Experimental Investigation of Local Properties and Statistics of Optical Vortices in Random Wave Fields

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We present the first direct experimental evidence of the local properties of optical vortices in a random laser speckle field. We have observed the Berry anisotropy ellipse describing the anisotropic squeezing of phase lines close to vortex cores and quantitatively verified the Dennis angular momentum rule for its phase. Some statistics associated with vortices, such as density, anisotropy ellipse eccentricity, and its relation to zero crossings of real and imaginary parts of the random field, are also investigated by experiments.

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Vortices or phase singularities in optical fields have been known for a long time, and extensive studies have been made on their basic properties since the seminal work of Nye and Berry in the early 1970s [1]. Recently, phase singularities have come to attract more attention because they are closely related to both the spiral solutions of the complex Ginzburg-Landau equation [2] in nonlinear optics [3] and in laser physics [4], and to defects in materials [5]. As pointed out by Freund [6], although phase singularities have found their major application in the study of nonlinear optical processes and particle manipulation [7], they are, in fact, of the greatest intrinsic importance in linear scattering of optical waves from random media. The density of optical vortices has been given theoretically by Berry [8], early experiments on the density of optical vortices have been conducted by Baranova *et al.* who first found out that the density of phase singularities is of the order of the density of speckles [9], and numerical simulation of the optical vortices in random wave fields has been performed by Freund and Staliunas *et al.* [6,10,11]. Subsequently, Berry and Dennis [12] have theoretically investigated the statistical characteristics of vortices in a random field in more detail. Here, we report the first experimental evidence of the core structure of phase singularities in a random speckle field, and conduct direct experimental verifications of the statistical properties of optical vortices as predicted by Berry and Dennis.

We detected the random wave field with a Mach-Zehnder type interferometer, as depicted in Fig. 1. Linearly polarized light from a He-Ne laser was split into two beams by the beam splitter (BS1). To adjust the speckle size and control the density of phase singularities, a 10$\times$ microscope objective (MO1) is slid back and forth to produce a proper illumination spot size on a 220-grit sandblasted ground glass plate (GG). To avoid aliasing errors in the sampling process, we chose the average speckle size carefully so that a single speckle includes 20 pixels to 100 pixels along the line traversing it. The adjustment of a 10$\times$ microscope objective (MO2) requires special care since the curvatures of the two wave fronts should match exactly in order to create the isotropic speckle field. This so-called lensless Fourier transform geometry produces an interferogram for the Fraunhofer field of the fully developed speckle when the light is scattered from the ground glass. From the interferogram recorded by a charge-coupled device (CCD) camera (For-A HMC-1170) with the pixel size 6.7 μm $\times$ 6.7 μm, we can directly reconstruct the complex random wave field by using the Fourier transform method [13].

Usually, a complex scalar wave field in two dimensions may be expressed as

$$\tilde{U}(x, y) = \mathrm{Re}(x, y) + j\,\mathrm{Im}(x, y) = A(x, y)\exp\{j\theta(x, y)\}, \quad (1)$$

where Re and Im are the real and imaginary parts of the optical field, $A \geq 0$ is the amplitude, and θ is the phase. Figure 2(a) shows an example of the reconstructed real part of the complex random wave field around a phase singularity, and Fig. 2(b) is the corresponding imaginary part with the contour lines $\mathrm{Re} = 0$ and $\mathrm{Im} = 0$ inserted, respectively. The phase singularity, occurring at the crossing of the zero-contour lines, is a point in the plane. The reconstructed phase is shown in Fig. 2(c) where the phase is a 2π helix around the phase singularity. Note that, in

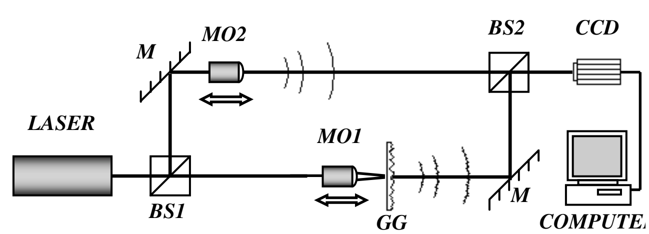

FIG. 1. Experimental setup for generation and detection of phase singularities in isotropic random waves. BS: beam splitters; MO: microscope objectives; M: mirrors; GG: ground glass plate.

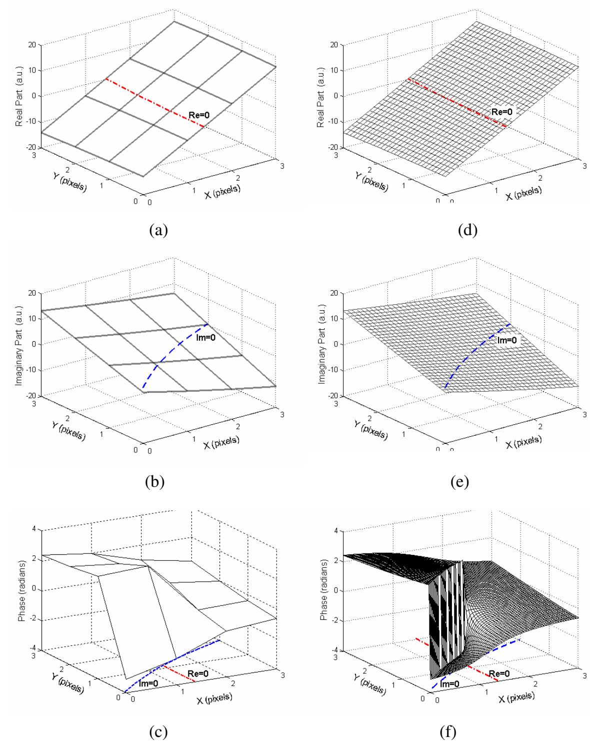

FIG. 2 (color online). Reconstruction of real and imaginary parts, and the corresponding phase structure around a phase singularity. Left column: before interpolation; right column: after interpolation.

contrast to the discontinuous and complicated phase structure, the real and imaginary parts have an extremely simple structure made of a smooth monotonic surface. Because of the limited size of the illumination area on the ground glass plate, the detected speckle field is strictly band limited. Based on the Whittaker-Shannon sampling theory, which assures that a band-limited signal can be recovered from its sampled data to arbitrarily fine details, we can reconstruct the complex random speckle field by a two-dimensional interpolation. The interpolated surfaces for the real and imaginary parts are shown in Figs. 2(d) and 2(e), respectively. From these interpolated real and imaginary parts, we can obtain the detailed phase profile around an optical vortex, as shown in Fig. 2(f). The use of the data interpolation enables us to observe the fine details of the phase structure around an optical vortex, and provides a convenient means to explore the core structure of an optical vortex without recourse to high magnification with an imaging system, which tends to reduce the field of view.

Generally, the phase change around a phase singularity is not uniform [12,14]. The current associated with the wave field is defined in the usual way by

$$\vec{J}(x, y) \equiv \mathrm{Im}\{\tilde{U}^*(x, y)\nabla\tilde{U}(x, y)\} = A^2(x, y)\nabla\theta(x, y), \quad (2)$$

with the vorticity given by $\vec{\omega}(x, y) \equiv (\nabla \times \vec{J})/2 = \nabla\mathrm{Re} \times \nabla\mathrm{Im}$.

Figure 3(a) shows an example of an intensity distribution in the neighborhood of a phase singularity obtained by experiment, and Fig. 3(b) is the corresponding local contours of the current for the same optical vortex. As predicted by Berry, elliptical local contours of intensity ($I = A^2$) and circular flow lines of current are observed around the phase singularity, which is located at the center of contour ellipses and contour circles, respectively. Figure 4 shows the relation between the magnitude of local current $\vec{J}$ and its distance R from the phase singularity along the line $x = 1.5$ in Fig. 3(b). With an increase in radial distance, the magnitude of the local current increases linearly. This demonstrates that the axis of the phase singularity is perpendicular to the observation plane along the optical vortex line in three-dimensional space.

Taking the optical vortex as the origin, and introducing a cylindrical coordinate system $(\hat{r}, \hat{\phi}, \hat{z})$, Dennis [12] has derived a new version of Kepler's law for phase singularity given by $R^2\partial\phi/\partial\theta = I/|\vec{\omega}(0)|$, which can be interpreted so that, along the contour line of intensity, equal area sectors of the ellipse are swept in equal intervals of phase θ. By the interpolation method introduced above, we obtain the phase distribution on the contour line of intensity (A) in Fig. 3(a). The relationship between the phase change per unit azimuth angle, $\Delta\theta/\Delta\phi$, and the square of the distance R^2 along this contour line is plotted in Fig. 5. The value $\Delta\theta/\Delta\phi$ increases linearly with R^2. Figure 5 demonstrates the conservation of angular momentum defined as $R^2\partial\phi/\partial\theta$.

Among the statistics associated with optical vortices for isotropic random wave, the density of vortices D is a quantity of great interest. Under the assumption of a Gaussian random process, this density has been given by Berry [8], and can be written as

$$D = \langle\delta(\mathrm{Re})\delta(\mathrm{Im})|\mathrm{Re}_x\mathrm{Im}_y - \mathrm{Im}_x\mathrm{Re}_y|\rangle, \quad (3)$$

where the subscripts denote partial derivatives and angular brackets denote the ensemble average. On the other hand, based on the prior knowledge that the vortices are necessarily constrained only in the dark coherence areas of a

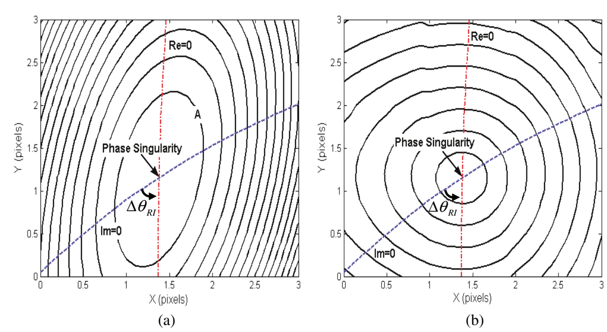

FIG. 3 (color online). Core structure around a phase singularity with zero crossings of real and imaginary parts of random field. (a) Intensity contours. (b) Current contours.

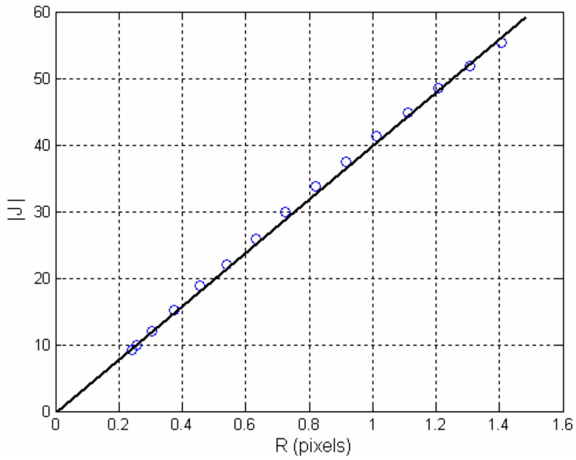

FIG. 4 (color online). Magnitude of current $\vec{J}$ versus distance from the phase singularity R. Open circles represent the points along $x = 1.5$ in Fig. 3(b).

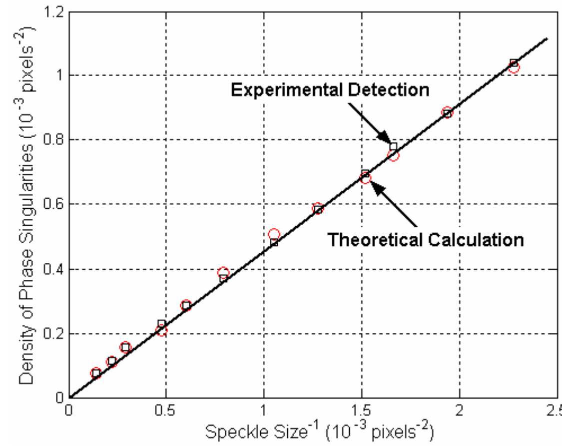

FIG. 6 (color online). Experimental detection (□) and theoretical calculation (○) for density of phase singularities versus speckle size^{-1}.

speckle pattern, Freund estimated the density of phase singularities with $D = 0.5A_{\mathrm{coh}}^{-1}$, where A_{coh} is the coherence area [6,10]. Although, Baranova *et al.* have qualitatively investigated the relation between the density of phase singularities and the radiation parameters in experiments, to our knowledge, the direct quantitative experimental verification of the theoretical predictions has not been conducted yet. Because the partial derivatives in Eq. (3) can be replaced by the finite difference of the reconstructed field, the proposed technique permits a direct experimental comparison with the theoretical predictions in Eq. (3). By changing the illumination spot size, we can control the density of phase singularities in the random field. The relation between the density of phase singularities and the reciprocal of the speckle size, i.e., A_{coh}^{-1}, is shown in Fig. 6. We found that the result of direct experimental detection was in good agreement with that predicted by Berry's formula. As expected, the density of phase singularities is exactly proportional to the reciprocal of the speckle size. The slope of the curve equals 0.46, which is in reasonable agreement with Freund's prediction. We attribute this small difference to the definition of speckle size. Figure 6 provides a direct experimental verification of Berry's theory about the density of phase singularities, and it also supports the prediction given by Freund.

Since the intensity contours around an optical vortex are elliptical, let us next find the corresponding statistics of eccentricity ε based on our experimental data. Figure 7 shows the histogram of eccentricity based on 1120 measurements; the histogram resembles an exponential probability density function. The high probability density is concentrated for large values of eccentricity, indicating that the typical core structure around a phase singularity is likely to be strongly anisotropic. We calculated the core eccentricity averages from these measured values and ob-

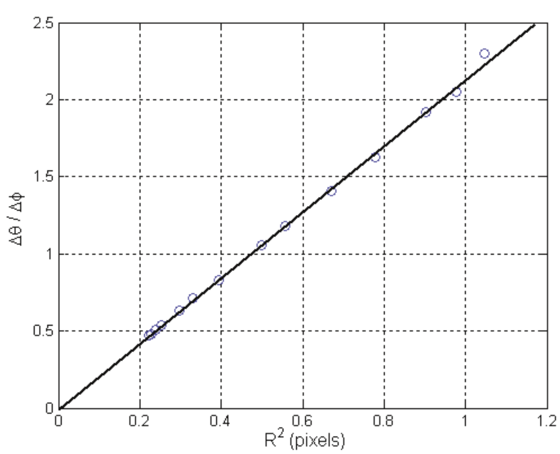

FIG. 5 (color online). Phase change along the azimuth angle $\Delta\theta/\Delta\phi$ versus R^2.

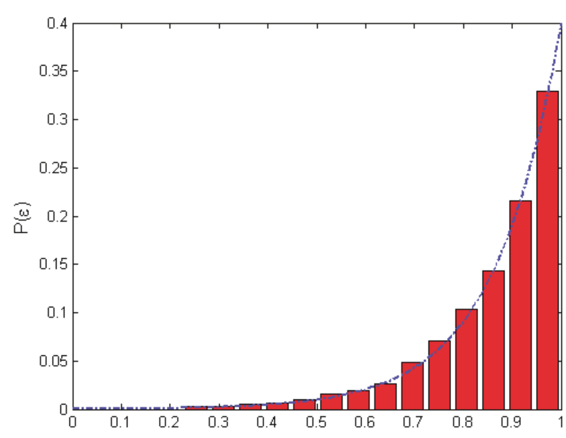

FIG. 7 (color online). Histogram of the result of 1120 measurements of eccentricity of anisotropic ellipses. Dash-dotted line represents an exponential distribution.

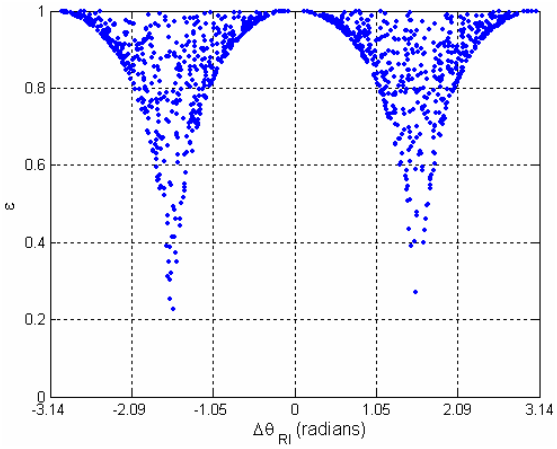

FIG. 8 (color online). Scatterplot of eccentricity of the anisotropic ellipse ε versus difference angle $\Delta\theta_{RI}$.

tained $\langle\varepsilon\rangle$ equal to 0.86, which agrees with Berry's theoretical value of 0.8697 [12] within the accuracy limitations imposed by the relatively small numbers of measurements.

It is well known that vortices will be created at the intersections of the zero crossings of the real and imaginary parts, which provide the topological information about the wave field [15,16]. An investigation of the relation between the local topological structures of zero crossings and the core anisotropy ellipses will give an insight into the evolution of morphology for optical vortices. Of the various parameters that determine the structure of the zero crossings, the most important is the difference angle ($\Delta\theta_{RI} \equiv \theta_R - \theta_I$) between the directions (tangents) of the two zero-crossing curves at the position of phase singularity. The angle θ_R (or θ_I), which is always positive as measured counterclockwise from the x axis, will be referred to as the zero-crossing angle. The scatterplot in Fig. 8 shows the relation between the zero-crossing difference angle $\Delta\theta_{RI}$ and its corresponding core eccentricity ε of the anisotropic ellipse. The difference angle $\Delta\theta_{RI}$ is seen to have more dispersed distributions for large eccentricity ε, while, as ε decreases, the distributions of $\Delta\theta_{RI}$ becomes more and more concentrated around the two peaks $\pm\pi/2$, indicating that the zero crossings for real and imaginary parts intersect more and more orthogonally when the anisotropic ellipse degenerates into a circle with its eccentricity approaching zero.

In summary, we have experimentally investigated the local properties and some statistics that describe the geometry of optical vortices for random waves with a novel method. As compared with the theories given by Berry, Dennis, and Freund, our experimental results demonstrate adequate consistency. Further, the reconstruction of the complex field from an interferogram, and phase retrieval from the interpolated real and imaginary parts of this reconstructed field make possible the observation of the detailed local properties of an optical vortex, and introduces new opportunities to explore other topological vortex phenomena.

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