3]).

A thermionic gridded electron gun [12] is utilized to generate low energy space-charge-dominated electron beams, with beam energies of several keV, and beam currents in the 100 mA range. The pulse length of the beam is about 100 ns. One key diagnostic tool is a high-resolution energy analyzer, with resolution of 0.2 eV for a 10 keV beam. Details of the design and operation of this analyzer are described elsewhere [13–15]. The analyzer is able to measure the time-resolved energy spread from the beam head to tail. In this Letter, we focus on the energy spread in the flattop of the beam. Two sets of experiments were conducted, with the energy analyzer located at two different locations. The first initial set of measurements was done at a distance of 25 cm away from the electron gun; the results have been reported elsewhere [16]. At this short distance, the contributions of the energy spread from longitudinal-transverse and longitudinal-longitudinal relaxations are comparable, and the scaling law of Eq. (1b) does not apply. The second set of experiments reported in this Letter was performed in an extended system of about 2 m in total length, as shown in Fig. 1, by adding a long solenoid (M4 in Fig. 1) of about 1.2 m in length as the main transport channel. Between the electron gun and the long solenoid, three short solenoids (M1, M2, and M3 in Fig. 1) are employed for matching the beam into the entrance of the long solenoid. The energy analyzer is located at a distance of 2.3 m from the electron gun. The calculated envelope for a 5 keV, 135 mA beam matched into the long solenoid with a radius of 8 mm is shown in Fig. 2.

For comparison, a typical energy spectrum from the previous measurement [16] for a 5 keV, 135 mA beam, measured 25 cm away from the electron gun is shown in Fig. 3(a). In this figure, the measured rms energy spread is 2.2 eV and the full width at half maximum (FWHM) is 3.9 eV. It is interesting to note that the ratio of the FWHM to rms energy spread in this spectrum is only about 1.8, narrower than the usually assumed Gaussian distribution. Figure 3(b) plots a measured beam energy spectrum in the new long system at the entrance of the energy analyzer (2.3 m from the electron gun), with the same beam parameters as in Fig. 3(a). Here, the measured rms energy spread is 4.0 eV and the FWHM is 8.3 eV. Compared with Fig. 3(a), we can see that, after 2.3 m of propagation, the beam energy spread grows by almost a factor of 2 and, more importantly, the energy spectrum is closer to a Gaussian distribution (the ratio is 2.1 at 2.3 m). This increased energy spread is mainly due to the energy transfer from the transverse direction into the longitudinal direction via Coulomb collisions. The relaxation of the energy spectrum into a Gaussian distribution is probably due to the statistical effects of a large number of scatterings between beam particles.

In order to study the scaling law of this energy relaxation between the longitudinal and transverse directions, we carried out more measurements at different beam parameters. We measured the energy spreads at beam energies of 3, 4, and 5 keV, and beam currents of 70, 100, and 135 mA, respectively. For each case, we also changed the beam

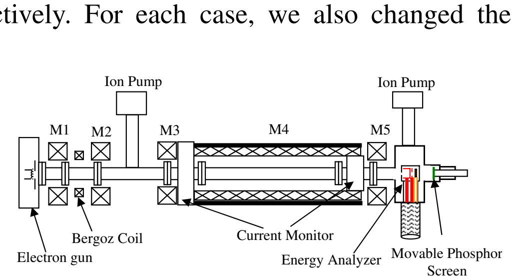

FIG. 1 (color online). Schematic of the experiment.

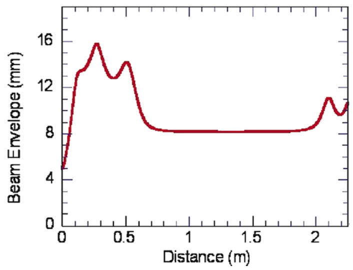

FIG. 2 (color online). The matched beam envelope for a 5 keV, 135 mA beam.

density, and hence the beam radius, inside the long solenoid by adjusting the focusing strength of the long solenoid and the focusing strengths of the three short solenoids to match the beam into the long solenoid. Figures 4(a)–4(c) depict the results for this experiment. In each figure, the beam energy spread is plotted against the beam radius for the 3 cases. The solid circles with error bars are the energy spreads measured from experiments and the triangles are the theoretical predictions from Eq. (1b). In the figure, we can see that the beam energy spread is higher with higher beam densities (smaller beam radius). This is because of the fact that the Coulomb collisions are stronger for higher beam density. The figures also show that the agreement between the experiment and the theory is remarkably good, especially at relatively large beam radii, hence low beam densities. At these points, the average difference between the experimental measurements and theoretical predictions is about 2%, within the experimental error bars. The error bars shown in the figure are determined based on the statistical errors observed during the measurements. The systematic error cannot be measured directly in the experiment, but, according to the theoretical error analysis of the analyzer, it should be smaller than 0.2 eV [17].

As discussed in the previous text, after the beams have been transported over several plasma periods, the energy spread due to the longitudinal-transverse relaxations becomes dominant. In this experiment, the distance of 2.3 m corresponds to about 4–12 plasma periods. From the scaling law in Eq. (3b), if the beam energy spread is due to the longitudinal-transverse relaxation, the beam energy spread is linearly proportional to the ratio of the beam current to the beam radius. In Fig. 5, we replot Fig. 4 so that the logarithm of the beam energy spread is plotted against the

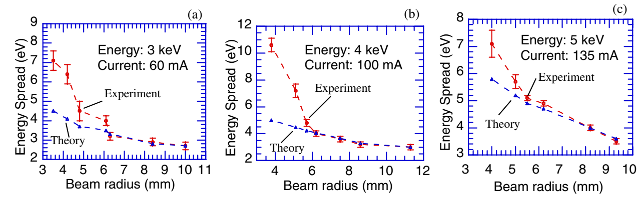

FIG. 4 (color online). Measured energy spread and theoretical predictions. (a) The 3 keV beam. (b) The 4 keV beam. (c) 5 keV beam.

logarithm of I/a. For clarity, theoretical calculations and error bars are not plotted. From this figure, we can see that the experimental points measured at relatively low beam densities (small I/a) are lying around a common linear base line. The straight line in the figure is the linear fit of those data points, with a slope of 0.46 and a correlation coefficient of 0.98. This indicates a good linear relationship between the logarithms of the beam energy spread and I/a at those points. More importantly, the measured slope of 0.46 is very close to the predicted slope of 0.5 from the scaling law in Eq. (3), which indicates that the measured energy spread is, indeed, due to the energy relaxation between the transverse and longitudinal directions. This plot, we believe, is the first direct experimental result that measures the growth of the beam energy spread due to the longitudinal-transverse relaxation via intrabeam scattering in a space-charge-dominated beam.

We also observed that at relatively high beam densities the measured increase of the beam energy spread starts to deviate from the linear base line and is always larger than the theoretical calculations from Eq. (1a), as shown in Figs. 4 and 5. This anomalous growth, in our opinion, could be caused by the collective instability discussed above. From Fig. 5, it is apparent that the increase of the energy spread starts to follow a different scaling law when I/a reaches a threshold. This threshold, or onset of the instability, occurs at a radius of about 6.2 mm, independent of beam current and voltage for the three cases in our experiment, as seen in Fig. 4. In order to understand the scaling governing this anomalous effect, the following analysis was performed. The matched beam envelope in the uniform magnetic field of the long solenoid is obtained

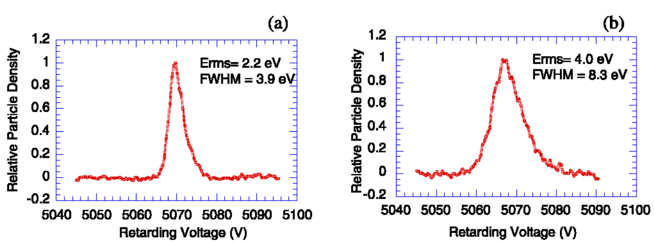

FIG. 3 (color online). Beam energy spectrums: (a) at location of 25 cm; (b) at location of 2.3 m.

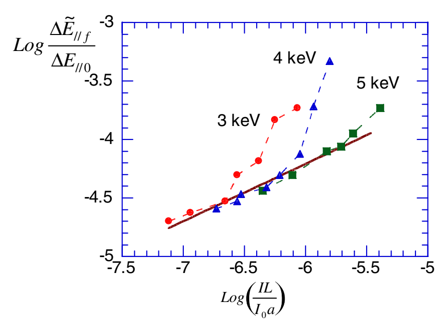

FIG. 5 (color online). Measured beam energy spread and its linear curve fit.

TABLE I. Parameter values for the three cases.

V_0 (kV)	1 (mA)	K	ε (μm)	k_0 (m^{-1})	χ	ν/ν_0	ν_p/ν_0	λ_p (m)
3	70	6.45×10^{-3}	90.77	13.17	0.968	0.179	1.391	0.343
4	100	5.99×10^{-3}	78.57	12.65	0.974	0.162	1.396	0.356
5	135	5.78×10^{-3}	70.24	12.40	0.978	0.147	1.399	0.362

from Eq. (6.2) in Ref. [1] in the alternate form $1 = \frac{K}{k_0^2 a^2} + \frac{\varepsilon^2}{k_0^2 a^4}$. Here, a is the beam radius, K is the generalized perveance, which is defined in Eq. (4.29c) of Ref. [1] for nonrelativistic electrons, k_0 is the wave number of the betatron oscillations without space charge, and λ_0 the wavelength of the betatron oscillations without space charge, and ε is the unnormalized ($4 \times$ rms) emittance. The first term in this equation defines the intensity parameter $\chi = K/k_0^2 a^2$, which is used to calculate the tune depression of the betatron oscillations, $k/k_0 = \nu/\nu_0 = (1 - \chi)^{1/2}$, and of the plasma oscillations, $k_p/k_0 = \nu_p/\nu_0 = (2\chi)^{1/2}$ [18]. The plasma frequency, $\omega_p = (e^2 n/\varepsilon_0 m)^{1/2}$, can be expressed in terms of the generalized perveance, as $\omega_p = (2K)^{1/2}\nu/a$ [see Eq. (4.24) in Ref. [1]] and the plasma wavelength as $\lambda_p = 2\pi\nu/\omega_p = 2\pi a/(2K)^{1/2}$. We note that Davidson independently defined a parameter that is identical to our χ [19]. From Fig. 4, we can see that the onset of the instability occurs at a beam radius of $a = 6.2$ mm for all three cases. We can therefore calculate the wave number without the space charge, k_0, the intensity parameter χ, and the tune depressions. Table I shows the experimental and calculated parameter values.

This is obviously a highly space-charge-dominated beam with about 3–4 plasma periods in the long solenoid to launch the anomalous energy spread, or instability.

We are planning an investigation, both theoretical and experimental, to understand more about the anomalous growth of longitudinal energy spread, which, we believe, is caused by the instability, and we will try to reproduce it by simulation studies.

In summary, we report in this Letter detailed energy spread measurements, which confirm the $(IL/a)^{1/2}$ scaling predicted for Coulomb collisions for parallel beam transport in matched solenoidal focusing. We have presented an alternative derivation leading to this scaling law, which is more thermodynamically based and better elucidates this scaling. We also observed abnormal growth of the energy spread at high beam densities, which, in our opinion, might be attributed to the longitudinal-transverse collective instabilities observed in the previous simulation studies [7,8]. The observations do not appear to support Knauer's potential energy conversion mechanism (2/3 power dependency on the beam current), as discussed on page 15 of Ref. [5]. Our results should also be of interest to the designers of high intensity linear accelerators and of high-energy physics rings, where the Boersch effect is known as intrabeam scattering.

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