# Characterization of the Initial Filamentation of a Relativistic Electron Beam Passing through a Plasma

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The linear instability that induces a relativistic electron beam passing through a plasma with return current to filament transversely is often related to some filamentation mode with the wave vector normal to the beam or confused with Weibel modes. We show that these modes may not be relevant in this matter and identify the most unstable mode on the two-stream or filamentation branch as the main trigger for filamentation. This sets both the characteristic transverse and longitudinal filamentation scales in the nonresistive initial stage.

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Inertial confinement fusion schemes commonly involve in their final stage the interaction between some highly energetic particle beams and a dense plasma target. This is, in particular, valid for the fast ignition scenario (FIS) [1] where some laser-produced relativistic electron beam would eventually propagate into the dense plasma where it would be stopped. This process would lead to strong local heating and the ignition of a fusion burn wave. In this respect, microscopic turbulence in beam-plasma systems is one of the main potentially deleterious effects for inertial fusion schemes since it may prevent the conditions for burn to be met by broadening the phase area where particles deposit their energy. Within the FIS framework, strong research effort has thus been put towards the interaction of a relativistic electron beam with a plasma with the focus on beam filamentation instability, that is microscopic in the transverse direction (see, e.g., [2–6]). The experimental evidence of filamentation of very high current laser-produced electron beams was recently reported for conditions relevant to the FIS [7]. More generally, filamentation is a potential instability in beam-plasma systems in frameworks ranging from accelerator physics to solar flares.

In the linear stage, filamentation is generally studied under some simplifying *ab initio* transverse approximation of the dielectric tensor, so that filamentation instability is attributed to the exponential growth of unstable electromagnetic purely transverse modes ($\mathbf{k} \cdot \mathbf{E} = 0$) with wave vector $\mathbf{k}$ normal to the beam [4,8–12]. It is also common to refer to this instability as Weibel instability [4,7,8], though the original mode studied by Weibel [13] would require some plasma temperature anisotropy to be driven. Figure 1 sketches the original definitions of various modes under the original Weibel scenario where $\mathbf{k}$ is parallel to the beam, along the low temperature axis. As long as the beam is not relativistic, the largest instability it undergoes is the two-stream one, where the second "stream" is the return current it generates in the plasma. But in the relativistic regime, the "filamentation" growth rate eventually exceeds the two-stream one and is supposed to induce beam filamentation.

In reality, the beam suffers much more instabilities at the same time. Indeed, filamentation, Weibel, or two-stream instabilities pertain to various orientations of the wave vector and various kinds of waves (transverse or longitudinal), but in the real world the beam-plasma system triggers every possible mode allowed by Maxwell equations with a wide range of wave vector orientation. Among all the triggered modes, the unstable ones start growing exponentially while the most unstable one mostly shapes the beam. When it comes to knowing how the beam is eventually affected when entering the plasma, one needs therefore to answer two questions: (i) which is the most unstable mode all over the $\mathbf{k}$ space for the system investigated? and (ii) how does this mode shape the beam? Following the guidelines built by these two questions, we assert that the so-called filamentation instability is not the

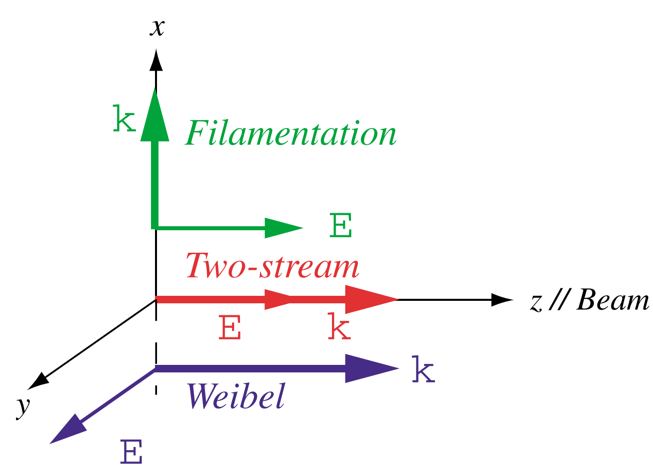


FIG. 1 (color online). Weibel, two-stream, and filamentation modes.

 

fastest growing instability, even in the relativistic regime, so that it is not the answer to the first question. As for the second question, we shall see that this instability would not produce the observed effects anyway, even if it were the stronger one. We conclude proposing a new ''candidate'' for beam filamentation and comparing our predictions with the experimental results presented in [7]. For clarity, we keep labeling the most unstable transverse mode with wave vector normal to the beam as the filamentation mode, though our point is precisely that it does not filament.

Let us consider a beam of electrons (having mass $m_e$ and charge $e$) of density $n_b$ and relativistic velocity $V_b$ passing through a return current of plasma electrons of density $n_p$, so that the system is unmagnetized. Both beam and plasma are infinite and homogenous and ions are supposed to form a fixed neutralizing background. Let us define the ratio $\alpha = n_b/n_p$ and introduce the plasma frequency $\omega_p = (4\pi n_p e^2/m_e)^{1/2}$. Here the beam will be assumed to be cold in the longitudinal direction, which is correct provided the ratio of its longitudinal thermal velocity $V_{tb\parallel}$ over the parallel phase velocity $\omega_p/k_\parallel$ is small compared to $\alpha^{1/3}$. The filamentation growth rate can then be evaluated in the weak beam density limit ($\alpha \ll 1$) through

$$\delta_F \simeq \beta\sqrt{\frac{\alpha}{\gamma_b}}\omega_p, \tag{1}$$

with $\beta = V_b/c$ and $\gamma_b = 1/\sqrt{1-\beta^2}$. Within the same weak beam density limit, the two-stream growth rate reads

$$\delta_{\mathrm{TS}} \simeq \frac{\sqrt{3}}{2^{4/3}}\frac{\alpha^{1/3}}{\gamma_b}\omega_p. \tag{2}$$

Since $\delta_F$ decreases like $\gamma_b^{-1/2}$ whereas $\delta_{\mathrm{TS}}$ decreases like $\gamma_b^{-1}$, the filamentation growth rate eventually exceeds the two-stream one when the beam is relativistic. Comparing filamentation growth rate with the Weibel one (transverse waves with the wave vector along the beam, as in [13]), one finds filamentation to be also dominant so that it eventually appears to be the largest instability [14].

However, this conclusion needs to be modified when accounting for every other unstable mode with the wave vector neither normal nor parallel to the beam. Investigating these modes demands a fully electromagnetic formalism which is the only way to capture longitudinal modes (two-stream) as well as transverse modes (Weibel and filamentation). Indeed, such a procedure shows that two-stream and filamentation modes pertain to the same branch of the dispersion equation so that it is possible to switch continuously from the former to the latter by increasing the angle $\theta_k$ between the beam and the wave vector from 0 to $\pi/2$. Consequently, the angle $\varphi_k$ between the wave vector and the electric field of the mode [$\varphi_k = \widehat{(\mathbf{k}, \mathbf{E})}$] needs to go continuously from 0 to $\pi/2$ to bridge between longitudinal two-stream modes and transverse filamentation modes. In a recent paper [15], we began to implement such an electromagnetic formalism using the relativistic Vlasov equation to describe the evolution of the electronic distribution function of the beam-plasma system. Using some simple water bag distribution functions for the beam and the plasma, we investigated the two-stream or filamentation (TSF) branch and found that the growth rate reaches a maximum for an intermediate orientation of the wave vector. This maximum scales like $\gamma_b^{-1/3}$ and reads

$$\delta_M \simeq \frac{\sqrt{3}}{2^{4/3}}\left(\frac{\alpha}{\gamma_b}\right)^{1/3}\omega_p. \tag{3}$$

It is noticeable that this result may be recovered under the electrostatic longitudinal approximation [16]. Such an approach cannot, however, sweep the whole $\mathbf{k}$ plane. Equation (3) shows that, even in the relativistic regime, the filamentation growth rate should not be the larger one. On the contrary, this mixed two-stream filamentation mode shall be all the more dominant over the usual filamentation mode that the beam is relativistic because of its $\gamma_b$ scaling. This trend amplifies even more when accounting for transverse beam temperature, since filamentation is damped [4,17] while $\delta_M$ is almost unaffected [17]. Therefore, one can say that the so-called filamentation instability may not be the fastest growing one.

Let us explore this further and move to our second point by questioning on what filamentation instability would do to the beam, if even it had the largest growth rate. Within the linear approximation, one restricts to small fluctuations of the electron charge density. If $\rho_1$ and $\mathbf{E}_1$ denote the first order perturbations, respectively, to the electron charge density and to the electric field, the Poisson equation written in Fourier space brings

$$\mathbf{k}\cdot\mathbf{E}_1(\mathbf{k}, \omega) = 4\pi\rho_1(\mathbf{k}, \omega). \tag{4}$$

It comes directly from this equation that a transverse mode with $\mathbf{k}\cdot\mathbf{E}_1 = 0$ has $\rho_1(\mathbf{k}, \omega) = 0$ and cannot yield density perturbations within the limits of the linear regime. As a consequence, the transverse filamentation instability with the wave vector normal to the beam cannot yield any charge density fluctuations from the linear stage. It is important to note that, for the same reasons, the original Weibel mode [13] cannot linearly induce density filamentation either.

As far as beam filamentation is concerned, experiments and simulations show that the electronic density varies transversely to the beam, producing the filaments [2,5–7]. Now, if the electronic density varies while background ions are (almost) at rest, there is necessarily a net charge perturbation which precisely cannot be accounted for by the mere exponential growth of a purely transverse wave. It seems therefore that even if it were the fastest growing instability, the so-called filamentation instability would not produce these filaments. It is worth noticing that it could produce *current* filaments, for Maxwell's equations allow such a wave to produce such perturbations. But these current filaments would have to preserve the neutrality of the system beam plasma, that is, to preserve electronic density since ions can be considered at rest.

Let us eventually determine which mode is responsible for the observed filamentation. We see here that the most natural candidate is the most unstable mode found along the TSF branch. Being the fastest growing mode, it is the one whose growth should ''shape'' the beam during the linear phase while the other modes create fluctuations around this basic shape. As for its ability to create filaments, it is quasilongitudinal [15,17] so that its divergence does not vanish. This mode, unlike the so-called filamentation mode, satisfies therefore the criteria to induce filamentation: It is the fastest growing one, it is microscopic in the transverse direction, and it is two-stream-like, that is, quasilongitudinal. Expressing the density perturbation in terms of the wave electric field yields $\rho_1(\mathbf{k},\omega) = kE_1(\mathbf{k},\omega)\cos\varphi_k/4\pi$, and one retrieves the density perturbation in the real space through

$$\rho_1(\mathbf{r},t) = \sum_{\mathbf{k},\omega_\mathbf{k}} \frac{kE_1(\mathbf{k},\omega_\mathbf{k})\cos\varphi_k}{4\pi}\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega_\mathbf{k}t). \quad (5)$$

The sum above runs over every wave vector and every proper frequency $\omega_\mathbf{k}$. Yet it will obviously be dominated by the contribution of the fastest growing [having $\delta_\mathbf{k} \equiv \mathrm{Im}(\omega_\mathbf{k}) > 0$] self-excited modes. Figure 2 displays the growth rates on the TSF branch in the $(k_\perp, k_\parallel)$ plane [18] for some zero or finite plasma thermal velocities $V_{tp} = V_{tp\parallel} = V_{tp\perp}$ and some zero or finite beam transverse thermal velocities $V_{tb\perp}$ [19]. It is important to note that the associated real parts $\mathrm{Re}(\omega_\mathbf{k})$ are in the vicinity of the resonance given by $\omega - k_\parallel V_b = 0$. To our knowledge, this is the first exact computation of TSF growth rates in the whole $k$ space including beam and plasma temperatures effects. These curves clearly show that when temperatures are accounted for they act to control the instability domain, damping the small wavelength perturbations along the filamentation direction ($k_\parallel = 0$) and deforming the growth rate surface so that a maximum growth rate appears for a finite oblique wave vector $\mathbf{k}_M$. In this respect, Fig. 2(b) shows the drastic influence of beam transverse temperature for a cold plasma. Yet, every physical plasma has a finite bulk temperature and, for $\alpha$ small enough, this plasma temperature can be shown [17] to control essentially the maximum growth rate location. Its $(k_\perp, k_\parallel)$ components are then

$$\mathbf{k}_M \sim \left(\pm\frac{\omega_p}{c}\sqrt{V_b/V_{tp}}, \pm\frac{\omega_p}{V_b}\right). \quad (6)$$

We can then roughly evaluate the density perturbation in Eq. (5) by retaining only the $\mathbf{k}_M$ contribution. As for the corresponding proper frequency, one has $\omega_{k_M} \sim \pm\omega_p + i\delta_M$, where $\delta_M$ is given by Eq. (3). Summing in the $\mathbf{k}$ space the four contributions associated to all the possible orientations of $\mathbf{k}_M$ in (6), one finds that the density perturbation behaves essentially as

$$\rho_1(\mathbf{r},t) \propto \exp(\delta_M t)\sin(k_{M\parallel}z - \omega_p t)\cos(k_{M\perp}x). \quad (7)$$

Equation (7) displays spacial modulation of electron density in the beam direction ($z$) as well as in the normal direction ($x$). In the normal direction, we witness the ''birth'' of the beam filamentation in the linear stage, with filament interspace

$$L_f \sim \pi\lambda_s\sqrt{V_{tp}/V_b}, \quad (8)$$

where $\lambda_s = c/\omega_p$ is the skin depth.

There are by now very few relevant experimental results available for quantitative comparisons with this result. We can consider Fig. 3 of Ref. [7], where plasma electronic density is about $10^{20}$ electrons/cm$^3$. This yields a plasma skin depth of about 53 $\mu$m while our Fig. 3 scale indicates the transverse space between filaments is somehow smaller. Indeed, the quantity $L_f$ introduced above is the skin depth times a $\pi\sqrt{V_{tp}/V_b}$ factor which is smaller than 1 for a nonrelativistic plasma since $V_b \sim c$ here. Taking account of the estimated plasma temperature (100 eV), we finally find $L_f \sim 23$ $\mu$m which is in good agreement with what is observed. Figure 3 displays the right-hand side of Eq. (7) for $t = 1/\omega_p$. Filaments are clearly visible,

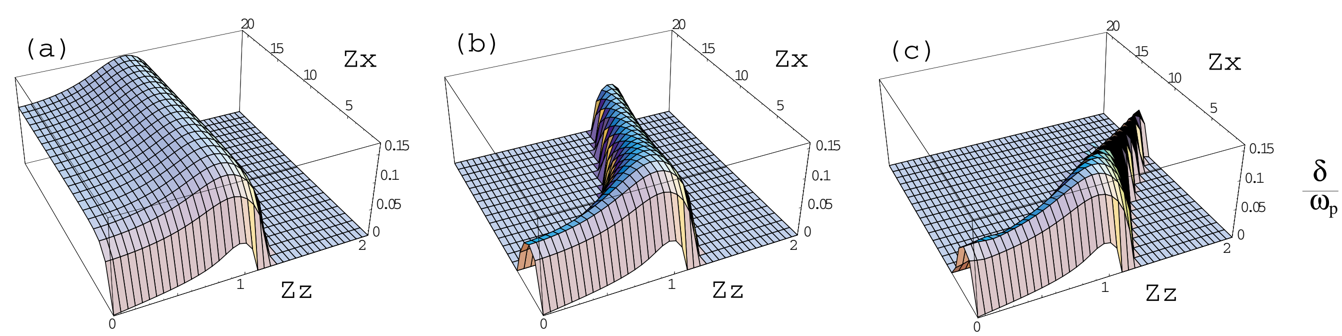


FIG. 2 (color online). Growth rates on the TSF branch in terms of $\mathbf{Z} = \mathbf{k}V_b/\omega_p$ with $\mathbf{V}_b \parallel \hat{\mathbf{z}}$. (a) Cold beam, cold plasma (see also [23]); (b) hot beam, cold plasma; and (c) hot beam, hot plasma. Parameters are $\alpha = 0.05$ and $\gamma_b = 4$ for (a)–(c), $V_{tb\perp} = V_b/10$ for (b),(c), and $V_{tp} = V_b/10$ for (c).

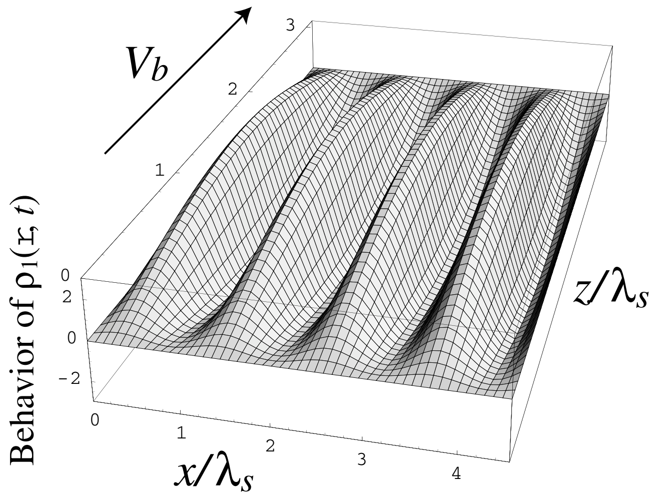


FIG. 3. Right-hand side of Eq. (7) for $t = 1/\omega_p$. Parameters are $V_{tp} = c/30$, $\alpha = 0.1$, and $\gamma_b = 7$ yielding $\delta_M \sim 0.16\omega_p$.

combined with a beam segmentation along the beam direction into segments $\pi\lambda_s V_b/c \sim \pi\lambda_s$ long. This parallel segmentation may not be easily distinguishable in Fig. 3 of Ref. [7] for its characteristic length (more than 150 $\mu$m) is comparable to the size of the entire picture.

Let us here briefly discuss the applicability of our study. As far as plasma (or beam) transverse temperatures are large enough, the above results should apply to a finite system where the beam has a finite radial extension $r_b$, the condition being that $r_b k_{M\perp} \gg 1$ (see [20] for filamentation instability in a finite size beam). As far as FIS quantitative applications are concerned, the major potential restriction of the present study is the fact that the longitudinal beam temperature has been neglected. Taking it into account would substantially increase its difficulty as it would require a full kinetic treatment and may render untractable the already demanding formal computations used in Fig. 2. A useful discussion on the onset of kinetic effects and the breakdown of the cold beam hypothesis may be found in Ref. [21]. Besides, we used there water bag distributions which were simpler to tackle than Maxwellian, but this should only marginally affect the quantitative results obtained.

Let us summarize our point as a conclusion. Relativistic beam filamentation is an observed phenomenon. It is usually associated with the exponential growth of an unstable mode called ''filamentation instability.'' It turns out that a thorough study of every unstable mode reveals that the ''filamentation mode'' should not be the most unstable. Furthermore, this mode is purely transverse and therefore unable to produce charge density perturbations. A better candidate to explain beam filamentation is the most unstable mode all over the **k** space, which turns out to be intermediate between filamentation and two-stream waves. Not only this mode appears to be the fastest growing one, it is also quasilongitudinal so that it can perfectly induce charge density perturbations. A simple evaluation of its growth shows how it creates beam filaments within a few plasma periods and agreement with experiment presented in [7] is found to be correct. It sets the characteristic transverse and longitudinal filamentation scales, at least during the linear initial stage when resistive (collisional) effects are still negligible [20]. Finally, we wish to mention that our study emphasizes the importance of quasilongitudinal modes in modeling filamentation which agrees with some considerations recently put forward by Macchi *et al.* [22] among others.

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