Parametric Instabilities and Their Control in Advanced Interferometer Gravitational-Wave Detectors

C. Zhao,* L. Ju, J. Degallaix, S. Gras, and D. G. Blair

School of Physics, The University of Western Australia, 35 Stirling Highway, Nedlands, Western Australia 6009, Australia

(Received 17 January 2005; published 1 April 2005)

A detailed simulation of Advanced LIGO test mass optical cavities shows that parametric instabilities will excite 7 acoustic modes in each fused silica test mass, with parametric gain R up to 7 and only 1 acoustic mode with $R \sim 2$ for alternative sapphire test masses. Fine-tuning of the test mass radii of curvature causes the instabilities to sweep through various modes with R as high as $\sim$2000. Sapphire test mass cavities can be tuned to completely eliminate instabilities using thermal g-factor tuning. In the case of fused silica test mass, instabilities can be minimized but not eliminated.

DOI: 10.1103/PhysRevLett.94.121102 PACS numbers: 04.80.Nn, 95.55.Ym

To achieve sufficient sensitivity to detect numerous predicted sources of gravitational waves, the three long baseline laser interferometer gravitational-wave detectors [1–4] need to achieve about 1 order of magnitude improved sensitivity. This improvement is planned to be achieved using larger lower acoustic loss test masses and substantially higher laser power [5]. It has already been pointed out that this improvement brings with it the risk of parametric instability [6–8]. The instability arises due to the potential for acoustic normal modes of the test masses to scatter light from the fundamental optical cavity mode into a nearby higher order mode, mediated by the radiation pressure force of the optical modes acting on the acoustic mode. The instability can occur if two conditions are met. First, there must be a substantial spatial overlap of the acoustic mode shape with the higher order cavity mode shape. Second, the optical frequency difference between the cavity fundamental mode and higher order mode must match the acoustic mode frequency.

Parametric instability was observed and controlled in the niobium bar gravitational-wave detector NIOBE [9]. If not controlled, instabilities cause acoustic modes to ring at very large amplitudes, sufficient to disrupt operation of a sensitive detector.

We show here that for the proposed Advanced LIGO (AdvLIGO) parameters, the conditions for instability are indeed met for a number of acoustic modes, specifically in the frequency ranges 28–35 kHz and 64–72 kHz. Because the Young's modulus and density are smaller for fused silica than for sapphire, the acoustic mode density of a fused silica test mass is much greater than that of a sapphire test mass with the same weight. As a consequence, the number of parametrically unstable modes is much greater for fused silica, and they generally have a higher parametric gain. After demonstrating the magnitude of the instabilities, we present a method by which the parametric instabilities may be detuned. Again, this is more effective for sapphire than for fused silica because the mode spacing in the relevant frequency range is about 6 times greater for sapphire than fused silica. We also show that it is unlikely to be possible to predesign against the parametric instabilities unless (a) the error of calculating normal mode frequencies in standard finite element modeling (FEM) software can be improved to less than the cavity bandwidth ($\sim$30 Hz), (b) the test mass density inhomogeneity is known, and (c) the mirror radius of curvature can be specified to better than 0.1%.

In an optical cavity, the frequency differences between the TEM_{00} mode and TEM_{mn} modes are

$$\Delta_- = \omega_0 - \omega_1 = \frac{\pi c}{L}\left\{k_1 - \frac{m + p \times n}{\pi} \arccos\sqrt{\left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right)}\right\}, \tag{1a}$$

$$\Delta_+ = \omega_{1a} - \omega_0 = \frac{\pi c}{L}\left\{k_{1a} + \frac{m + p \times n}{\pi} \arccos\sqrt{\left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right)}\right\}. \tag{1b}$$

Here ω_0 is the fundamental mode frequency, ω_1 is the Stokes mode frequency, ω_{1a} is the anti-Stokes mode frequency, L is the cavity length, R_1 and R_2 are the mirror radii of curvature, k_1 and k_{1a} are longitudinal mode indices, m and n are transverse mode indices, $p = 1$ for the Hermite-Gaussian mode, and $p = 2$ for the Laguerre-Gaussian mode.

By inspection of Eqs. (1a) and (1b), the fundamental modes ($m + n = 0$) are symmetrically distributed around the carrier modes and the Stokes mode is compensated by an anti-Stokes mode. Higher order transverse modes ($m + n > 0$) are not symmetrically distributed and do not compensate each other.

Braginsky *et al.* [7] have shown that the effective parametric gain R in a power recycled interferometer is given by

$$R = \frac{2PQ_m}{mcL\omega_m^2}\left(\frac{Q_1\Lambda_1}{1 + (\Delta\omega_1/\delta_1)^2} - \frac{Q_{1a}\Lambda_{1a}}{1 + (\Delta\omega_{1a}/\delta_{1a})^2}\frac{\omega_{1a}}{\omega_1}\right). \tag{2}$$

[In the unusual case that the Stokes mode is within the very narrow bandwidth ($\delta_{\rm pr}$ [7]) of the coupled cavity of the power recycling cavity and arm cavities (~4 Hz for AdvLIGO nominal parameters), the formula of R will be different from Eq. (2) [7].] When the parametric gain exceeds unity, the acoustic mode will be excited. Here P is the total power inside the cavity, Q_1 and Q_{1a} are the quality factors of the Stokes and anti-Stokes modes, Q_m is the quality factor of acoustic mode, $\delta_{1(a)} = \omega_{1(a)}/2Q_{1(a)}$, m is the test mass's mass, L is the cavity length, $\Delta\omega_{1(a)} = \omega_0 - \omega_{1(a)} - \omega_m$ is the possible detuning from the ideal resonance case, and Λ_1 and Λ_{1a} are the overlap factors between optical and acoustic modes. The overlap factor is defined as [6]

$$\Lambda_{1(a)} = \frac{V(\int f_0(\vec{r}_\perp) f_{1(a)}(\vec{r}_\perp) u_z d\vec{r}_\perp)^2}{\int |f_0|^2 d\vec{r}_\perp \int |f_{1(a)}|^2 d\vec{r}_\perp \int |\vec{u}|^2 dV}. \quad (3)$$

Here f_0 and $f_{1(a)}$ describe the optical field distribution over the mirror surface for the fundamental and Stokes (anti-Stokes) modes, respectively, $\vec{u}$ is the spatial displacement vector for the mechanical mode, u_z is the component of normal to the mirror surface. The integrals $\int d\vec{r}_\perp$ and $\int dV$ correspond to integration over the mirror surface and the mirror volume V, respectively.

Using FEM (ANSYS®) to calculate mode shapes, we have evaluated the overlap factors and calculated R for AdvLIGO test mass [10,11] acoustic modes close to the first and the second order transverse modes. The test mass and cavity parameters used are listed in Tables I and II. This Letter is not to predesign the real situation but to prove the principle that the error of the FEM has not been taken into account in the simulation. This assumption will not affect the final results as it is only the frequency difference between the acoustic mode and the optical mode that affects the parametric gain, and we tune the optical mode frequencies across a 3–5 kHz range. The simulation for acoustic modes close to cavity transverse modes higher than second order is not included, because higher order modes generally have lower parametric gain due to their diffraction losses. Figure 1(a) shows those modes with $R > 1$ [12]. Figure 1(b) shows a particular acoustic mode structure with a frequency (31.251 kHz) close to the frequency difference between TEM_{00} and TEM_{10} modes. Figure 1(c) shows the TEM_{10} mode optical field distribution. The similarity of these mode structures is apparent. The overlap factor [Eq. (3)] for these two modes is ~2.9.

TABLE I. Test mass material parameters [11].

Parameters	Sapphire	Fused silica
Nominal Q	200×10^6	100×10^6
Poisson ratio	0.23	0.17
Young's modulus	4×10^{11} N/m^2	7×10^{10} N/m^2
Density	3983 kg/m^3	2200 kg/m^3
Size (diameter × thickness)	31.4 × 13 cm	38.8 × 15.4 cm
Test mass's mass	40 kg	40 kg

Because of the large acoustic mode density there are 7 acoustic modes which have the potential ($R > 1$) to be unstable for a fused silica test mass, compared with only 1 mode for a sapphire test mass. For the nominal AdvLIGO parameters, the maximum parametric gain R for a fused silica test mass is up to ~7, compared with ~2 for a sapphire test mass.

It is possible that the parametric gain could be much larger than the values mentioned above for several reasons: (a) Standard FEM methods for calculating the acoustic mode frequencies have errors much larger than the cavity bandwidth [7]. (b) The suspension system may change acoustic mode frequencies. (c) An error of 2 m (0.1%) in the mirror radius of curvature results in a cavity mode spacing error of ~30 Hz which is equal to the cavity bandwidth. Finally, the thermal expansion of the high reflective coating surface due to the coating absorption [10,13] (as well as some small bulk absorption contribution) causes the radii of curvature of test masses to vary from their nominal values. Thus the worst case, where the acoustic mode frequency is very close to the frequency difference between the fundamental mode and a high order transverse mode, cannot be definitely avoided. For instance, in a fused silica test mass, if the acoustic mode at the frequency of 33.354 kHz [the third dot from the top in Fig. 1(a)] has a 459 Hz error, the parametric gain R could be as large as ~2000.

The fact that the cavity mode spacing changes with the mirror radius of curvature [see Eq. (1)] also provides an opportunity to tune out the most unstable acoustic modes. Lawrence *et al.* [14] and Degallaix *et al.* [15] have demonstrated that by using a heating ring near the front of the test mass one can adjust the test mass radius of curvature to effectively compensate for thermal lensing. The German-British Collaboration for the Detection of Gravitational Waves (GEO 600) project [16] has used this method to compensate the mismatch of radii of curvature of two interferometer mirrors. Here we propose a similar method, with the heating ring at the back of the test mass to tune the cavity mode frequencies. Substantial changes in the radius of curvature can be achieved. Figure 2 shows the AdvLIGO end test mass radius of curvature and the relative R,

$$\frac{R_{\triangle T}}{R_0} = \frac{1}{[1 + (\Delta\omega_1/\delta_1)^2]}, \quad (4)$$

as a function of the maximum temperature difference across the test mass when heated by a heating ring with variable heating power. Here we assume that there exists only a single acoustic mode with a frequency equal to the frequency difference between the fundamental mode and the first high order mode when without heating. The change of the radius of curvature from ~2.076 to ~2.066 km corresponds to the maximum temperature difference across the test mass from 0 to ~0.11 K for sapphire (average mirror temperature changed from 300 to ~302.5 K) and from 0 to ~1.2 K for fused silica (average

TABLE II. Cavity parameters [10].

Parameters	Sapphire	Fused silica
Cavity length	4 km	4 km
Laser wavelength	1064 nm	1064 nm
Radius of curvature	2.076 km	2.076 km
Cavity g factor	$g_1 = g_2 = -0.9265$	$g_1 = g_2 = -0.9265$
$\delta_1/2\pi(m + p * n = 0, 1, 2)$	15 Hz	15 Hz
$\delta_{1a}/2\pi(m + p * n = 6)$	191 Hz	191 Hz
$\delta_{1a}/2\pi(m + p * n = 7)$	535 Hz	535 Hz
Power recycling mirror transmission	6%	6%
Laser power inside arm cavities	830 kW	830 kW

mirror temperature changed from 300 to ~301 K). If one considers only a single acoustic mode, this tuning is sufficient to reduce R to 1% of its original value. Unfortunately, there are many potential acoustic modes. When tuning the cavity modes away from a particular acoustic mode, we generally increase the coupling to nearby acoustic modes. In sapphire test masses, the frequency gap is ~1 kHz. Tuning the cavity mode to a point between two acoustic modes minimizes the parametric gain of both. The acoustic mode gap of ~200 Hz for fused silica test masses makes such tuning much less effective. Thus we see that in fused silica [Fig. 3(b)] it is impossible to tune the parametric gain R to less than 2. In sapphire [Fig. 3(a)] it is possible to tune the cavity away from the instabilities ($R < 1$) at the radius of curvature around 2.092 km. Figure 4 shows the total numbers of acoustic modes whose parametric gains R are greater than 1 as a function of mirror radius curvature for sapphire and fused silica, respectively. The optimum tuning of fused silica leads to 2 modes with R of 1.5 and 2.5

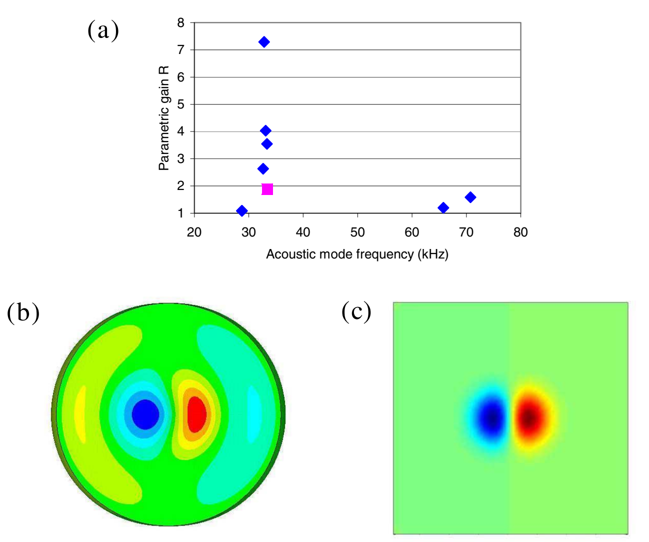

FIG. 1 (color online). (a) All the acoustic modes with $R > 1$ for sapphire (square) and fused silica (diamond), respectively, assuming nominal AdvLIGO parameters [10,11]. (b) A typical test mass acoustic mode structure. (c) The field distribution of the cavity TEM_{10} mode showing high overlap of the acoustic and optical mode structures.

when the radius curvature increased to 2.135 km. In sapphire there are no modes with R greater than 1 at the optimum tuning point corresponding to about 2.092 and 2.127 km radius of curvature.

Tuning the arm cavity radius of curvature also changes the TEM_{00} mode waist size and may mismatch the arm cavity with the recycling cavities. Over modest tuning ranges, this effect is small. For example, when the radius of curvature of the AdvLIGO end test mass changes from 2.076 to 2.066 km, the arm cavity beam waist changes from 1.15 to 1.13 cm. The introduced loss due to the mode mismatching is ~300 ppm which is acceptable in relation to the recycling mirror transmission (6% for the power recycling mirror and 7% for the signal recycling mirror).

In summary, it is inevitable that parametric instabilities will appear in AdvLIGO. By thermally tuning the arm cavity mirror radius of curvature we can tune the cavity

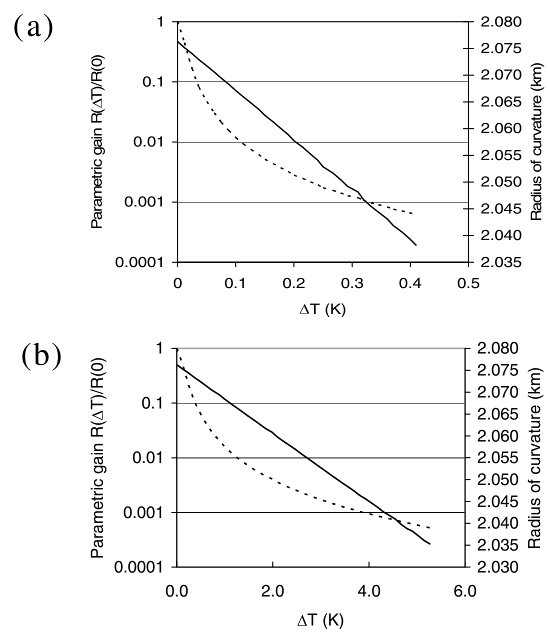

FIG. 2. The dependence of the relative parametric gain R (dotted line) and the mirror radius of curvature (solid line) on the maximum temperature difference across the test mass if considering only one acoustic mode, (a) for sapphire and (b) for fused silica.

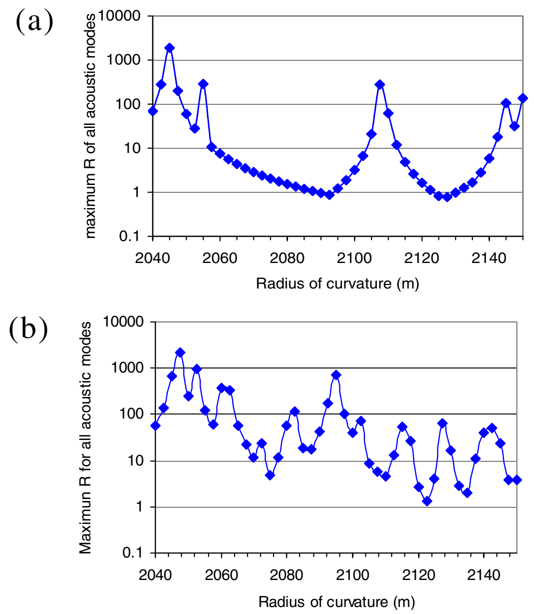

FIG. 3 (color online). The maximum parametric gain R of all acoustic modes as a function of mirror radius of curvature, (a) for sapphire and (b) for fused silica.

away from instability in the case of sapphire test masses or minimize the instability gain in the case of fused silica test masses. Thermal tuning is feasible and need not introduce extra noise. While the data have been applied to AdvLIGO parameters, it is also directly relevant to the VIRGO interferometer.

We note that the instabilities discussed here refer to single cylindrical test masses. Each test mass will experience instability, and for noncylindrical symmetry there will be twice as many acoustic modes. The optimum tuning for suspended pairs of test masses requires further study. Braginsky *et al.* [17] has proposed the use of small but high finesse detuned cavities as a means of low noise ''tranquilizing'' of parametric instabilities. The extra cavities needed in the scheme create extra complexity into an already complex system. Feedback schemes similar to the demonstrated cold damping of thermal noise [18] could be another solution, again adding complexity.

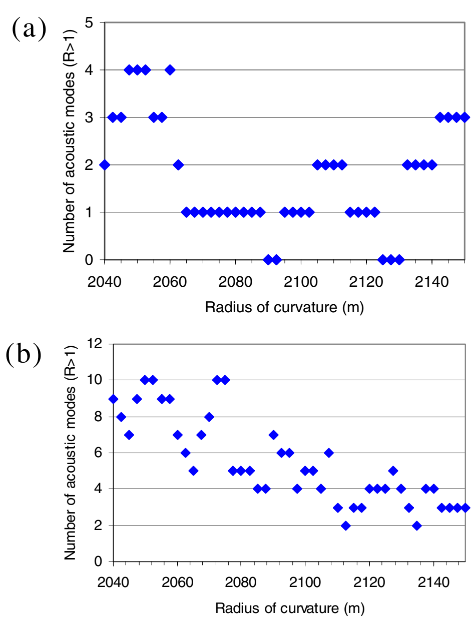

FIG. 4 (color online). The numbers of acoustic modes with $R > 1$ when tuning the radius of curvature for (a) sapphire and (b) fused silica.

A similar analysis for the Gingin High Optical Power Facility shows that this facility is ideally suited for experimental study of parametric instability. Results will be presented elsewhere.

This is a project of Australian Consortium for Gravitational Astronomy (ACIGA), supported by Australian Research Council.

*Electronic address: zhao@physics.uwa.edu.au

[1] A. Abramovici *et al.*, Science **256**, 325 (1992).
[2] F. Acernese *et al.*, Classical Quantum Gravity **19**, 1421 (2002).
[3] B. Willke *et al.*, Classical Quantum Gravity **19**, 1377 (2002).
[4] M. Ando *et al.*, Phys. Rev. Lett. **86**, 3950 (2001).
[5] A. Buonanno and Y. Chen, Phys. Rev. D **64**, 042006 (2001).
[6] V. B. Braginsky, S. E. Strigin, and S. P. Vyatchanin, Phys. Lett. A **287**, 331 (2001).
[7] V. B. Braginsky, S. E. Strigin, and S. P. Vyatchanin, Phys. Lett. A **305**, 111 (2002).
[8] W. Kells and E. D'Ambrosio, Phys. Lett. A **299**, 326 (2002).
[9] D. G Blair *et al.*, Phys. Rev. Lett. **74**, 1908 (1995).
[10] G. Billingsley, G. Harry, and W. Kells, LIGO Report No. T000127-01-D, 2004, available at http://www.ligo.caltech.edu at the Document Control Center page.
[11] http://www.ligo.caltech.edu/ligo2/scripts/l2refdes.htm
[12] See EPAPS Document No. E-PRLTAO-94-037515 for two tables that list all acoustic modes in the frequency range with overlap factors for Stokes modes greater than 0.0001. A direct link to this document may be found in the online article's HTML reference section. The document may also be reached via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html) or from ftp.aip.org in the directory /epaps/. See the EPAPS homepage for more information.
[13] W. Winkler, K. Danzmann, A. Rudiger, and R. Schilling, Phys. Rev. A **44**, 7022 (1991).
[14] R. Lawrence, M. Zucker, P. Fritschel, P. Marfuta, and D. Shoemaker, Classical Quantum Gravity **19**, 1803 (2002).
[15] J. Degallaix, C. Zhao, L. Ju, and D. Blair, Appl. Phys. B **77**, 409 (2003).
[16] H. Luck, A. Freise, S. Goßler, S. Hild, K. Kawabe, and K. Danzmann, Classical Quantum Gravity **21**, S985 (2004).
[17] V. B. Braginsky and S. P. Vyatchnin, Phys. Lett. A **293**, 228 (2002).
[18] P. F. Cohadon, A. Heidmann, and M. Pinard, Phys. Rev. Lett. **83**, 3174 (1999).