Nonadiabatic Landau-Zener Tunneling in Waveguide Arrays with a Step in the Refractive Index

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Landau-Zener tunneling is discussed in connection with optical waveguide arrays. Light injected in a specific band of the Bloch spectrum in the propagation constant can be transmitted to another band, changing its physical properties. This can be achieved using two coupled waveguide arrays with different refractive indices. The step in the refractive index causes wave "acceleration" and thus induces strongly nonadiabatic Landau-Zener tunneling. Theoretically, the analysis is performed by considering a Schrödinger equation in a periodic potential with a step. The region of physical parameters where this phenomenon can occur is analytically determined and a realistic experimental setup is suggested. Its application could allow the realization of light filters.

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When a quantum system is subject to an external force, a nonadiabatic crossing of energy levels can occur. This phenomenon is known as *Landau-Zener tunneling* [1,2], and some of its recent observations are for Josephson junctions [3] and optical effective two-level systems [4]. On the other hand, the problem of quantum motion in a periodic potential was solved already in the 1920s (see, e.g., [5]) and gives rise to band spectra and Bloch states. Nowadays, the observation of Landau-Zener tunneling between Bloch waves is at the frontier of research in Bose-Einstein condensates (BEC) in optical lattices [6–11]. The external forcing, responsible for Landau-Zener tunneling, is created by either placing the BEC in a gravitational potential [6] or accelerating the optical lattice itself [7].

In this Letter we propose a new and experimentally feasible way of generating Landau-Zener tunneling. We consider waveguide arrays [12], where the periodic potential of the Schrödinger equation is provided by the spatial oscillation of the refractive index in the transversal direction. Tunneling is caused by combining two waveguide arrays with different refractive indices (see Fig. 1). As we will see, this corresponds to creating a *step* in a periodic potential. For arrays of coupled waveguides, the longitudinal direction z (see Fig. 1), along which the refractive index is constant, plays the role of "time" in the stationary regime. The refractive index varies only along the transversal direction x, which represents space. Various linear and nonlinear phenomena have been observed in waveguide arrays: discrete spatial optical solitons [13], diffraction management [14], excitation of Bloch modes [15], generation of multiband optical breathers [16] and of single band-gap solitons [17], and anomalous band-gap transmission regimes [18]. Fast progress in discovering various nonlinear effects in waveguide arrays has been possible due to the introduction of the *tight-binding* approximation [19–21], which reduces the nonlinear Schrödinger equation to the *discrete* nonlinear Schrödinger equation [12]. However, such a reduction eliminates the rich band structure of the periodic medium and only a single Bloch band is left. On the contrary, we want to maintain the band structure and, hence, study transitions between the bands. Indeed, the coupling of two waveguides with different refractive indices introduces a step in the periodic potential and, consequently, Landau-Zener tunneling. We consider here a strongly nonadiabatic limit (sharp steps), because the Landau-Zener tunneling rate is exponentially small in the adiabatic limit [6–11]. For the adiabatic case, we address the reader to Ref. [22], where the observation of Bloch oscillations in waveguide arrays subject to a temperature gradient has been reported. In their Letter, the possibility of producing Landau-Zener tunneling has been mentioned, without suggesting a spe-

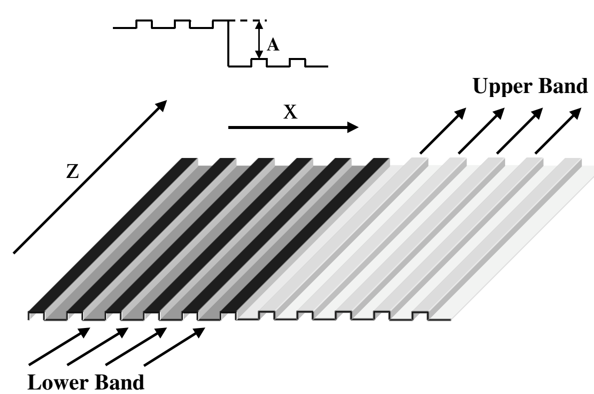

FIG. 1. Schematic picture of the combination of two waveguide arrays of the same spatial period but with different refractive indices (different gray levels in the main plot). The refractive index profile across the array with a step of size A is shown in the upper inset.

cific mechanism. In our Letter we propose a new and efficient mechanism to produce this phenomenon. In the following we study only transitions between the first two bands, denoting them *upper* and *lower* bands. In particular, we propose to inject light into the left waveguide array with a given angle in order to populate the lower Bloch band and retrieve it from the right waveguide array. Below, we derive analytically the step size bounds inside which most of the intensity of the lower band mode is transferred to the upper band mode, creating also a spatial separation between lower and upper band light. We demonstrate this effect by performing numerical simulations of the Schrödinger equation in a periodic potential with a step. The waveguide array refractive index profile in the transversal direction x, which has a periodic rectangular shape (see the upper inset in Fig. 1), is approximated by a harmonic potential, while the step in the refractive index is substituted by a slope $-\alpha$ of height A. Such an approximation allows a simple analytical treatment of the problem. Thus, in the linear regime, the adimensionalized Schrödinger equation of the optical system can be written as follows (see, e.g., Ref. [23]):

$$i\frac{\partial \Psi}{\partial z} + \frac{1}{2}\frac{\partial^2 \Psi}{\partial x^2} + [V(x) + 2w\sin^2 x]\Psi = 0, \qquad (1)$$

where Ψ stands for the complex envelope of the electric field and w is the height of the harmonic potential. Moreover,

$$\begin{aligned} V(x) &= 0 \quad \text{for } x < 0, \\ V(x) &= -A \quad \text{for } x > A/\alpha, \qquad (2) \\ \text{and} \quad V(x) &= -\alpha x \quad \text{for } 0 < x < A/\alpha. \end{aligned}$$

Via the simple transformation $\Psi \to \Psi \exp[izV(x)]$, wave equation (1) gets a well-known form (see Refs. [9–11]):

$$i\frac{\partial \Psi}{\partial z} + \frac{1}{2}\left(\frac{\partial}{\partial x} - i\alpha z\right)^2 \Psi + 2w\sin^2(x)\Psi = 0, \qquad (3)$$

where α plays the role of acceleration in the Landau-Zener phenomenon. Let us note that if the potential would depend only on the z coordinate, then it could be gauged away via the transformation $\Psi \to \Psi \exp[iW(z)]$, with $W'(z) = V(z)$. We should keep in mind that acceleration takes place only within the step region $0 < x < A/\alpha$, unlike the previously considered cases of BEC's, where the whole condensate is accelerated (see, e.g., Ref. [11]).

Outside the step, where acceleration is absent, one can write down the wave functions and the dispersion relations in simple approximate form (see, e.g., [5]):

$$\Psi_{\pm} = \left[\frac{2\kappa \pm \sqrt{w^2 + 4\kappa^2}}{w} e^{-ix} - e^{ix}\right] e^{i(\beta_{\pm} z + \kappa x)}, \qquad (4)$$

$$\beta_{\pm}(\kappa) = \frac{\pm\sqrt{w^2 + 4\kappa^2} - 1 - \kappa^2}{2}, \qquad (5)$$

where $\kappa = K - 1$ is the wave number detuning from the zone boundary and the $+$ $(-)$ sign indicates the upper (lower) band. β is the dimensionless propagation constant. Note that with our conventions the zone-boundary mode wave number is $K = 1$ and that the two mode approximation works better just in the vicinity of zone boundaries ($K \to 1$), exactly where Landau-Zener tunneling takes place.

The dispersion relations (5) for the two bands are schematically shown in Fig. 2. The picture is similar to the one observed in experiments [17]. Let us remark that at the zone boundary ($\kappa \to 0$), the amplitudes of the upper and lower band modes are $|\Psi_+| = 2\sin x$ and $|\Psi_-| = 2\cos x$, respectively. This means that the light intensity in the lower band is concentrated in between waveguide centers, while, in the upper band, intensity is concentrated on waveguide centers. This property is a clear experimental indication whether the wave is in the lower or upper band.

In the step region the wave function Ψ is written as follows:

$$\Psi = [a(z)e^{iKx} + b(z)e^{i(K-2)x}]. \qquad (6)$$

Following Ref. [9], we substitute Eq. (6) into the wave equation (3). Assuming $K = 1$ and removing a common phase dependence in $a(z)$ and $b(z)$ via the transformation $(a, b) \to (a, b) \cdot \exp[-i(z + z^3\alpha/3)]$ (which allows us to consider also the large α limit), we get the Landau-Zener model [1,2] in its original form:

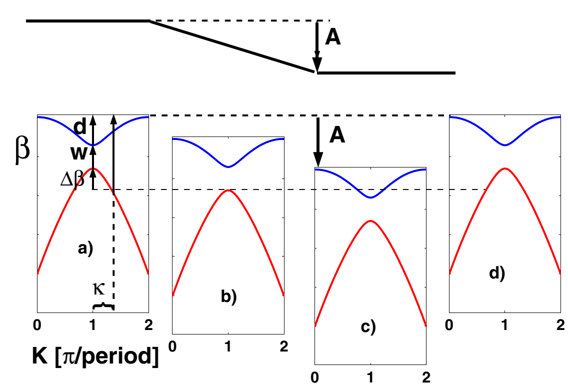

FIG. 2 (color online). Schematic band-gap structure and picture of the Landau-Zener tunneling process. w is the gap between the bands (the height of the period potential), d is the width of the upper band, and κ is the initial detuning of the lower band mode wave number from zone boundary. β is the dimensionless propagation constant [see formula (5)] and $\Delta\beta = \beta_-(0) - \beta_-(\kappa)$. A is the height of the step. Initially, light is injected in the left array, populating the lower band mode (a). Going across the step, κ decreases, reaching zero as the mode approaches the zone boundary causing Landau-Zener tunneling (b). After tunneling, most of the light intensity is transferred to the upper band mode, whose wave number decreases until the end of the step is reached. This is the light we observe in the right array (c). A smaller light intensity remains in the lower band and is observed in the left array (d).

$$i\frac{\partial a}{\partial z} = -\alpha z a + \frac{w}{2} b; \qquad i\frac{\partial b}{\partial z} = \alpha z b + \frac{w}{2} a. \tag{7}$$

Thus, according to Landau-Zener's result, tunneling from the lower zone-boundary mode to the upper band takes place with the following rate [1,2]:

$$r = \exp[-\pi w^2/(4\alpha)]. \tag{8}$$

The experimental setup could be as follows. One should inject a lower band mode with nonzero but small relative wave number κ. This is accomplished by choosing for the light beam a direction forming an angle θ with respect to the z direction, such that $\tan\theta = \kappa$. Hence, the wave front will move towards the step. An analysis of the dependence of the tunneling mechanism on the physical parameters appearing in Fig. 2 shows that the transition to the upper band mode is verified only if the step in the refraction index A fulfills the following inequalities:

$$\Delta\beta + w < A < \Delta\beta + w + d, \tag{9}$$

where $\Delta\beta = \beta_-(0) - \beta_-(\kappa)$ is the variation of the propagation constant between the initial state and the zone boundary and d is the width of the upper band.

Let us try to justify this result by commenting at the same time on the results of some numerical simulations. These are performed by fixing $w = 0.5$, $\kappa = 0.2$ and taking the step size $A = 0.8$ within the above limits (9). Waveguide centers are placed every period, with the first waveguide at half a period from the left boundary. The refractive index step with a slope $\alpha = 2.4$ is placed in the middle of the array. In such a setup the lower band mode cannot overcome the step and when it reaches the zone boundary Landau-Zener tunneling to the upper band occurs. Light is partially transmitted to the right in the upper band and reflected to the left in the lower band. This regime is demonstrated in Fig. 3, which evidences, even visually, the fact that light in the left array is concentrated in between waveguides, while it lies at waveguide centers on the right array. Let us emphasize here that neither the transmitted nor the reflected wave undergo Bloch oscillations. This is due to the fact that the average refractive index is constant everywhere outside the step and changes nonadiabatically only in the step region. An adiabatic regime is needed in order to observe Bloch oscillations in waveguide arrays [22].

In order to prove quantitatively the validity of the use of Landau-Zener formula (8), let us mention that only the right going wave intensity of the Bloch wave function (4), which is normalized to 1, reaches the step. Thus, the transmitted wave intensity will be equal to r. Hence, we can calculate the transmission coefficient T as the ratio of the transmitted wave intensity to the total. Using again formula (4), we finally get

$$T = \frac{w^2}{w^2 + (2\kappa - \sqrt{w^2 + 4\kappa^2})^2} \exp\left[-\frac{\pi w^2}{4\alpha}\right]. \tag{10}$$

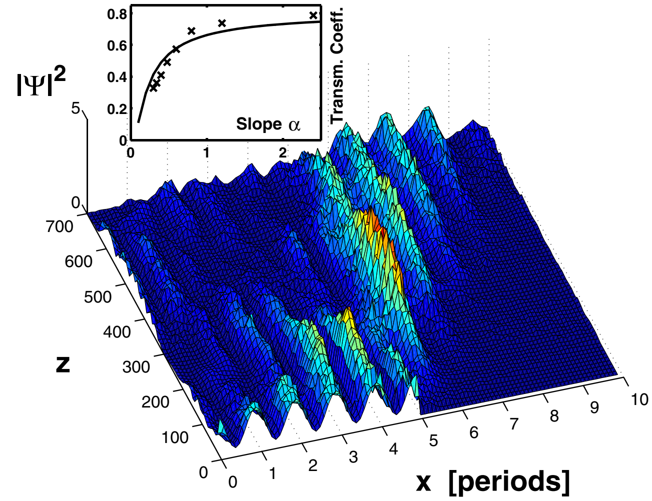

FIG. 3 (color online). Landau-Zener tunneling from the lower to the upper band. The step size $A = 0.8$ is taken within the limits of Eq. (9). A step with a slope $\alpha = 2.4$ is placed at $x = 5$ and the lower band mode is injected into the first five periods. The inset shows the dependence of the transmission coefficient on the step slope α. The solid line corresponds to the analytical formula (10) and the crosses are numerical simulations.

The comparison of this formula with numerical simulations is presented in the inset of Fig. 3. The agreement is very good and, as expected, the transmission coefficient decays exponentially fast in the adiabatic limit $\alpha \rightarrow 0$.

If the step size is higher than the upper bound of Eq. (9), the lower band mode cannot overcome the step and total reflection takes place (see the upper graph of Fig. 4).

On the other hand, if

$$A < \Delta\beta, \tag{11}$$

the lower band mode is able to overcome the step and to penetrate to the right side without tunneling (see the lower graph of Fig. 4). Indeed, wave intensity is now concentrated in between the waveguides and one can conclude that only lower band modes are present in the array.

Finally, if the step is located within the following limits,

$$\Delta\beta < A < \Delta\beta + w, \tag{12}$$

the wave on the left array has no counterpart with the same propagation constant on the right array. Hence, no stationary penetration of the light through the step is possible, more or less as in the upper graph of Fig. 4.

Concluding, a novel type of linear optical tunneling effect is discovered. This is discussed in connection with waveguide arrays which have a spatially oscillating refractive index. We explain this effect resorting to the Landau-Zener model, which is commonly used for accelerated quantum-mechanical two-level systems. In our case, tunneling between different bands takes place while a wave passes through a step in the refractive index. In waveguide arrays, Bloch bands in the propagation constant (light wave number) play the role of energy bands in quantum me-

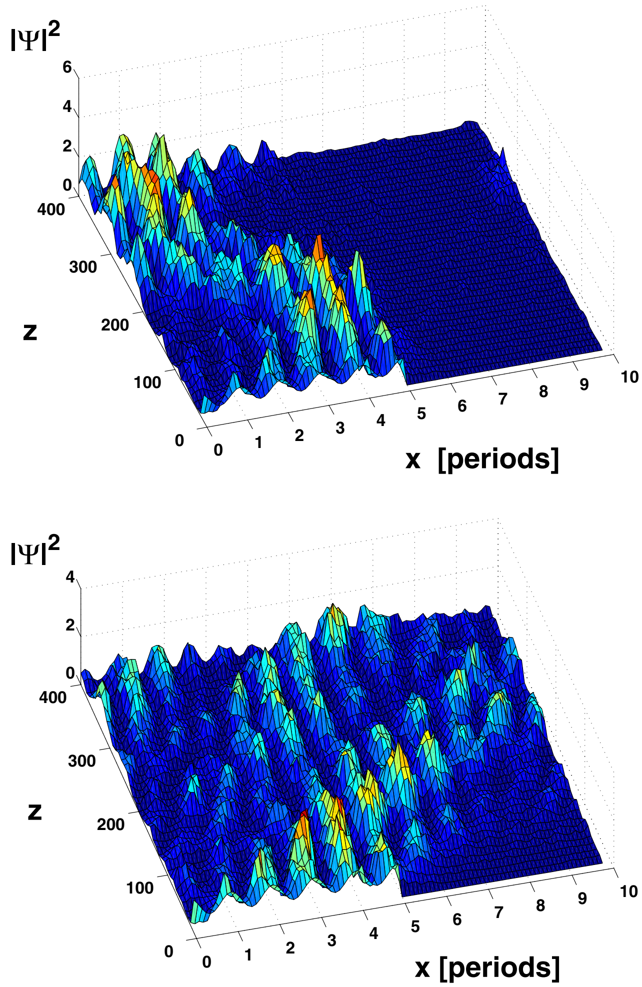

FIG. 4 (color online). Upper graph: total reflection from the step. The step size, $A = 1$, is greater than the upper bound of Eq. (9). Lower graph: penetration through the step without tunneling. The step size, $A = 0.07$, fulfills the inequality (11). In both cases the step slope is infinite.

chanics. Numerical simulations show that, if certain limits in the refractive index step are respected, a spatial separation of light in different bands can be achieved. More interesting for applications is, perhaps, the use of this mechanism to build *light wave number filters*. Indeed, by appropriately choosing the physical parameters, it could be possible to shift the central wave number of a light beam and to reduce the spread in both frequency or wave number. The inclusion of a weak nonlinearity does not qualitatively alter the picture, while strongly nonlinear cases need a separate treatment. In particular, the study of the scattering process of optical gap solitons with the step could be of special interest. Moreover, the effect discussed in this Letter should be generic for systems with periodic potential and applications in different fields could be found. For instance, the analysis of a similar process in Bose-Einstein condensates is in progress.

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