Breached Superfluidity via *p*-Wave Coupling

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Anisotropic pairing between fermion species with different Fermi momenta opens two-dimensional areas of gapless excitations, thus producing a spatially homogeneous state with coexisting superfluid and normal fluids. This breached pairing state is stable and robust for arbitrarily small mismatch and weak *p*-wave coupling.

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Introduction.—Recently there has been considerable interest in the possibility of new forms of superfluidity that could arise when one has attractive interactions between species with different Fermi surfaces. This is stimulated by experimental developments in cold atom systems [1] and by considerations in high-density QCD [2]. Possible coexistence of superfluidity with gapless excitations is an especially important qualitative issue. Spatially homogeneous superfluid states that coexist with gapless modes at isolated points or lines in momentum space arise in a straightforward way when BCS theory is generalized to higher partial waves [3]. Gapless states also are well known to occur in the presence of magnetic impurities [4] and, theoretically, in states with spontaneous breaking of translation symmetry [5], where the gapless states span a two-dimensional Fermi surface. Strong coupling between different bands also may lead to zeros in quasiparticle excitations and gapless states [6]. For spherically symmetric (s-wave) interactions a spherically symmetric *ansatz* of this type naturally suggests itself when one attempts to pair fermions of two different species with distinct Fermi surfaces, and a pairing solution can be found [7–11]. The stability of the resulting state against phase separation [12] or the appearance of a tachyon in the gauge field (negative squared Meissner mass) [13] is delicate, however [14,15]. It appears to require some combination of unequal masses, momentum-dependent pairing interactions, and long-range neutrality constraints. Here we demonstrate another possibility: direction-dependent interactions, specifically p-wave interactions. In this case, stability appears to be quite robust. It seems quite reasonable, intuitively, that expanding an existing (lower-dimensional) locus of zeros into a two-dimensional zone should be significantly easier than producing a sphere of gapless excitations "from scratch." We shall show that it occurs even for arbitrarily small coupling and small Fermi surface mismatch.

Interactions relevant to pairing can be dominated by p-wave (or higher) harmonics under several circumstances. If the s-wave interaction is repulsive, it will not be subject to the Cooper instability, and will not induce pairing. The Cooper instability can be regarded as an enhancement of the effective interaction for attractive channels as one integrates out high-energy modes near the Fermi surface. Thus the effective Hamiltonian will come to resemble the form we assume if the interspecies interactions are repulsive in s wave but attractive in p wave. Fermi statistics forbids diagonal (intraspecies) s-wave interactions; if the higher partial waves are repulsive, or weakly attractive, the model discussed here will apply. In the context of cold atom systems, tuning to an appropriate p-wave Feshbach resonance, as recently reported in [16], can ensure interspecies p-wave dominance.

A crucial difference between the model we consider and the conventional p-wave superfluid system, ^{3}He, lies in the distinguishability of the paired species. So, although there are two components, there is no approximate quasispin symmetry and no analogue of the fully gapped B phase [17]. In the absence of a magnetic field ^{3}He has an approximate $SO(3)_L \times SO(3)_S \times U(1)$ symmetry under separate spatial rotations, spin rotations, and number, which is spontaneously broken to the diagonal $SO(3)_{L+S}$ in the B phase. The residual symmetry enforces a gap of uniform magnitude in all directions in momentum space. The systems we consider have quite different symmetry and breaking patterns, for example, $U(1)_{L_z} \times U(1)_A \times U(1)_B \rightarrow U(1)_{L_z+A+B}$ for two spin-polarized species A, B in a magnetic field, or $SO(3)_L \times U(1)_A \times U(1)_B \rightarrow U(1)_{L_z+A+B}$ if the magnetic field can be neglected. The reduced residual symmetry allows for an interesting direction-dependent structure in momentum space. (In the A phases ^{3}He pairs effectively as two separate single-species systems, which again is quite different from our setup.)

Experimental realizations of p-wave interaction in cold atom systems have been reported recently in Ref. [18]. Feshbach resonance in the p wave occurs between ^{40}K atoms in $f = 9/2$, $m_f = -7/2$ hyperfine states. This is in contrast to the s-wave resonance, which occurs between nonidentical $f = 9/2$, $m_f = -9/2$ and $f = 9/2$, $m_f = -7/2$ states [19]. A promising system for possible observation of the p-wave breached pairing superconductivity is a mixture of $f = 9/2$, $m_f = -9/2$ and $f = 9/2$, $m_f = -7/2$ atoms ^{40}K tuned into the repulsive side of the s-wave Feshbach resonance. Different densities (Fermi momenta) of $m_f = -9/2$ and $m_f = -7/2$ particles can be prepared using different magnitudes of an initial additional magnetic field, which is then removed. Large atomic relaxation times ensure that the created (metastable) states will exist long enough to allow formation of a superfluid phase.

Model.—Having in mind cold atoms in a magnetic trap with atomic spins fully polarized by a magnetic field, we consider a model system with the two species of fermions having the same Fermi velocity v_F, but different Fermi momenta $p_F \pm I/v_F$. The effective Hamiltonian is

$$H = \sum_{\mathbf{p}} [\epsilon_p^A a_{\mathbf{p}}^\dagger a_{\mathbf{p}} + \epsilon_p^B b_{-\mathbf{p}}^\dagger b_{-\mathbf{p}} - \Delta_{\mathbf{p}}^* a_{\mathbf{p}}^\dagger b_{-\mathbf{p}}^\dagger - \Delta_{\mathbf{p}} b_{-\mathbf{p}} a_{\mathbf{p}}] \quad (1)$$

with $\epsilon_p^A = \xi_p + I$, $\epsilon_p^B = \xi_p - I$, $\xi_p = v(p - p_F)$, $\Delta_{\mathbf{p}} = \sum_{\mathbf{k}} V_{\mathbf{p}-\mathbf{k}} \langle a_{\mathbf{k}}^\dagger b_{-\mathbf{k}}^\dagger \rangle$. Here the attractive interspecies interaction is $-V_{\mathbf{p}-\mathbf{k}}$ within the "Debye" energy $2\omega_D$ around the Fermi surface ($\omega_D \gg I$), and the intraspecies interaction is assumed to be either repulsive or negligibly small. Excitations of the Hamiltonian (1), $E_{\mathbf{p}}^{\pm} = \pm\sqrt{\xi_p^2 + \Delta_{\mathbf{p}}^2} + I$, are gapless provided that there are areas on the Fermi surface where $I > \Delta_{\mathbf{p}}$. The gap equation at zero temperature,

$$\Delta_{\mathbf{p}} = \frac{1}{2}\sum_{\mathbf{k}} V_{\mathbf{p}-\mathbf{k}} \frac{\Delta_{\mathbf{k}}}{\sqrt{\xi_k^2 + \Delta_{\mathbf{k}}^2}} \theta\left(\sqrt{\xi_k^2 + \Delta_{\mathbf{k}}^2} - I\right), \quad (2)$$

can be simplified by taking the integral over $d\xi_p$,

$$\Delta_{\mathbf{n}} = \nu \int \frac{d\o_{\mathbf{n}'}}{4\pi} V(\mathbf{n}, \mathbf{n}')\Delta_{\mathbf{n}'}\left[\ln\frac{1}{|\Delta_{\mathbf{n}'}|} + \Theta(I - |\Delta_{\mathbf{n}'}|) \times \ln\frac{|\Delta_{\mathbf{n}'}|}{I + \sqrt{I^2 - \Delta_{\mathbf{n}'}^2}}\right], \quad (3)$$

where $\nu = p_F^2/(2\pi^2 v_F)$ is the density of states. In the last expression it is assumed that I and $\Delta_{\mathbf{n}}$ are dimensionless, and scaled with the Debye energy: $I \to 2\omega_D I$, $\Delta_{\mathbf{n}} \to 2\omega_D \Delta_{\mathbf{n}}$. In deriving Eq. (3) we neglect the dependence of $V_{\mathbf{p}-\mathbf{k}}$ on the absolute values of $\mathbf{p}$ and $\mathbf{k}$; this is valid for $\omega_D \ll E_F$. At weak coupling we may linearize in the partial wave expansion $V(\mathbf{n}, \mathbf{n}') = \sum_{l,m} V_l Y_{lm}(\mathbf{n}) Y_{lm}^\star(\mathbf{n}')$. Assuming p-wave dominance, we parametrize $V(\mathbf{n}, \mathbf{n}') = g(\mathbf{n} \cdot \mathbf{n}')$ with $g > 0$. p-wave gap parameters can arise in the forms Y_{10} and $Y_{1\pm 1}$, describing polar and planar phases, respectively.

Polar phase.—$\Delta_{\mathbf{n}} \sim Y_{10}(\mathbf{n})$. We look for a solution in the form $\Delta_{\mathbf{n}} = \Delta \cos(\mathbf{n}, \mathbf{z})$ where $\mathbf{z}$ is a fixed but arbitrary direction (rotational symmetry is broken). The gap equation becomes

$$-\frac{1}{\nu g} = \int_0^{\pi/2} d\theta \sin\theta \cos^2\theta \ln(\Delta\cos\theta) + \int_{\theta^*}^{\pi/2} d\theta \sin\theta\cos^2\theta \ln\left(\frac{z + \sqrt{z^2 - \cos^2\theta}}{\cos\theta}\right), \quad (4)$$

where $\theta^* = \arccos z$, for $z = I/\Delta < 1$, and $\theta^* = 0$ for $z > 1$. Performing the integrations (detailed calculations will be given elsewhere [20]), we obtain the algebraic gap equation,

$$\ln(1/y) = z^3\frac{\pi}{4} \quad (z < 1),$$
$$\ln\left(\frac{1/y}{z + \sqrt{z^2 - 1}}\right) = -\frac{z}{2}\sqrt{z^2 - 1} + \frac{z^3}{2}\arcsin[z^{-1}] \quad (z > 1), \quad (5)$$

where $y = \Delta/\Delta_0$ is the relative magnitude of the gap compared to its value at $I = 0$,

$$\Delta_0^{\rm pol} = \exp\left(-\frac{3}{\nu g} + \frac{1}{3}\right) \approx 1.40\exp\left(-\frac{3}{\nu g}\right). \quad (6)$$

for weak coupling. There is a factor of 3 in the exponent with anisotropic interaction instead of 1 as in the s-wave BCS. For small values of z the solution to the gap equation is $y = 1 - \frac{\pi x^3}{4}$ with $x = I/\Delta_0$. We depict the solution of the polar phase gap equation in Fig. 1, with the following numerical values of the characteristic points: $x_A = (4/3\pi e)^{1/3} = 0.538$, $y_A = e^{-1/3} = 0.717$ [at point A, $y'(x) \to \infty$], $y_C = e^{-1/3}/2 = 0.358$ [at point C, $y(x) = 0$].

Planar phase.—$\Delta_{\mathbf{n}} \sim Y_{11}(\mathbf{n})$, $\Delta_{\mathbf{n}} \sim Y_{1-1}(\mathbf{n})$. We now look for a solution in the form $\Delta_{\mathbf{n}} = \Delta \sin(\mathbf{n}, \mathbf{z})e^{i\phi}$, where

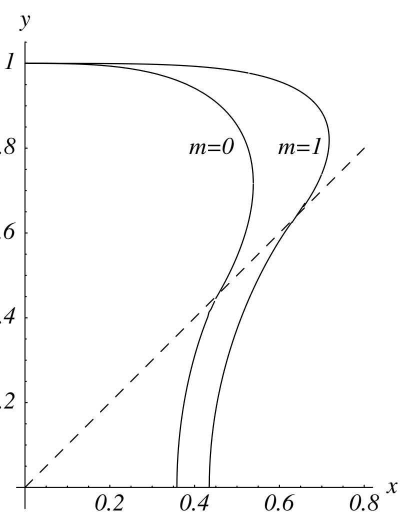

FIG. 1. Solutions $y(x)$ of the gap equation in the polar, $m = 0$, and planar, $m = 1$, phases. The lower branch corresponds to the unstable state. The branches merge at the points where $y'(x) \to \infty$, beyond which there are no nonzero solutions of the gap equation. The broken line $\Delta = I$ is included to guide the eye.

ϕ is the polar angle in the plane perpendicular to $\mathbf{z}$. The gap equation becomes

$$-\frac{2}{\nu g} = \int_0^{\pi/2} d\theta \sin^3\theta \ln(\Delta \sin\theta) + \int_0^{\theta^*} d\theta \sin^3\theta \ln\left(\frac{z + \sqrt{z^2 - \sin^2\theta}}{\sin\theta}\right), \quad (7)$$

where $\theta^* = \arcsin z$, for $z < 1$, and $\theta_0 = \pi/2$, for $z > 1$. Performing the integration we obtain the algebraic gap equation,

$$\ln(1/y) = -\frac{z^2}{4} + \frac{z}{8}(3 + z^2)\ln\left|\frac{1+z}{1-z}\right| + \frac{1}{2}\ln|1 - z^2|, \quad (8)$$

where again y is the relative magnitude of the gap compared to its value for a zero mismatch. For the planar phase,

$$\Delta_0^{pl} = \frac{1}{2}\exp\left(-\frac{3}{\nu g} + \frac{5}{6}\right) \approx 1.15\exp\left(-\frac{3}{\nu g}\right). \quad (9)$$

For small values of z the solution to the gap equation has the form $y = 1 - \frac{3x^4}{4}$ and x is defined as before. Note that the planar phase gap is more robust than the polar phase, being perturbed by the fourth power instead of the third. Solution of the gap equation for the planar phase is depicted in Fig. 1, with the following numerical values of the characteristic points: $x_A = 0.674$, $y_A = 0.787$, $z_A = x_A/y_A = 0.856$; $x_C = e^{-5/6} = 0.435$.

Stability.—The condensation energy is given by (at $T = 0$)

$$\Omega_s - \Omega_n = \nu \int \frac{d\phi_{\mathbf{n}}}{4\pi}\left[-\frac{|\Delta_{\mathbf{n}}|^2}{2} + I^2 - I\sqrt{I^2 - |\Delta_{\mathbf{n}}|^2}\Theta(I - |\Delta_{\mathbf{n}}|)\right]. \quad (10)$$

Evaluating this expression for $z = I/\Delta < 1$, we obtain for polar phase

$$\Omega_s - \Omega_n = \nu\Delta^2\left(-\frac{1}{6} - \frac{\pi z^3}{4} + z^2\right), \quad (11)$$

which is negative for $z < 0.537$, and for planar phase

$$\Omega_s - \Omega_n = \nu\Delta^2\left(-\frac{1}{3} + \frac{z^2}{2} + \frac{z(1-z^2)}{4}\ln\frac{1+z}{1-z}\right), \quad (12)$$

which is negative for $z < 0.623$. For our specific model Hamiltonian, at weak coupling, the planar phase is more stable. For $I > \Delta$ the condensation energy is always positive, indicating that the lower branches are unstable.

Following the standard methods in the theory of superconductivity [21] we calculate the supercurrents in our system under the influence of the homogeneous in space vector potential $\mathbf{A}$. The supercurrent is anisotropic, $j_i = \frac{e^2N}{m}\kappa_{ik}A_k$ with the components given by ($\kappa_{xx} = \kappa_{yy}$)

$$\begin{bmatrix}\kappa_{zz}\\ \kappa_{xx}\end{bmatrix} = 1 - \frac{3I}{2}\int\frac{d\phi_{\mathbf{n}}}{4\pi}\begin{bmatrix}\cos^2\theta\\ \sin^2\theta\end{bmatrix}\frac{\Theta(I - |\Delta_{\mathbf{n}}|)}{\sqrt{I^2 - |\Delta_{\mathbf{n}}|^2}}. \quad (13)$$

For the polar phase, assuming $z > 1$ we find

$$\begin{bmatrix}\kappa_{zz}\\ \kappa_{xx}\end{bmatrix} = \begin{matrix}1 - 3z^3/4\pi,\\ 1 - 3\pi z/4 + 3\pi z^3/8.\end{matrix} \quad (14)$$

The coefficient κ_{xx} becomes negative at $z \geq 0.480$ (κ_{zz} at higher values of $z \geq 0.752$), indicating an instability with respect to a transition into some inhomogeneous state [probably similar to a Larkin-Ovchinnikov-Fulde-Ferrel (LOFF) state]. For the planar phase,

$$\begin{bmatrix}\kappa_{zz}\\ \kappa_{xx}\end{bmatrix} = 1 \mp \frac{3z^2}{4} - \frac{3z}{8}(1 \mp z^2)\ln\left(\frac{1+z}{1-z}\right). \quad (15)$$

The coefficients κ_{xx} and κ_{zz} always remain positive for the whole range of $z < 0.623$ where the gap Eq. (8) has stable solutions. Thus, we find that the planar phase has a lower energy and a higher density of Cooper pairs than the polar phase and is therefore more stable.

Specific heat.—The important manifestation of the BCS states with gapless excitations is the appearance of the term linear in temperature in the specific heat, which is characteristic for a normal Fermi liquid. The specific heat is given by

$$C = \sum_{\mathbf{p}}\left(E_{\mathbf{p}}^+\frac{\partial n(E_{\mathbf{p}}^+)}{\partial T} + E_{\mathbf{p}}^-\frac{\partial n(E_{\mathbf{p}}^-)}{\partial T}\right), \quad (16)$$

where $E_{\mathbf{p}}^\pm = \pm\sqrt{\xi_p^2 + \Delta_{\mathbf{n}}^2} + I$. At low temperatures $T \ll I$ the first term in Eq. (16) gives an exponentially small contribution. The second term, with E^-, in Eq. (16) is

$$C = \frac{\nu}{4T^2}\int_{-\infty}^{\infty} d\xi \int\frac{d\phi_{\mathbf{n}}}{4\pi}\frac{(\sqrt{\xi^2 + |\Delta_{\mathbf{n}}|^2} - I)^2}{\cosh^2[\frac{\sqrt{\xi^2+|\Delta_{\mathbf{n}}|^2}-I}{2T}]}. \quad (17)$$

Performing the integration, we calculate the contribution of the gapless modes to the specific heat at $T \ll I$ to be

$$C = \nu T\frac{\pi}{6}\begin{cases}\pi z, & \text{polar phase,}\\ 4z^2, & \text{planar phase.}\end{cases} \quad (18)$$

As expected, the "normal" contribution to the specific heat is proportional to the area occupied by the gapless modes, i.e., the I/Δ strip around the equator for the polar phase and the I^2/Δ^2 islands around the poles for the planar phase.

Conclusion and comments.—We have presented substantial evidence that our simple model supports the planar phase gapless superfluidity in the ground state. For $I \ll \Delta$ the gapless modes contribute high powers in terms of mismatch, $\sim I^4$ for the solution and $\sim I^2$ for the heat capacity; i.e., they represent small perturbations. The residual continuous symmetry of this state, and its favorable energy relative to plausible competitors (normal state, polar phase) suggest that it is a true ground state in this model. The planar phase is symmetric under simultaneous axial rotation and gauge (i.e., phase) transformation. Also, we obtain a positive density of superconducting electrons, suggesting that inhomogeneous LOFF phases are disfavored at small I.

In some respects the same qualitative behavior we find here in the p wave resembles what arose in the s wave [14]. Namely, isotropic s-wave superconductivity has two branches of solution: the upper BCS which is stable and—for simple interactions— fully gapped, and the lower branch which has gapless modes but is unstable. The striking difference is that in the p wave the upper branch retains stability while developing a full two-dimensional Fermi surface of gapless modes. Thus, the anisotropic p-wave breached pair phase, with coexisting superfluid and normal components, is stable already for a wide range of parameters at weak coupling using the simplest (momentum-independent) interaction. This bodes well for its future experimental realization.

In our model, which has no explicit spin degree of freedom, gapless modes occur for either choice of order parameter with residual continuous symmetry. By contrast, for ^{3}He in the B phase the p-wave spin-triplet order parameter is a 2×2 spin matrix, containing both polar and planar phases components, there are no zeros in the quasiparticle energies, and the phenomenology broadly resembles that of a conventional s-wave state [17]; in the A phase (which arises only at $T \neq 0$ [22]) the separate up and down spin components pair with themselves, in an orbital p wave, and no possibility of a mismatch arises.

Experimentally, the microscopic nature of the pairing state can be revealed most directly by probing the momentum distribution of the fermions, including angular dependence. Time of flight images, obtained when trapped atoms are released from the trap and propagate freely, reflect this distribution.

It is possible that the emergent Fermi gas of gapless excitations develops, as a result of residual interactions, secondary condensations. Also, one may consider analogous possibilities for particle-hole, as opposed to particle-particle, pairing. In that context, deviations from nesting play the role that the Fermi surface mismatch plays in the particle-particle case. We are actively investigating these issues.

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