Dual Meissner Effect and Magnetic Displacement Currents

Tsuneo Suzuki, Katsuya Ishiguro, Yoshihiro Mori, and Toru Sekido

Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan
RIKEN, Radiation Laboratory, Wako 351-0158, Japan
(Received 1 October 2004; published 7 April 2005)

The dual Meissner effect is observed without monopoles in quenched $SU(2)$ QCD with Landau gauge fixing. Magnetic displacement currents that are time-dependent Abelian magnetic fields act as solenoidal currents squeezing Abelian electric fields. Monopoles are not always necessary for the dual Meissner effect. A mean-field calculation suggests that the dual Meissner effect through the mass generation of the Abelian electric field is related to a gluon condensate $\langle A_\mu^a A_\mu^a \rangle \neq 0$ of mass dimension 2.

The understanding of the color confinement mechanism is an important problem that is yet to be solved. It is believed that the dual Meissner effect is the mechanism [1,2]. However, the cause of the dual Meissner effect has not yet been clarified. A possible cause is the appearance of magnetic monopoles after projecting $SU(3)$ QCD to an Abelian $U(1)^2$ theory by partial gauge fixing [3]. If such monopoles condense, the dual Meissner effect could explain the color confinement. In fact, an Abelian projection adopting a special gauge called maximally Abelian gauge (MA) [4,5] leads us to interesting results [6,7] that support importance of monopoles.

Now, the following question arises: What happens when other general Abelian projections are adopted or when no Abelian projection is adopted? For example, consider an Abelian projection diagonalizing Polyakov loops. Monopoles exist in the continuum limit at a point where eigenvalues of Polyakov loops are degenerate [3]. However, it can be easily shown that such a point moves only in a timelike direction. In other words, there exist only timelike monopoles that do not contribute to the string tension [8]. Let us discuss another simple case of the Landau gauge. Vacuum configurations are smooth and monopoles arising from singularities do not exist. In these cases, monopole condensation does not occur. We have to find a more general confinement mechanism that is realized in QCD.

This study shows that the dual Meissner effect in an Abelian sense works well even when monopoles do not exist. Monte Carlo simulations of quenched $SU(2)$ QCD in the Landau gauge are adopted. Instead of monopoles, time-dependent Abelian magnetic fields regarded as magnetic displacement currents squeeze the Abelian electric fields. The dual Meissner effect implies the dual London equation and mass generation of the Abelian electric fields that may be related to the existence of a dimension 2 gluon condensate. The present numerical results are not perfect since the continuum limit, infinite-volume limit, and gauge-independence have not yet been examined. These discussions use the Abelian components based only on the assumption that Abelian components are dominant in infrared QCD (Abelian dominance [6,7,9,10]); however, this assumption has not yet been clarified. Nevertheless, the authors believe that the results obtained here are very interesting, since they show for the first time that the Abelian dual Meissner effect works in lattice non-Abelian QCD without requiring monopoles from a singular gauge transformation [11]. The gauge adopted here is the simple one and is used only for the purpose of obtaining smooth configurations. Hence, these results suggest that the Abelian dual Meissner effect is the actual universal mechanism of color confinement. Moreover, the probable relation between the Abelian dual Meissner effect and the dimension 2 gluon condensate sheds new light on the importance of the gluon condensate [12–18].

We use the following improved gluonic action found by Iwasaki [19] as a lattice $SU(2)$ QCD:

$$S = \beta\{c_0 \sum \mathrm{Tr(plaquette)} + c_1 \sum \mathrm{Tr(rectangular)}\},$$

which enables us to obtain better scaling behaviors of physical quantities. The mixing parameters are fixed as $c_0 + 8c_1 = 1$ and $c_1 = -0.331$.

In order to directly measure the correlations of gauge-variant electric and magnetic fields, we adopt the simplest gauge, called Landau gauge, which maximizes $\sum_{s,\mu} \mathrm{Tr}[U_\mu(s) + U_\mu^\dagger(s)]$. After the gauge fixing, we attempt to measure electric and magnetic flux distributions by evaluating correlations of Wilson loops and field strengths. For comparison, we also discuss the MA gauge where $\sum_{s,\mu} \mathrm{Tr}[U_\mu(s)\sigma_3 U_\mu^\dagger(s)\sigma_3]$ is maximized. In order to obtain a good signal to noise ratio, the APE smearing technique [20] is used when evaluating the Wilson loops $W(R, T) = W^0 + iW^a\sigma^a$.

The measurements of the string tension are used to fix the scale when we use $\sqrt{\sigma} = 440$ [MeV]. We adopt a coupling constant $\beta = 1.2$ in which the lattice distance $a(\beta = 1.2)$ is 0.0792(2) [fm]. This lattice distance is chosen based on the comparison between our results and those of the MA gauge [21,22]. The lattice size is 32^4 and after 5000 thermalizations, we have taken 5000 thermalized configurations per 100 sweeps for measurements.
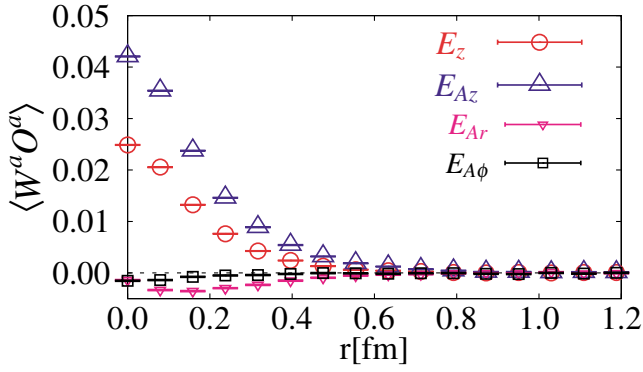
FIG. 1 (color online). Abelian $\vec{E}_A$ and non-Abelian $\vec{E}$ electric field profiles in Landau gauge. $W(R \times T = 6 \times 6)$ is used.

Non-Abelian electric and magnetic fields are defined from 1×1 plaquette $U_{\mu\nu}(s) = U^0_{\mu\nu} + iU^a_{\mu\nu}\sigma^a$, similar to Ref. [23]:

$$E^a_k(s) \equiv \frac{1}{2}[U^a_{4k}(s - \hat{k}) + U^a_{4k}(s)],$$

$$B^a_k(s) \equiv \frac{1}{8}\epsilon_{klm}[U^a_{lm}(s - \hat{l} - \hat{m}) + U^a_{lm}(s - \hat{l})$$
$$+ U^a_{lm}(s - \hat{m}) + U^a_{lm}(s)].$$

We also define the Abelian electric (E^a_{Ai}) and magnetic fields (B^a_{Ai}) in a similar manner using the Abelian pla-quettes $\theta^a_{\mu\nu}(s)$ defined with the link variables $\theta^a_\mu(s)$:

$$\theta^a_{\mu\nu}(s) \equiv \theta^a_\mu(s) + \theta^a_\nu(s + \hat{\mu}) - \theta^a_\mu(s + \hat{\nu}) - \theta^a_\nu(s), \quad (1)$$

where $\theta^a_\mu(s)$ is given by $U_\mu(s) = \exp(i\theta^a_\mu(s)\sigma^a)$. In the MA gauge, the Abelian link variable $\theta^{MA}_\mu(s)$ is defined by a phase of the diagonal part of a non-Abelian link field:

$$U^0_\mu(s) = \sqrt{1 - |c_\mu(s)|^2}\cos\theta^{MA}_\mu(s),$$

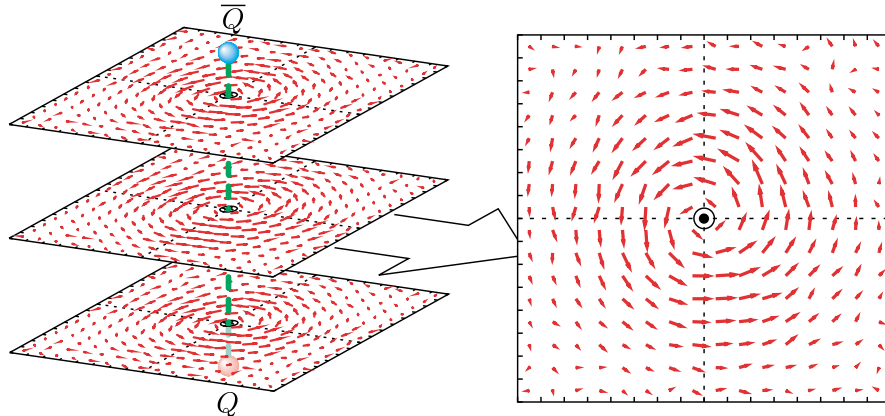$$U^3_\mu(s) = \sqrt{1 - |c_\mu(s)|^2}\sin\theta^{MA}_\mu(s).$$

Since the off-diagonal part $|c_\mu(s)|$ is small [6], $\theta^{MA}_\mu(s) \sim \theta^3_\mu(s)$ in MA gauge. As a source corresponding to a static quark and antiquark pair, we adopt only non-Abelian Wilson loops in this study.

First, we show Abelian and non-Abelian electric field profiles around a quark pair in the Landau gauge in Fig. 1. The profiles are studied mainly on a perpendicular plane at the midpoint between the two quarks. It must be noted that the electric fields perpendicular to the $Q\bar{Q}$ axis are found to be negligible. It is very interesting to observe from Fig. 1 that the Abelian electric field E_{Az} defined here is also squeezed although short-range Coulombic contributions are different [24]. It is, thus, essential to ascertain the reason for the squeezing of the Abelian flux.

Let us now discuss only the flux distributions of Abelian fields. It is numerically verified [25] that no DeGrand-Toussaint monopoles [26] are present. Hence, the Abelian fields satisfy the simple Abelian Bianchi identity kinematically, as demonstrated below:

$$\vec{\nabla} \times \vec{E}^a_A = \partial_4\vec{B}^a_A, \qquad \vec{\nabla} \cdot \vec{B}^a_A = 0. \quad (2)$$

In the case of MA gauge, there exist additional monopole current $(\vec{k}, k_4)$ contributions:

$$\vec{\nabla} \times \vec{E}^{MA} = \partial_4\vec{B}^{MA} + \vec{k}, \qquad \vec{\nabla} \cdot \vec{B}^{MA} = k_4. \quad (3)$$

Here, $\vec{E}^{MA}$ and $\vec{B}^{MA}$ are defined in terms of plaquette variables $\theta^{MA}_{\mu\nu}(s)$ (mod 2π) that are constructed with $\theta^{MA}_\mu(s)$.

The Coulombic electric field arising from the static source is expressed in terms of the gradient of a scalar potential in the lowest perturbation theory. Hence, it contributes neither to the curl of the Abelian electric field nor to the Abelian magnetic field in the Abelian Bianchi iden-tity, Eq. (2). According to the dual Meissner effect, the squeezing of the electric flux occurs due to the cancellation of Coulombic electric fields and those due to the solenoidal magnetic currents. In the case of the MA gauge, magnetic
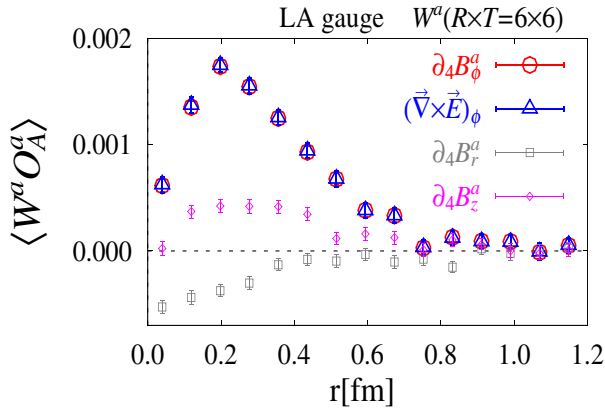

FIG. 2 (color online). Magnetic displacement currents in Landau gauge as a solenoidal current.

FIG. 3 (color online).   Curl of Abelian electric fields and magnetic displacement currents around a static quark pair in Landau gauge.
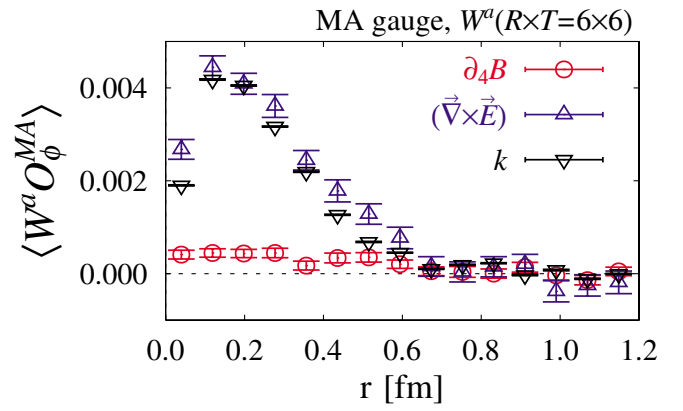

FIG. 4 (color online).   Curl of Abelian electric fields, monopole currents, and magnetic displacement currents around a static quark pair in MA gauge.

monopole currents $\vec{k}$ serve as the solenoidal current [21,22,27].

What happens in a smooth gauge like the Landau gauge where monopoles do not exist? It can be seen from Eq. (2) that only $\partial_4 \vec{B}_A$, which is regarded as a magnetic displacement current, can play the role of the solenoidal current. It is very interesting to see Fig. 2 that demonstrates the occurrence of this phenomenon in the Landau gauge. Note that the solenoidal current is in a direction that squeezes the Coulombic electric field. Let us also see the R dependence shown in Fig. 3. The components of the magnetic displacement current $\partial_4 B_{Ar}$ and $\partial_4 B_{Az}$ do not vanish; however, they are extremely suppressed. In comparison, we present the case of the MA gauge in Fig. 4. Here, $\partial_4 B_{A\phi}$ is found to be numerically negligible as already expected from the cited literature [10,21,27]. Instead monopole currents are found to circulate [21,22,27]. In this case, k_r is nonvanishing, although it is also suppressed in comparison with k_ϕ; k_z is almost zero. The authors believe that the nonvanishing of the radial and z components of $\partial_4 \vec{B}$ in the Landau gauge and k_r in the MA gauge is due to lattice artifacts and the small size of the Wilson loop used here. It is interesting that the shapes of $\partial_4 B_{A\phi}$ in the Landau gauge and k_ϕ in the MA gauge appear to be similar even though the strengths are different. These shapes have a peak at almost the same distance at approximately 0.2 [fm] and almost vanish at approximately 0.7 [fm].

It can be speculated whether the above consideration of Abelian fields is sufficient to understand the nonperturbative confinement problem in infrared QCD in terms of Abelian quantities [6,7,28]. Let us attempt to verify the Abelian dominance in the Landau gauge using a controlled cooling [29] under which the string tension remains nonvanishing. It is found that the profile of the non-Abelian electric field E_z^a tends to that of the Abelian one of E_{Az}^a in the long-range region, as shown in Fig. 5. This is consistent with the previous result [10] using a different approach.

It has been shown that the magnetic displacement currents are important in the dual Meissner effect when there are no monopoles. In such a case, how can the origin of the dual Meissner effect without monopole condensation be understood? The Abelian dual Meissner effect indicates the massiveness of the Abelian electric field as an asymptotic field:

$$(\partial_\rho^2 - m^2)\vec{E}_A \sim 0. \tag{4}$$

This leads us to a dual London equation that is a key to the dual Meissner effect. Let us evaluate the curl of the magnetic displacement current. Using Eq. (2), we obtain

$$\vec{\nabla} \times \partial_4 \vec{B}_A = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}_A) - \vec{\nabla}^2 \vec{E}_A.$$

From Eq. (4), we obtain the dual London equation:

$$\vec{\nabla} \times \partial_4 \vec{B}_A \sim (\partial_4^2 - m^2)\vec{E}_A. \tag{5}$$

Let us use the simple mean-field approach developed by Fukuda [30]. By neglecting the gauge-fixing and Faddeev-Popov terms, we obtain the equation of motion
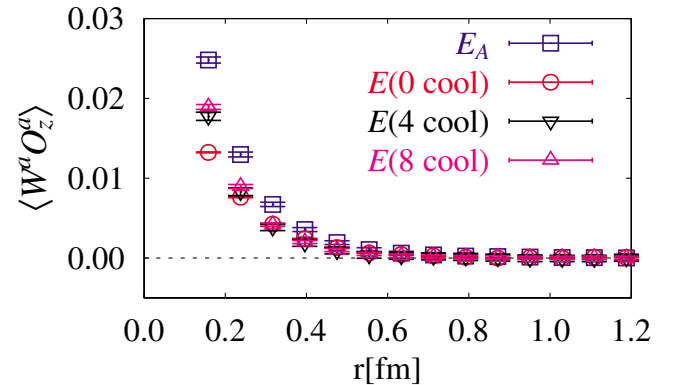

FIG. 5 (color online).   Profiles of non-Abelian electric field E_z^a after cooling. E_{Az}^a represents the Abelian electric field. $W(R \times T = 6 \times 6)$ is used.

PHYSICAL REVIEW LETTERS

$D_\mu^{ab} F_{\mu\nu}^b = 0$ and the (non-Abelian) Bianchi identity $D_\mu^{ab*} F_{\mu\nu}^b = 0$. By applying D operator to the Bianchi identity and using the Jacobi identity and the equations of motion, we obtain $(D_\rho^2)^{ab} F_{\mu\nu}^b = 2g \epsilon^{abc} F_{\mu\alpha}^b F_{\nu\alpha}^c$. Note that $(D_\rho^2)^{ab} = \partial_\rho^2 \delta^{ab} + g\epsilon^{acb}(\partial_\rho A_\rho^c) + g^2(A_\rho^a A_\rho^b - \delta^{ab}(A_\rho^c)^2)$. Hence, if $\langle A_\mu^a A_\nu^b \rangle = \delta^{ab} \delta_{\mu\nu} v^2 \neq 0$, we see asymptotically that the electric fields become massive $(\partial_\rho^2 - m^2)E_k^a \sim 0$ with $m^2 = 8g^2 v^2$. Now, the Abelian electric field is also massive asymptotically $(\partial_\rho^2 - m^2)E_{Ak}^a \sim 0$. Hence, the dual London Eq. (5) is obtained.

The importance of the dimension 2 gluon condensate has been stressed by Zakharov and his collaborators [12] and also by many authors in Refs. [13,14]. Recent discussions on the value of the gluon condensate can be seen in Ref. [15]. Some discuss the mass generation of the gluon propagator [31]. However, it is found [18] that the gluon propagator has no on-mass-shell poles in the presence of the dimension 2 gluon condensate. This does not directly imply that the electric field propagator has no on-mass-shell pole since a gluon field $\vec{A}$ and an electric field $\vec{E}$ are different canonical variables [30]. The lattice study of gluon propagators in the Landau gauge demonstrates the importance of the dimension 2 condensate and its relevance to instantons [16]. Although the operator of the gluon condensate is gauge variant, there have been various discussions concerning gauge-invariant contents in the expectation value of the operator [12–14,17].

[1] G. 't Hooft, in *Proceedings of the EPS International Conference*, edited by A. Zichichi (Editorice Compositori, Bologna, 1976), p. 1225.
[2] S. Mandelstam, Phys. Rep. **23C**, 245 (1976).
[3] G. 't Hooft, Nucl. Phys. B **190**, 455 (1981).
[4] T. Suzuki, Prog. Theor. Phys. **69**, 1827 (1983).
[5] A. S. Kronfeld, M. L. Laursen, G. Schierholz, and U. J. Wiese, Phys. Lett. B **198**, 516 (1987); A. S. Kronfeld, G. Schierholz, and U. J. Wiese, Nucl. Phys. B **293**, 461 (1987).
[6] T. Suzuki and I. Yotsuyanagi, Phys. Rev. D **42**, R4257 (1990).
[7] T. Suzuki, Nucl. Phys. B, Proc. Suppl. **30**, 176 (1993); M. N. Chernodub and M. I. Polikarpov, in *Confinement, Duality, and Nonperturbative Aspects of QCD*, edited by P. van Baal (Plenum Press, New York, 1997), p. 387.
[8] M. N. Chernodub, Phys. Rev. D **69**, 094504 (2004).
[9] J. Giedt and J. Greensite, Phys. Rev. D **55**, 4484 (1997).
[10] P. Cea and L. Cosmai, Phys. Rev. D **52**, 5152 (1995).
[11] The dual London equation was discussed gauge invariantly with a definition of a non-Abelian monopole current in P. Skala, M. Faber, and M. Zach, Nucl. Phys. B **494**, 293 (1997); Phys. Lett. B **424**, 355 (1998).
[12] L. Stodolsky, Pierre van Baal, and V. I. Zakharov, Phys. Lett. B **552**, 214 (2003), and references therein.
[13] K-I. Kondo, Phys. Lett. B **514**, 335 (2001); Phys. Lett. B **572**, 210 (2003).
[14] D. Dudal *et al.*, J. High Energy Phys. 01 (2004) 044, and references therein.
[15] E. R. Arriola, P. O. Bowman, and W. Broniowski, Phys. Rev. D **70**, 097505 (2004).
[16] P. Boucaud *et al.*, Phys. Rev. D **66**, 034504 (2002).
[17] A. A. Slavnov, hep-th/0407194.
[18] X. Li and C. M. Shakin, hep-ph/0411234, and references therein.
[19] Y. Iwasaki, Nucl. Phys. B **258**, 141 (1985); Univ. Tsukuba Report No. UTHEP-118, 1983 (to be published).
[20] APE Collaboration, M. Albanese *et al.*, Phys. Lett. B **192**, 163 (1987).
[21] G. S. Bali, V. Bornyakov, M. Muller-Preussker, and K. Schilling, Phys. Rev. D **54**, 2863 (1996).
[22] Y. Koma, M. Koma, E.-M. Ilgenfritz, T. Suzuki, and M. I. Polikarpov, Phys. Rev. D **68**, 094018 (2003).
[23] G. S. Bali, K. Schilling, and Ch. Schlichter, Phys. Rev. D **51**, 5165 (1995).
[24] The gluon mass determined from Fig. 1 is roughly 1.35 [GeV]. However, the distance R between the quark pair in Fig. 1 is only $R = 0.48$ [fm] and it is too short to correctly fix the penetration length and the gluon mass in the confinement force. In fact, the penetration length observed increases up to $R = 0.8$ [fm] which is consistent with the results in Ref. [23]. The details will be published elsewhere.
[25] T. Suzuki *et al.*, hep-lat/0410039.
[26] T. A. DeGrand and D. Toussaint, Phys. Rev. D **22**, 2478 (1980).
[27] V. Singh, D. A. Browne, and R. W. Haymaker, Phys. Lett. B **306**, 115 (1993).
[28] K-I. Kondo, Phys. Lett. B **600**, 287 (2004).
[29] M. Campostrini *et al.*, Phys. Lett. B **225**, 403 (1989).
[30] R. Fukuda, Prog. Theor. Phys. **67**, 648 (1982); **67**, 655 (1982).
[31] J. A. Gracey, Eur. Phys. J. C **39**, 61 (2005).