Teleportation of Electronic Many-Qubit States Encoded in the Electron Spin of Quantum Dots via Single Photons

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We propose a teleportation scheme that relies only on single-photon measurements and Faraday rotation, for teleportation of many-qubit entangled states stored in the electron spins of a quantum dot system. The interaction between a photon and the two electron spins, via Faraday rotation in microcavities, establishes Greenberger-Horne-Zeilinger entanglement in the spin-photon-spin system. The appropriate single-qubit measurements, and the communication of two classical bits, produce teleportation. This scheme provides the essential link between spintronic and photonic quantum information devices by permitting quantum information to be exchanged between them.

The information contained in a quantum two-level system cannot be fully copied. This limitation, which is a special case of the no-cloning theorem [1], describes a fundamental difference between classical and quantum information. For a quantum information processor the operation that can partially replace copying is the ability to transfer quantum information from one system to another. When the transfer is separated into a channel of classical information and one of Einstein-Podolsky-Rosen (EPR) [2] correlations it is called "quantum teleportation". One of the most spectacular achievements in the manipulation of quantum information [3] is the successful teleportation of the quantum superposition of a single-photon state [4] by means of entangled photon pairs [5]. EPR teleportation of a single photonic qubit, as originally introduced theoretically [5], and realized experimentally [4,6], requires both the generation of one maximally entangled two-photon pair (a "Bell state") and a two-photon entangled measurement of one member of this pair along with the original photonic qubit (a "Bell measurement") [7]. The original teleportation scheme only permitted the transfer of a single-qubit state, but recently, this scheme has been extended theoretically to the case of single-qudit states [8] (the higher-dimensional version of single-qubit states), two-qubit states [9], and three-qubit states [10]. It has also recently been shown experimentally that the entanglement between two qubits can be teleported, again by means of Bell measurements [11]. Teleportation of single ionic spin states [12,13] has also been demonstrated recently, and the Bell measurements were implemented by a Raman phase gate applied to two ions, followed by single-ion spin measurements.

Here we propose a teleportation scheme for an arbitrary number of electronic qubits that does not require an intermediate electronic qubit, or the use of external lasers to implement a phase gate. Instead of generating and measuring Bell states between the electronic qubits, this scheme relies on entangling both of the qubits with a single photon, yielding three-particle entanglement [a Greenberger-Horne-Zeilinger (GHZ) state] [14] of the qubit-photon-qubit Hilbert space. Any qubit that can be entangled with a photon can be used, but for specificity we consider here qubits encoded in the electron spin of individual quantum dots. Figure 1 shows our teleportation scheme for teleporting a many-qubit state from D to D'. In the specific approach we describe, the establishment of spin-photon entanglement occurs naturally through conditional Faraday rotation in a microcavity. We emphasize that the entanglement of the destination qubit and the photon can be performed first, and this photon can be retained at the origin before it is entangled with the origin qubit; thus our pro-
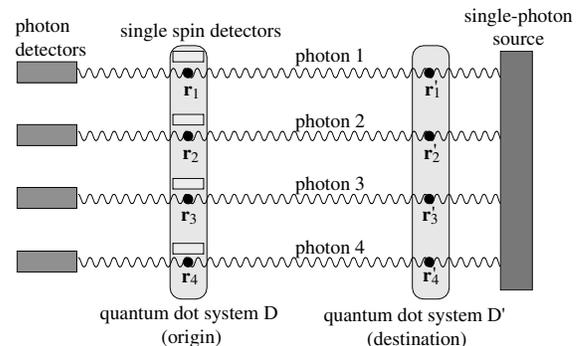

FIG. 1. Teleportation of the many-qubit state located initially in the quantum dot system D to the quantum dot system D'. Each quantum dot of D at $\mathbf{r}_j$ is connected to the quantum dot of D' at $\mathbf{r}'_j$ through the photon j. This requirement can be satisfied using quantum dots of different sizes, where each pair of dots at $\mathbf{r}_j$ and $\mathbf{r}'_j$ have the same size. Then each pair of dots can be connected by a photon with the proper resonant frequency. Another approach could use fiber optics to ensure a unique connection between each pair of dots at $\mathbf{r}_j$ and $\mathbf{r}'_j$. The independent single-spin detection apparatus is sketched in Fig. 2.

cedure is teleportation, not quantum transmission. The
sending of the photon from the destination to the origin,
after it is entangled with the destination qubit, is the step
corresponding to the distribution of EPR pairs in telepor-
tation. Our EPR pairs, however, are a hybrid consisting of
an entangled electronic spin and photon polarization. We
find teleportation can be implemented using only single-
photon measurements: measurement of the polarization of
the photon entangled with both qubits, and measurement of
the spin orientation of the origin qubit via a single photon.

We describe below in detail our teleportation scheme for
one qubit (see Fig. 2). We consider one excess electron
in one quantum dot in a general single-spin state $|\psi_{\mathrm{e}}^{(1)}\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$, where the quantization axis is the z axis. In
order to distinguish clearly each step of the teleportation,
we introduce the times $t_A < t_A + T < t_B < t_C < t_D$, where
$t_A = 0$. The photon propagating in the $-z$ direction is
initially linearly polarized in the x direction, and interacts
first with the destination spin, which is initialized parallel
to x. Thus the destination spin-photon wave function at $t =
0$ is $|\psi_{\mathrm{pe'}}^{(1)}(0)\rangle = |\leftrightarrow\rangle|\leftarrow'\rangle$. The orientation of the spin af-
fects the possible virtual processes excited by the photon,
two of which are shown in Fig. 3. For spin-$\uparrow$, light of $\sigma_{(z)}^+$
polarization excites an electron and heavy hole (upper
right, Fig. 3), and light of $\sigma_{(z)}^-$ polarization excites an
electron and a light hole (lower left, Fig. 3). Selection rules
and Pauli blocking force the light subsequently emitted
from electron-hole recombination to have the same polar-
ization as was absorbed, and the final occupations of
electron states in the dot (lower right, Fig. 3) are the
same as the initial occupations. These virtual processes,
however, lead to Faraday rotation: a phase shift $e^{iS_0^{\mathrm{hh}}}$ of $\sigma_{(z)}^+$
and $e^{iS_0^{\mathrm{lh}}}$ of $\sigma_{(z)}^-$. If, however, the spin were $\downarrow$ instead the
phase shift of $\sigma_{(z)}^+$ light would be $e^{iS_0^{\mathrm{lh}}}$ and of $\sigma_{(z)}^-$ would be

$e^{iS_0^{\mathrm{hh}}}$. Thus there is a *conditional Faraday rotation* of the
components of the electron-photon state depending on the
photon polarization and spin orientation.

After the interaction of the initially unentangled photon
with the spin in the quantum dot, the state is

$$|\psi_{\mathrm{pe'}}^{(1)}(T)\rangle = e^{iS_0^{\mathrm{hh}}}|\psi_{\mathrm{hh}}^{(1)}\rangle + e^{iS_0^{\mathrm{lh}}}|\psi_{\mathrm{lh}}^{(1)}\rangle, \quad (1)$$

where $|\psi_{\mathrm{hh}}^{(1)}\rangle = (|\sigma_{(z)}^+\rangle|\uparrow'\rangle + |\sigma_{(z)}^-\rangle|\downarrow'\rangle)/2$ originates from
the virtual process where a photon creates an electron
and a heavy hole, and $|\psi_{\mathrm{lh}}^{(1)}\rangle = (|\sigma_{(z)}^-\rangle|\uparrow'\rangle + |\sigma_{(z)}^+\rangle|\downarrow'\rangle)/2$
originates from the virtual process where the photon cre-
ates an electron and a light hole. Both $|\psi_{\mathrm{hh}}^{(1)}\rangle$ and $|\psi_{\mathrm{lh}}^{(1)}\rangle$ are
EPR states. We now define the photon state $|\varphi\rangle = \cos\varphi|\leftrightarrow\rangle + \sin\varphi|\updownarrow\rangle$ with a linear polarization rotated by
φ around the z axis with respect to the state $|\leftrightarrow\rangle$ of linear
polarization in the x direction. We can also write $|\varphi\rangle = (e^{-i\varphi}|\sigma_{(z)}^+\rangle + e^{i\varphi}|\sigma_{(z)}^-\rangle)/\sqrt{2}$. Consequently,

$$|\psi_{\mathrm{pe'}}^{(1)}(T)\rangle = \frac{e^{i(S_0^{\mathrm{hh}}+S_0^{\mathrm{lh}})/2}}{\sqrt{2}}(|-S_0/2\rangle|\uparrow'\rangle + |+S_0/2\rangle|\downarrow'\rangle), \quad (2)$$

where $S_0 = S_0^{\mathrm{hh}} - S_0^{\mathrm{lh}}$. Thus the spin-photon interaction
produces a conditional single-photon Faraday rotation
around the z axis by the angle $\pm S_0/2$. If $S_0 = \pi/2$,
the linear polarization of the incoming photon is rotated
$-\pi/4$ by the spin up component, and at the same time
is rotated $+\pi/4$ by the spin down component, yielding
two orthogonal photon polarizations. Thus $|\psi_{\mathrm{pe'}}^{(1)}(T)\rangle = (|\searchadjust\rangle|\uparrow'\rangle + |\nearrow\rangle|\downarrow'\rangle)/\sqrt{2}$, which is maximally entangled.
To enhance the spin-photon interaction sufficiently to
achieve $S_0 = \pi/2$, each quantum dot should be placed in
its own microcavity (as shown in Fig. 2). Using a switch-
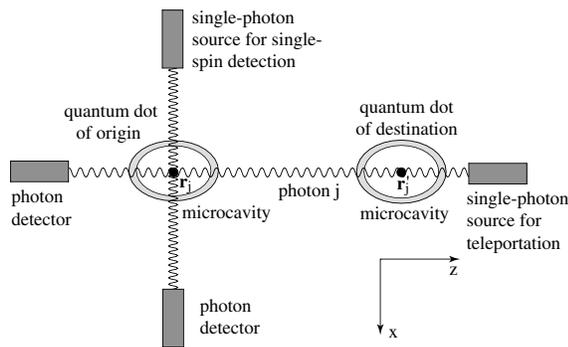able cavity, as will be described below, permits the precise

FIG. 2.   Teleportation of the many-qubit state can be regarded
as the independent teleportation of each spin state from $\mathbf{r}_j$ to $\mathbf{r}_j'$,
mediated by photons traveling in the $-z$ direction. Each dot is
embedded in a microcavity with different lengths in z and x, so
that the scattered photon cannot change its direction. Single-spin
detection is performed by photons propagating along x. All the
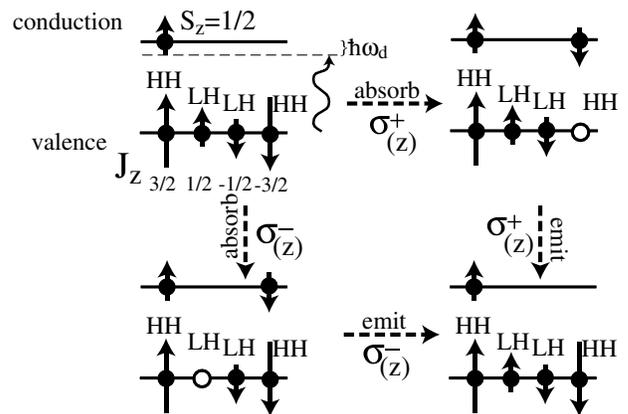photons are measured independently from each other.

FIG. 3.   Selection rules for a photon propagating in the z
direction and an excess spin-$\uparrow$ electron in the dot. $\sigma_{(z)}^+$ can be
virtually absorbed (upper right) by exciting an electron and a
heavy hole, and $\sigma_{(z)}^-$ by exciting an electon and a light hole
(lower left). $\hbar\omega_d$ is the detuning of the nonresonant photon.

control of the Faraday rotation angle $S_0/2$ necessary for high fidelity teleportation. After interacting with the spin at D' the photon is sent to D, and can be retained as a resource for teleportation from D to D' until the spin at D decoheres. For coherence times of 100 ns [15] the maximum tele-

portation distance (determined by the speed of light) would be $\sim$15 m and for 10 ms [16] it is 1.5×10^6 m.

When it is time to teleport the spin at D to D' we let the photon interact with the quantum dot at D, giving rise to a GHZ state in the hybrid spin-photon-spin system,

$$|\psi_{\text{epe}'}^{(1)}(t_C)\rangle = e^{i(S_0^{\text{hh}} + S_0^{\text{lh}})/2}(\alpha|\uparrow\rangle| - S_0/2 - \pi/4\rangle|\uparrow'\rangle + \alpha|\uparrow\rangle| - S_0/2 + \pi/4\rangle|\downarrow'\rangle$$
$$+ \beta|\downarrow\rangle| + S_0/2 - \pi/4\rangle|\uparrow'\rangle + \beta|\downarrow\rangle| + S_0/2 + \pi/4\rangle|\downarrow'\rangle)/\sqrt{2}. \tag{3}$$

Choosing $S_0 = \pi/2$, we obtain

$$|\psi_{\text{epe}'}^{(1)}(t_C)\rangle = [|\updownarrow\rangle(-\alpha|\uparrow\rangle|\uparrow'\rangle + \beta|\downarrow\rangle|\downarrow'\rangle) + |\leftrightarrow\rangle(\alpha|\uparrow\rangle|\downarrow'\rangle + \beta|\downarrow\rangle|\uparrow'\rangle)]/\sqrt{2}. \tag{4}$$

Changing to the x representation for the spin at D yields

$$|\psi_{\text{epe}'}^{(1)}(t_C)\rangle = \{|\updownarrow\rangle[|\leftarrow\rangle(-\alpha|\uparrow'\rangle + \beta|\downarrow'\rangle) + |\rightarrow\rangle(-\alpha|\uparrow'\rangle - \beta|\downarrow'\rangle)] + |\leftrightarrow\rangle[|\leftarrow\rangle(\beta|\uparrow'\rangle + \alpha|\downarrow'\rangle) + |\rightarrow\rangle(\beta|\uparrow'\rangle - \alpha|\downarrow'\rangle)]\}/2. \tag{5}$$

Measurements will complete the teleportation. If the linear polarization of the photon is measured first, then depending on the two initial spin orientations [see Eq. (4)], collapse of the wave function leaves the qubits at D and at D' in one of the four Bell states. After performing a single-spin measurement in the x direction of the spin at D, the spin state at D' is projected onto [see Eq. (5)] $|\psi_{\text{e1}}^{(1)}(t_D)\rangle = -\alpha|\uparrow'\rangle + \beta|\downarrow'\rangle$, $|\psi_{\text{e2}}^{(1)}(t_D)\rangle = -\alpha|\uparrow'\rangle - \beta|\downarrow'\rangle$, $|\psi_{\text{e3}}^{(1)}(t_D)\rangle = \beta|\uparrow'\rangle + \alpha|\downarrow'\rangle$, or $|\psi_{\text{e4}}^{(1)}(t_D)\rangle = \beta|\uparrow'\rangle - \alpha|\downarrow'\rangle$ with equal probability. These projections correspond exactly to the states obtained in Ref. [5]. After communicating classically the outcome of the measurements of the linear polarization of the photon and D's spin orientation along x to D', the original spin state of D can be reconstructed at D' and teleportation is complete. The same amount of classical communication (2 bits) is required for our approach as was required in Ref. [5]. However, no intermediate electronic qubit was required, and the measurements are simply performed with single photons.

Teleportation for two or more dots can still be performed bit by bit, so long as photon j ($j = 1, 2, 3, \dots$) coming from the dot at $\mathbf{r}'_j$ of D' travels to the dot at $\mathbf{r}_j$ of D (see Figs. 1 and 2). Teleportation is mediated by photons that scatter independently off the dots and the conditional phase shifts from each spin can be treated independently. This approach provides a method of teleporting a many-qubit state of an arbitrary number of qubits, always relying only on single-photon measurements.

Faraday rotation to entangle the photon and electron spin also provides the way to measure the spin with a single photon. We assume that our microcavities have an additional resonant mode at a different frequency for photons propagating in the x direction. Equation (1) shows that if the spin on the quantum dot points in the $+x$ ($-x$) direction, an incoming linearly polarized photon is converted into an outgoing circularly polarized photon $\sigma_{(x)}^+$ ($\sigma_{(x)}^-$). Measuring the circular polarization of the photon

after it escapes yields the spin orientation along x. Electrical single-spin measurements at D could use instead a single electron transistor, converting the spin information to charge information [17,18]. Each of these steps could be performed with high fidelity (time-correlated single-photon counting permits a counting efficiency $\sim$1 [19]).

The spin-selective coupling between the electron spins and the photons, which leads to their mutual entanglement, is enhanced by surrounding each of the dots by its own individual high-Q microcavity [20]. Each microcavity has a single well-defined left-circularly polarized photon mode (and a right-circularly polarized photon mode of identical frequency) nearly resonant with the fundamental optical transition of the dot. As we rely on nonresonant interaction of photons both in the z and x directions, the four highest-energy valence states should be nearly degenerate (corresponding to nearly spherical dots of zincblende or wurtzite material); i.e., their energy difference is much smaller than the detuning energy $\hbar\omega_d$. Then, in both z and x, we get a spin-selective conditional Faraday rotation.

The Faraday rotation from the virtual process in which a photon creates an electron and heavy hole is 3 times larger than that from the virtual process in which a photon creates an electron and light hole, and is in the opposite direction [21]. The transition matrix elements for exciting an electron and heavy hole (V_{hh}) or an electron and light hole (V_{lh}) are calculated from the electron-photon interaction Hamiltonian $e\mathbf{A} \cdot \mathbf{p}/2mc$, where $\mathbf{p}$ is the momentum of an electron and $\mathbf{A}$ is the vector potential of the electromagnetic field. The full Hamiltonian in the rotating frame,

$$\mathcal{H} = \begin{pmatrix} E_c - \hbar\omega_d & V_{\text{hh}} & 0 & 0 \\ V_{\text{hh}} & E_c & 0 & 0 \\ 0 & 0 & E_c - \hbar\omega_d & V_{\text{lh}} \\ 0 & 0 & V_{\text{lh}} & E_c \end{pmatrix}, \tag{6}$$

where E_c is the energy of the excess electron. The basis is $\{|\uparrow\rangle|\sigma_{(z)}^+\rangle; |\uparrow\downarrow, \text{hh}\rangle; |\uparrow\rangle|\sigma_{(z)}^-\rangle; |\uparrow\downarrow, \text{lh}\rangle\}$; the states $|\uparrow\downarrow, \text{hh}\rangle$ and

$| \uparrow\downarrow, \text{lh} \rangle$ are shown in Fig. 3. The Hamiltonian for spin-down is similar. The rates of Faraday rotation for these virtual transitions are $\Omega_{\text{hh}} = V_{\text{hh}}^2/\hbar^2\omega_{\text{d}} = 3V_{\text{lh}}^2/\hbar^2\omega_{\text{d}}$ and $\Omega_{\text{lh}} = V_{\text{lh}}^2/\hbar^2\omega_{\text{d}}$. Thus the phase shift accumulated by the photon state during the photon's residence time T in the microcavity is given by $S_0^{\text{hh}} = \Omega_{\text{hh}}T$ and $S_0^{\text{lh}} = \Omega_{\text{lh}}T$ for heavy and light holes, respectively. If the total phase shift is $S_0 = S_0^{\text{hh}} - S_0^{\text{lh}} = \pi/2$ (modulo 2π), the photon and electron spin become maximally entangled.

We desire the interaction strength between the photon and the quantum dot transition to be weak, i.e., $V_{\text{hh}}, V_{\text{lh}} \ll \hbar\omega_{\text{d}}$. The frequency of the photon is tuned below the bandgap E_{gap}, which leads to nonresonant interaction. Typical values for the bandgap and the level broadening are $E_{\text{gap}} = 1$ eV and $\Gamma = 10\ \mu\text{eV}$ (see Ref. [22]), respectively. The interaction time between the photon and the electron spin is about $T = 1$ ns (much smaller than the limiting spin decoherence time in semiconductor nanostructures [15,16]), leading to a bandwidth of $\Gamma_{\text{photon}} = 0.7\ \mu\text{eV}$. If the size of the microcavity is $3.5\ \mu\text{m}^3$ [23], V_{hh} is typically $50\ \mu\text{eV}$. Thus for a reasonable choice $\hbar\omega_{\text{d}} \approx 1.5$ meV $\gg \Gamma, \Gamma_{\text{photon}}$, we have $\Omega_0 = \frac{\pi}{2} \times 10^9\text{s}^{-1}$, and consequently $S_0 = \pi/2$.

To control the interaction time T precisely the microcavity should be actively Q switched. Response times of electro-optic modulators can be <1 ps (see, e.g., Ref. [24]), which leads to a phase error of the order of 1 ps/1 ns = 0.1%. The Q factor can be as high as $Q = 1.25 \times 10^8$, which is equivalent to a photon lifetime of $\tau = 43$ ns (see Ref. [25]). Although this Q was achieved for a $10^5\ \mu\text{m}^3$ cavity, the same processing could be applied to $3.5\ \mu\text{m}^3$ cavities. The theoretical limit [26] is $Q \sim 10^{13}$. Insertion of a spherical (colloidal) quantum dot into a 2D or even 3D photonic crystal with holes [27,28] would likely be simplest. After $T = 1$ ns in the small cavity, before Q switching, the escape probability is $1 - e^{-T/\tau} = 2\%$. Thus the entangled state can be produced with high fidelity. This estimate also applies to the fidelity of the required single-shot photon measurements of the electron spin orientation. Transmission into or out of the cavity should have much greater than 99% fidelity—commercial highly transmissive coatings have reflectivities $\ll 0.1\%$. Thus the total fidelity of the teleportation may exceed 97%.

Our teleportation scheme also provides a general link between spintronic quantum information devices and photonic ones. Letting the photon interact only with the spin at D gives the possibility to transfer $|\psi_{\text{e}}(0)\rangle$ onto the photon state, and back. As one example of a general class of optospintronic quantum information devices we suggest a quantum dynamic RAM memory [3]. In such a device the many-spin state would be transferred to the many-photon state and back. As the decoherence time of photons exceeds that of the spins of electrons, it would be useful to keep the quantum information encoded as photons between error-correcting operations acting on the electron spins.

Thus the refresh time could be much longer than the decoherence time of the electrons. This could enhance the maximum distance possible for teleportation beyond that determined from the electron spin coherence time. Now that an efficient method of transferring quantum information between spintronic and photonic systems is available, many other such devices can be imagined which also exploit the complementary advantages of spintronic and photonic quantum information processing.

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