Helicity Conservation in Gauge Boson Scattering at High Energy

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We remark that the high energy gauge boson scattering processes involving two-body initial and final states satisfy certain selection rules described as helicity conservation of the gauge boson amplitudes (GBHC). These rules are valid at the Born level, as well as at the level of the leading and subleading 1-loop logarithmic corrections, in both the standard model and the minimal supersymmetric standard model (MSSM). A ''fermionic equivalence'' theorem is also proved, which suggests that GBHC is valid at all orders in the MSSM at sufficiently high energies, where the mass suppressed contributions are neglected.

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Many people may have noticed that, at high energy where masses are neglected, two-body processes involving transverse gauge bosons ($V =$ gluon, photon, Z, $W^{\pm}$) satisfy certain selection rules implying asymptotic helicity conservation in the s channel. This can easily be seen at the Born level in either the standard model (SM) or its renormalizable supersymmetric (SUSY) extensions; e.g., the minimal supersymmetric standard model (MSSM). For example, considering the processes $V_{\lambda_V} + V'_{\lambda_{V'}} \to A_{\lambda_A} + A'_{\lambda_{A'}}$, and computing the diagrams of Fig. 1 corresponding to A, A' beings scalars, one observes that the high energy helicity amplitudes $F_{\lambda_V \lambda_{V'} \lambda_A \lambda_{A'}}$ vanish for $\lambda_V = \lambda_{V'}$, while for the fermion production case of Fig. 2 the vanishing of the high energy amplitudes is guaranteed whenever either of the relations $\lambda_V = \lambda_{V'}$ or $\lambda_A = \lambda_{A'}$ is satisfied. Correspondingly, the amplitudes for the crossed process $V_{\lambda_V} + A_{\lambda_A} \to V'_{\lambda_{V'}} + A'_{\lambda'_A}$ vanish when $\lambda_V = -\lambda_{V'}$ for the case of Fig. 1, or when either of the relations $\lambda_V = -\lambda_{V'}$ or $\lambda_A = -\lambda_{A'}$ is satisfied for the fermion case of Fig. 2.

Similar asymptotic rules also exist for the purely gauge helicity amplitudes $F_{\lambda_1\lambda_2\lambda_3\lambda_4}$ of the processes $V_{1\lambda_1}V_{2\lambda_2} \to V_{3\lambda_3}V_{4\lambda_4}$ involving four gauge bosons. Thus, it has been observed in [1] that these amplitudes satisfy asymptotically

$$\begin{aligned} F_{+++-} &= F_{++-+} = F_{+-++} = F_{-+++} = F_{---+} \\ &= F_{--+-} = F_{-+--} = F_{+---} = F_{++--} \\ &= F_{--++} = 0 \end{aligned} \quad (1)$$

at the Born level, in either the SM or MSSM. Consequently, only the helicity amplitudes satisfying $\lambda_1 + \lambda_2 = \lambda_3 + \lambda_4$ can survive asymptotically at this level.

These properties of gauge boson helicity conservation (GBHC) are *a priori* different and complementary to the well-known fermion-helicity conservation in processes involving external fermions. The later is an essentially kinematical consequence of the fermionic vertices in SM or MSSM, valid at a diagram by diagram basis, provided that the energy is sufficiently high so that all masses can be neglected. (See below for the discussion of the effects of Yukawa couplings.)

The GBHC property though, referring specifically to the external gauge boson helicities, is more subtle. Contrary to the fermionic case, detail cancellation among the contributions of various diagrams must take place before GBHC is established. This can be seen from the Born processes described by Figs. 1 or 2, where the asymptotic vanishing of the helicity amplitudes for $\lambda_V = \lambda_{V'}$ is established through the occurrence of ''large gauge cancellations'' among the Vff and VVV vertices; or among the Vss, VVV, and $VVss$ vertices, with s describing generic scalar particles. It should also be emphasized that such cancellations are only realized when the minimal gauge couplings, characterizing the renormalizable gauge theories, are used. They would be violated if, e.g., higher dimensional operators are inserted the theory, even though $SU(3) \times SU(2) \times U(1)$ gauge symmetry is still respected [1]. Renormalizability of the theory is therefore crucial, for these rules to be valid. (The simplest illustration is the scalar coupling of the type $\phi F^{\mu\nu}F_{\mu\nu}$. It is perfectly gauge invariant but, if used in a scalar exchange diagram, it violates the above rules. Another simple example is the ''anomalous'' quadruple coupling [1,2]. The complete list of such anomalous gauge invariant couplings can be found in [3].)

Thus far, we have only considered tree diagrams, and one may wonder whether these high energy helicity con-

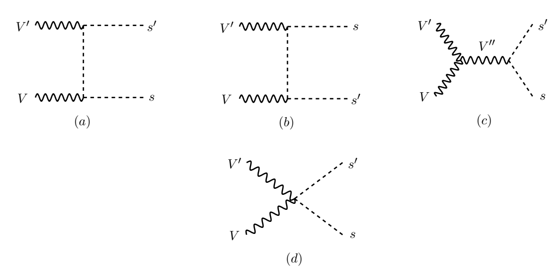

FIG. 1. Born diagrams for $VV' \to ss'$, with VV' being gauge bosons and ss' being scalar particles.

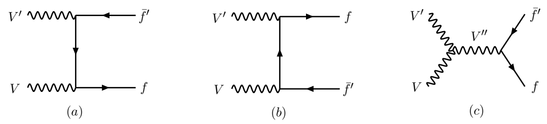

FIG. 2. Born diagrams for $VV' \to f\bar{f}'$ with VV' being gauge bosons and $f\bar{f}'$ being fermions.

servation properties remain true beyond the Born approximation. Indeed, for processes receiving a Born contribution, one can immediately check that these properties remain true at the level of the 1-loop leading $\ln^2 s$ and subleading $\ln s$ logarithmic corrections, according to the theory developed in [4–6]. We have also checked this explicitly for $e^-e^+ \to \gamma\gamma, ZZ, \gamma Z$ using the complete 1-loop results of [7] and for $e^+e^- \to W^+W^-$ using [8].

We have also looked at the process $\gamma\gamma \to \gamma\gamma$ [9], $\gamma\gamma \to ZZ$ [10], and $\gamma\gamma \to \gamma Z$ [11], where there is no Born term and the high energy 1-loop behavior is known. The validity of GBHC for the leading and subleading logarithmic terms is again observed in both SM and MSSM. However, at the level of the sub-sub-leading (constant) 1-loop contributions, GBHC is generally violated within SM, but it is still preserved in MSSM.

Motivated by this observation and the surprising analogy between the fermionic helicity conservation and GBHC, we have looked at its justification on the basis of supersymmetric invariance and renormalizability. The aim of the present paper is to release this justification.

We work in the framework of the exact supersymmetric limit of MSSM, assuming in addition that the Higgs-bilinear μ-term of the superpotential is also vanishing. In such a theory, all particles are massless, and the electroweak gauge symmetry is not broken. We denote the leptons and quarks by the chiral spin $= 1/2$ fields (ψ_L, ψ_R), the sleptons and squarks by the corresponding scalar fields $(\tilde{\psi}_L, \tilde{\psi}_R)$, the gauge bosons by V_j^μ, their gaugino partners by $\chi_j = \chi_{jL} + \chi_{jR}$, the Higgsino doublets by $\tilde{H}_{(1,2)L}$, and the corresponding Higgs doublets by $H_{(1,2)L}$. The later include also the Goldstone bosons.

In fact, since all particles are massless in this theory, the notation of the fermionic fields may be further simplified by denoting them as $(\psi_\lambda, \chi_\lambda)$, with λ being the helicity of the particle the field absorbs. The corresponding scalar fields may also be defined by this helicity and written as $\tilde{\psi}_\lambda$; in fact it is advantageous to think of this scalar field as carrying a "formal helicity" 2λ. The same definition applies also to Higgsino and Higgs fields. In this massless theory, all purely scalar self-interactions consist of 4-leg-vertices arising either from the F terms generated by the superpotential or from D terms. In each of these vertices, the total formal helicity defined above is conserved.

The sum of fermion helicity and formal helicity of the scalar fields is also conserved in all gaugino-fermion-sfermion and gaugino-Higgsino-Higgs MSSM vertices. Thus, e.g., a massless quark of a definite helicity can be transformed to an opposite gaugino helicity, emitting at the same time a scalar field, which remembers it; so that the sum of the fermion-helicity and the formal helicity is conserved at each vertex separately.

The fermion helicity in each of the gauge-fermion vertices is of course also conserved for all kinds of fermions, including gauginos and Higgsinos. In this respect, we think of the massless gauge bosons of our theory as carrying vanishing formal helicity, and claim that all gauge-fermion vertices also conserve the sum of fermion and formal helicities.

It might be useful to think of this conservation of the sum of fermion and formal helicities as a new global U(1) symmetry respected by all vertices in our framework, except the fermion vertices induced by the Yukawa terms in the superpotential.

However, if we restrict to processes determined by diagrams in which the Yukawa terms can only appear in Hermitian conjugate pairs, then this overall generalized helicity conservation rule will not be affected. Since we only consider two-body scattering amplitudes, this is achieved, e.g., by restricting to processes involving an even number of external transverse gauge bosons and/or an even number of external gauginos. In such amplitudes, the number of external Higgs fields as well as the number of external Higgsinos are also always even. These are in fact the processes which constitute our main interest.

With these definitions, it is straightforward to check helicity conservation for any 2-fermion to 2-fermion process at high energy, when all masses are neglected. More explicitly, in any allowed such process, the helicities of the incoming and outgoing particles in an amplitude, which is not forced to vanish asymptotically, should satisfy

$$F(f_\lambda f'_{\lambda'} \to f_\mu f'_{\mu'}) \Leftrightarrow \lambda + \lambda' = \mu + \mu', \qquad (2)$$

to all orders in our framework. We emphasize that this result is valid separately for each contributing diagram, independently of the nature of the fermions involved; i.e., whether some or all of them are quarks or leptons or their antiparticles, or gauginos, or Higgsinos.

The same result (2) remains true, if two of the fermions (irrespective of whether they are incoming or outgoing) are replaced by scalars. In this case, of course, the helicities for the scalar particles actually refer to their formal helicities defined above. Since these are ± 1 though, while the fermionic ones are half-integers, it is immediately seen that the only relevant amplitudes, which may be asymptotically nonvanishing, have the structure

$$F(f_\lambda s \to f'_\lambda s') \quad \text{or} \quad F(f_\lambda f'_{-\lambda} \to s s'), \qquad (3)$$

where (s, s') denote any kind of scalars (including of course also the Goldstone bosons), and (f, f') are fermions with their helicities indicated as indices in (3).

It is important to realize that (2) and (3) imply conservation of physical helicities at asymptotic energies, for any processes involving only external fermions and/or scalars. The physical helicities of all scalars are, of course, vanishing.

For proving GBHC for the physical helicities of the transverse gauge bosons, we just rely upon the validity of (2) and (3), and the supersymmetric transformation properties of the external fields. (The notion of formal helicity is not needed for this.) For simplicity we start from the 2-fermion to 2-fermion amplitudes in (2), for the case where all incoming and outgoing fermions describe gauginos. We then remark that the supersymmetric transformation for the gaugino fields is

$$\delta\chi^j = \frac{1}{2}\sigma^{\mu\nu}F^j_{\mu\nu}\gamma_5\epsilon - D^j\epsilon, \tag{4}$$

where j is the gaugino group index, $F^j_{\mu\nu}$ and D^j are the corresponding gauge-strength and auxiliary fields, and ϵ is the usual SUSY Majorana constant [12]. This implies that a massless incoming gaugino state of helicity μ and momentum p along the $\hat{z}$-axis transforms completely into a massless gauge state with helicity λ and the same momentum and gauge quantum numbers. The explicit result is

$$\begin{aligned}\delta\chi_\mu &= \delta\left(\frac{(1+2\mu\gamma_5)}{2}\chi^j\right)\\ &= \frac{(1+2\mu\lambda)}{2}\frac{ip}{\sqrt{2}}(1+\lambda\gamma_5)(i\lambda\sigma^{23}+\sigma^{13})\epsilon. \end{aligned}\tag{5}$$

(The derivation of this relation only involves the standard algebra for the massless fermionic and gauge states, for the aforementioned momenta and helicities.) The crucial term in (5) is the factor $(1 + 2\mu\lambda)$ on the right-hand side, which guarantees that the helicities of the transverse gauge bosons generated under a SUSY transformation will always have the same signs as those of the initial gauginos. [The D term in (4), being always a product of 4 fields in an unbroken SUSY theory, gives no contribution to the single particle projection in (5).] Thus, any asymptotic helicity structure of the 2-gaugino to 2-gaugino process will be transformed into a 2-gauge to 2-gauge process having the same structure. Starting therefore from (2) applied to gauginos, we conclude that the physical helicities of the asymptotically nonvanishing 2-transverse gauge to 2-transverse gauge amplitudes satisfy

$$F(V_\lambda V'_{\lambda'} \to V_\mu V'_{\mu'}) \Leftrightarrow \lambda + \lambda' = \mu + \mu', \tag{6}$$

to all orders in our framework.

This procedure can be straightforwardly extended to amplitudes involving any even number of gauginos. Thus, the only asymptotically nonvanishing amplitudes involving two transverse gauge bosons should have the helicity structure

$$\begin{aligned} &F(V_\lambda f_\mu \to V'_\lambda f'_\mu), \qquad F(V_\lambda V'_{-\lambda} \to f_\mu f'_{-\mu}),\\ &F(V_\lambda s \to V'_\lambda s'), \qquad F(V_\lambda V'_{-\lambda} \to s s'), \end{aligned}\tag{7}$$

with (f, f') and (s, s') being fermions and scalars, respectively, with the appropriate quantum numbers of course, so that the process is allowed.

In the above study we have proven the "physical helicity" conservation rules (2), (3), (6), and (7), in an exactly supersymmetric theory, where all particles are massless and electroweak symmetry (EW) is not broken. Longitudinal gauge bosons do not exist in this theory, but the Goldstone boson (Higgs) fields do appear, among the scalar external states of (3) and (7).

After EW breaking and masses are generated, (2), (3), (6), and (7) will of course remain asymptotically true for transverse gauge bosons. At the same time, the equivalence theorem guarantees that the external Goldstone bosons may readily be replaced by longitudinal gauge bosons in these amplitudes [13]. Thus, if, e.g., the scalars in the last of the amplitudes (7) are Goldstone bosons, then the equivalence theorem guarantees also the existence of the asymptotic amplitudes

$$F(V_\lambda V'_{-\lambda} \to V''_0 V'''_0).$$

Doing such replacements, in all possible ways, it is easy to see that the complete set of the asymptotically allowed gauge-involving amplitudes is again described by (6) and (7) (the purely Goldstone four-body amplitude will, of course, also be needed here), with the vector bosons helicities now allowed to acquire vanishing values, while (s, s') are now interpreted as sfermions or physical Higgs particles only. (In principle, it could even be possible to have asymptotically nonvanishing amplitudes of the form $V_0 s \to s' s''$, where the vector boson is longitudinal. Conservation of other quantum numbers like, e.g., CP, forbids the appearance of such terms in MSSM.) Equations (2) and (3) will of course also remain true under this interpretation.

The above proof of "fermionic equivalence" assumes that SUSY is indeed realized in Nature at a moderate scale, such that the corresponding selection rules can be observed at high energy. In such a case in fact, Eqs. (2), (3), (6), and (7) can be extended to any two-body process which is not determined by diagrams of odd order in the Yukava couplings. Thus, the asymptotically nonvanishing amplitudes should satisfy

$$F(a_{\lambda_1} b_{\lambda_2} \to c_{\lambda_3} d_{\lambda_4}) \Leftrightarrow \lambda_1 + \lambda_2 = \lambda_3 + \lambda_4, \tag{8}$$

to all orders in α, for any kind of particles (a, b, c, d) with physical helicities $(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$, provided the process is of even order on the Yukawa couplings, and it is of course allowed. As already mentioned, a sufficient condition for this is that the process involves an even number of transverse gauge and an even number of gaugino states. If both initial particles have spin $1/2$, and the final are gauge or scalar bosons (or vice versa), the helicity constraint in (8) is further restricted as $\lambda_1 + \lambda_2 = \lambda_3 + \lambda_4 = 0$; while if one of the particles in each of the initial and final state has spin $1/2$ and the other is boson, helicity is conserved separately for the fermions and the bosons of the process [compare (7)].

In case SUSY would not be realized at a moderate scale, or not realized at all, then SM will provide the appropriate framework. In this framework, GBHC would remain valid only at the Born approximation, including the leading and subleading 1-loop logarithmic corrections. (Occasionally it may be possible to extend this rule to nonasymptotic energies also. As an example, we mention the tree level observation in [14], where the projections of the t and $\bar{t}$ spins along the "off-diagonal axis" in the $e^-e^+ \to t\bar{t}$ c.m. frame must be equal for any energy. This off-diagonal axis coincides asymptotically with the $t - \bar{t}$ helicity axis.) Depending on the process, it may be broken at the sub-sub-leading (constant) level though. We have already mentioned that this is the case in 2-gauge boson to 2-gauge boson processes. Specific studies of other processes should be done in order to see if this is a general feature, i.e., if indeed there is a residual GBHC-violating term in SM, which is only canceled when the supersymmetric partner contributions are added. *A priori*, there could also be cases in which the sub-sub-leading terms cancel separately in SM and in SUSY contributions.

Incidently, one should also mention that the cancellation of the GBH-violating amplitudes leads to a remarkable simplification of the actual theoretical description of the processes; about half of the helicity amplitudes disappear and the expressions of the remaining ones are noticeably simplified.

Theoretically, GBHC looks like an appealing simple rule. Experimentally, it may be possible to check it at the CERN Large Hadron Collider (LHC) or International Linear Collider (ILC) by looking at processes involving gluons, photons, Z, or W's in processes like

$$q\bar{q} \to gg, g\gamma, gZ, gW, \gamma\gamma, \gamma Z, ZZ, W^+W^-, \gamma W, ZW,$$
$$gq \to gq, \gamma q, Zq, Wq, \qquad gg \to gg, q\bar{q},$$
$$e^+e^- \to \gamma\gamma, \gamma Z, ZZ, W^+W^-, \qquad \gamma e \to \gamma e, Ze, W\nu,$$
$$\gamma\gamma \to f\bar{f}, \gamma\gamma, \gamma Z, ZZ, W^+W^-,$$

as well as processes involving external supersymmetric particles, like, e.g., $gg \to \tilde{g}\tilde{g}, \tilde{q}\bar{\tilde{q}}$ and $\gamma\gamma \to \tilde{f}\bar{\tilde{f}}, \chi\chi, H^+H^-, H^0H'^0$. These checks can be done either through a direct measurement of the polarization of the initial or the final states, whenever possible, or by looking at the agreement between the differential cross section measured experimentally and the theoretical predictions based on the leading helicity conserving amplitudes.

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