Globally Polarized Quark-Gluon Plasma in Noncentral $A + A$ Collisions

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Produced partons have a large local relative orbital angular momentum along the direction opposite to the reaction plane in the early stage of noncentral heavy-ion collisions. Parton scattering is shown to polarize quarks along the same direction due to spin-orbital coupling. Such global quark polarization will lead to many observable consequences, such as left-right asymmetry of hadron spectra and global transverse polarization of thermal photons, dileptons, and hadrons. Hadrons from the decay of polarized resonances will have an azimuthal asymmetry similar to the elliptic flow. Global hyperon polarization is studied within different hadronization scenarios and can be easily tested.

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Strong transverse polarization of hyperons has been observed in unpolarized $p + p$ and $p + A$ collisions since the 1970s[1]. Given the beam and hyperon momenta $\vec{p}$ and $\vec{p}_H$, hyperons produced in the beam fragmentation region are found transversely polarized in the direction perpendicular to the hyperon production plane, $\vec{n}_H = \vec{p} \times \vec{p}_H / |\vec{p} \times \vec{p}_H|$. Polarizations of Λ, Ξ, and $\bar{\Xi}$ are negative while Σ and $\bar{\Sigma}$'s are positive. In the meantime, $\bar{\Lambda}$ and Ω are not transversely polarized. Although the origin for such striking transverse hyperon polarization is still in debate, one can relate it to the single-spin left-right asymmetries observed in hadron-hadron collisions with transversely polarized beam [2], which in turn can be attributed to the orbital angular momenta (OAM) of the valence quarks in a polarized nucleon [3–5], or fragmentation functions of transverse polarized quarks [6] as well as twist-3 parton correlations in nucleons [7]. It has also been suggested [8,9] that hyperon polarization could disappear due to the formation of a quark-gluon plasma (QGP).

In this Letter, we show that parton interaction in *noncentral* heavy-ion collisions leads to a global quark polarization along the opposite direction of the reaction plane,

$$\vec{n}_b = \vec{p} \times \vec{b} / |\vec{p} \times \vec{b}|, \tag{1}$$

as determined by the nuclear impact parameter $\vec{b}$. This global polarization is essentially a local manifestation of the global angular momentum of the colliding system through interaction of spin-orbital coupling in QCD. It will have far reaching consequences in noncentral heavy-ion collisions, such as left-right asymmetry of hadron spectra in the reaction plane, global transverse polarization of direct photons, and dileptons and hadrons with spin. Within different hadronization scenarios, we will discuss hyperon polarization as a result of the global quark polarization. Possible contributions from final state hadronic interaction will also be discussed.

Let us consider two colliding nuclei with the beam projectile moving in the direction of the z axis, as illustrated in Fig. 1. We define the impact parameter $\vec{b}$ (along $\hat{x}$) as the transverse distance of the projectile from the target nucleus and the reaction plane as given by $\vec{n}_b$ (along $\hat{y}$) in Eq. (1). Partons produced in the overlapped region of the collision will carry a global angular momentum along the direction opposite to the reaction plane ($-\hat{y}$). A thermalized QGP requires final state parton interaction. Given the nature of partonic interaction at high energy, the global angular momentum would never lead to a collective rotation of the system. It will, however, be manifested in the finite transverse (along $\hat{x}$) gradient of the average longitudinal momentum $p_z(x, y, b)$ per produced parton. We assume for the moment that $p_z(x, y, b)$ is independent of the longitudinal position and is just an average value. Given the range of interaction Δx, two colliding partons will have relative longitudinal momentum $\Delta p_z = \Delta x dp_z/dx$ with OAM $L_y \sim -\Delta x \Delta p_z$ along the direction of $\vec{n}_b$. This relative OAM L_y will lead to global quark polarization due to spin-orbital coupling.

The initial collective longitudinal momentum can be calculated as the total momentum difference between participant projectile and target nucleons, whose transverse

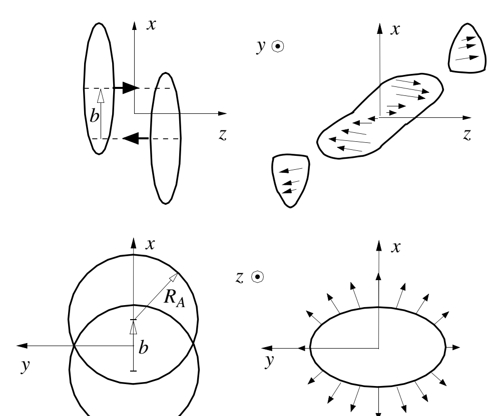

FIG. 1. Illustration of noncentral heavy-ion collisions with impact parameter $\vec{b}$. The global angular momentum of the produced matter is along $-\hat{y}$, opposite to the reaction plane.

distributions (integrated over y) are

$$\frac{dN_{\rm part}^{P,T}}{dx} = \frac{3A}{2\pi R_A}\{\tilde{y}_{\max}\sqrt{1-\tilde{y}_{\max}^2-(\tilde{x}\mp\tilde{b}/2)^2} + [1-(\tilde{x}\mp\tilde{b}/2)^2]\arcsin[\tilde{y}_{\max}/\sqrt{1-(\tilde{x}\mp\tilde{b}/2)^2}]\};$$
$$\tilde{y}_{\max} = \min\{\sqrt{1-(\tilde{x}+\tilde{b}/2)^2}, \sqrt{1-(\tilde{x}-\tilde{b}/2)^2}\}, \quad (2)$$

where $\tilde{x} = x/R_A$, $\tilde{b} = b/R_A$, and $R_A = 1.12A^{1/3}$ is the nuclear radius in a hard-sphere distribution.

Since the measured total multiplicity in $A + A$ collisions is proportional to the number of participant nucleons [10], we can assume the same for the produced partons with a proportionality $c(s)$. The average collective longitudinal momentum per parton is then

$$p_z(x, b) = \left(\frac{\sqrt{s}}{2c(s)}\right)\frac{dN_{\rm part}^P/dx - dN_{\rm part}^T/dx}{dN_{\rm part}^P/dx + dN_{\rm part}^T/dx}. \quad (3)$$

The distribution $p_z(x, b)$ is an odd function in both x and b and therefore vanishes at $x = 0$ or $b = 0$. As shown in Fig. 2 (as dashed lines) in units of $4p_0 \equiv 4\sqrt{s}/2c(s)$ and as a function of $\tilde{x}$ for different values of $\tilde{b}$, it is a monotonically increasing function of $\tilde{x}$ until the edge of the overlapped region $|\tilde{x} \pm \tilde{b}/2| = 1$ where it drops to zero. The transverse gradient dp_z/dx, shown as solid lines in unit of $dp_0/dx \equiv \sqrt{s}/2c(s)R_A$, is an even function of x and vanishes at $b = 0$. It increases almost linearly with b for small and intermediate values of b. Except the singular behavior at the boundary of the overlapped region which is caused by the assumed hard-sphere nuclear distribution, dp_z/dx is approximately uniform across the transverse x direction. The classical relative OAM is then $L_y \equiv -(\Delta x)^2 dp_z/dx$ for partons separated by Δx in the transverse direction. Because of transverse expansion, dp_z/dx will decrease with time according to $(R_A - b/2)/[R_A - b/2 + v_x(b) \times (\tau - \tau_0)]$, where $v_x(b)$ is the transverse flow velocity in the reaction plane.

In Au + Au collisions at $\sqrt{s} = 200$ GeV, the number of charged hadrons per participant nucleon is about 15 [10].

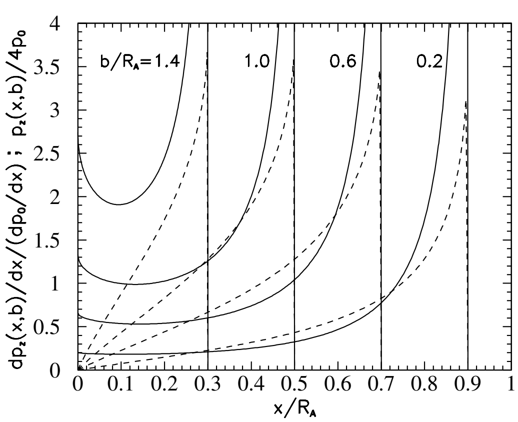

FIG. 2. $(dp_z/dx)/(dp_0/dx)$ (solid) and $p_z(x, b)/4p_0$ (dashed) as functions of x/R_A for different values of b/R_A.

Assuming the number of partons per (meson dominated) hadron is about 2, then $c(s) \simeq 45$. Given $R_A = 6.5$ fm, $dp_0/dx \simeq 0.34$ GeV/fm and $L_0 \equiv -(\Delta x)^2 dp_0/dx \simeq 1.7$ for $\Delta x = 1$ fm.

We can relax the approximation of uniform distribution of $p_z(x, b)$ in the longitudinal direction by identifying pseudorapidity with the spatial rapidity $\eta = 0.5\ln(t + z)/(t - z)$. According to experimental studies of hadron production in $p + A$ and $A + B$ collisions, the collective longitudinal momenta p_z and dp_z/dx are distributed across a broad range of rapidity and peaks in the forward (backward) region for a given layer of dense matter at positive (negative) x. The position of the peak in rapidity should increase with $|x|$. Note that even though $p_z(x, b)$ vanishes at around $x = 0$, dp_z/dx and L_y are still finite for $b \neq 0$ as shown in Fig. 2. Averaging over the x direction will result in finite dp_z/dx and L_y around central rapidity region in noncentral heavy-ion collisions.

To study quark polarization due to parton collisions with a fixed direction of OAM, we consider quark scattering with fixed impact parameter $\vec{x}_T$. For given initial relative momentum $(E, \vec{p})$ and final spin $\lambda/2$ of the quark along $\vec{n}_b$, the cross section is

$$\frac{d\sigma_\lambda}{d^2x_T} = C_T\int\frac{d^2q_T}{(2\pi)^2}\frac{d^2k_T}{(2\pi)^2}e^{i(\vec{k}_T-\vec{q}_T)\cdot\vec{x}_T}\mathcal{I}_\lambda(\vec{q}_T, \vec{k}_T, E),$$
$$\mathcal{I}_\lambda = \frac{g^2}{2(2E)^2}\bar{u}_\lambda(p_q)\not{A}(\vec{q}_T)(\not{p} + m_q)\not{A}(\vec{k}_T)u_\lambda(p_k) \quad (4)$$

within the screened potential model [11], where $A_0(q_T) = g/(q_T^2 + \mu^2)$ is the screened static potential with Debye screen mass μ and C_T is the color factor associated with the target. Average over spin polarization is implied for the initial quark and $\vec{p}_{q(k)} = \vec{p} + \vec{q}_T(\vec{k}_T)$ is the final quark momentum. For small angle scattering, $q_T, k_T \sim \mu \ll E$,

$$\frac{\mathcal{I}_\lambda}{g^2} \approx \frac{1}{2}A_0(q_T)A_0(k_T)\left[1 - i\lambda\frac{(\vec{q}_T - \vec{k}_T)\cdot(\vec{n}_b \times \vec{p})}{2E(E + m_q)}\right]. \quad (5)$$

The first term gives the unpolarized cross section

$$\frac{d\sigma}{d^2x_T} \equiv \frac{d\sigma_+}{d^2x_T} + \frac{d\sigma_-}{d^2x_T} = 4C_T\alpha_s^2K_0^2(\mu x_T), \quad (6)$$

or $d\sigma/dq_T^2 = C_T 4\pi\alpha_s^2/(q_T^2 + \mu^2)^2$ in momentum space. The second term gives rise to a polarized cross section $d\Delta\sigma/d^2x_T \equiv d\sigma_+/d^2x_T - d\sigma_-/d^2x_T$,

$$\frac{d\Delta\sigma}{d^2x_T} = -\mu\frac{\vec{p}\cdot(\hat{x}_T \times \vec{n}_b)}{E(E + m_q)}4C_T\alpha_s^2K_0(\mu x_T)K_1(\mu x_T), \quad (7)$$

where $\hat{x}_T = \vec{x}_T/x_T$ and K_n's are modified Bessel functions. It is evident that parton scattering polarizes quarks along the direction opposite to the parton reaction plane determined by the impact parameter $\vec{x}_T$, the same direction of the relative OAM. This is essentially the manifest of spin-orbital coupling in QCD. Ordinarily, the polarized cross section along a fixed direction $\vec{n}_b$ vanishes when averaged over all possible direction of the parton impact parameter $\vec{x}_T$. However, in noncentral heavy-ion collisions, the local relative OAM L_y provides a preferred average reaction plane for parton collisions. This will lead to a global quark polarization opposite to the reaction plane of nucleus-nucleus collisions. This conclusion should not depend on our perturbative treatment of parton scattering as far as the effective interaction is mediated by the vector coupling in QCD.

Averaging over the relative angle between parton $\vec{x}_T$ and nuclear impact parameter $\vec{b}$ from $-\pi/2$ to $\pi/2$ and over x_T, one can obtain the global quark polarization

$$P_q = -\pi\mu p/4E(E + m_q) \tag{8}$$

via a single scattering for given E. If we take $p = \Delta p_z/2 = \vec{x}_T \cdot \hat{x} dp_z/dx$ before averaging, the result will be similar but numerical evaluation is needed. In the limit $m_q = 0$ and $p \ll \mu$, one expects $P_q \sim -\Delta p_z/\mu$. Given an average range of interaction $\Delta x^{-1} \sim \mu \sim 0.5$ GeV and $dp_0/dx = 0.34$ GeV/fm for semiperipheral ($b = R_A$) collisions (see Fig. 2) at BNL Relativistic Heavy-Ion Collider (RHIC), $P_q \sim -0.3$. Multiple scattering will further increase the polarization.

In nonrelativistic limit for massive quarks, $m_q \gg p, \mu$,

$$P_q \approx -\pi\mu p/8m_q^2. \tag{9}$$

In the same limit, the spin-orbital coupling energy is $E_{LS} = (\vec{L} \cdot \vec{S})(dV_0/dr)/(rm_q^2)$. Given the range of interaction $r \sim 1/\mu$, $dV_0/dr \sim -\mu^2$, and $L \sim p/\mu$, $E_{LS}/\mu \sim -\mu p/m_q^2$ is just the above quark polarization.

The global quark polarization opposite to the reaction plane will have many observable consequences in noncentral heavy-ion collisions if an interacting QGP is formed. One expects to see left-right asymmetry in hadron spectra at large rapidity similar to the single-spin asymmetry in $p + p$ collisions [3–7]. Thermal photons, dileptons, and final hadrons with spin will be similarly polarized. Since hadrons from the strong decay of polarized resonances have angular distributions that prefer the direction perpendicular to the resonances' polarization, they will result in an azimuthal asymmetry with respect to the reaction plane, similar to the asymmetry due to elliptic flow. In the following, we will discuss global hyperon polarization since it can be easily measured through the weak decay [12].

To demonstrate the robustness of the qualitative features of the predicted hyperon polarization due to global quark polarization, we consider several hadronization scenarios. We consider first hadronization via parton recombination. In this case, not only the spin of polarized quarks but also the relative OAM can contribute to the final hadrons' polarization. Given hadron size Δx, the classical estimate of the relative OAM is $L_y \sim -(\Delta x)^2 dp_z/dx$. In the nonrelativistic quark model, however, constituent quarks in the ground state are all in the s-wave state. The contribution of OAM to hadron polarization resides in the total angular momentum of the constituent quark. Overall, the effective polarization of a constituent quark should be proportional to that of the valence quark. Alternatively, polarization of constituent quarks can be similarly estimated, e.g., Eq. (9), assuming massive constituent quarks as pointlike particles.

We can categorize recombination into exclusive $qqq \to H$ and inclusive $qqq \to H + X$ processes. The production cross section of polarized hyperons from the exclusive recombination can be written as

$$\sigma_H^\lambda = \sum_{\lambda_1,\lambda_2,\lambda_3} |\langle q_1^{\lambda_1} q_2^{\lambda_2} q_3^{\lambda_3} | H^\lambda\rangle|^2 \sigma_{q_1q_2q_3} R_{q_1}^{\lambda_1} R_{q_2}^{\lambda_2} R_{q_3}^{\lambda_3}, \tag{10}$$

where $R_q^\lambda = (1 + \lambda P_q)/2$ is the quark polarization probability. Extracting R_H^λ from $\sigma_H^\lambda \equiv \sigma_H R_H^\lambda$ that only depends on quark polarization P_q, one can calculate the recombination probability $R_H = R_H^\uparrow + R_H^\downarrow$ and the hyperon polarization $P_H = (R_H^\uparrow - R_H^\downarrow)/R_H$. Given baryons' SU(6) wave functions in the quark model [4] and the polarization for strange (P_s) and nonstrange quarks (P_q), one can obtain $P_\Lambda = P_s$, $R_\Lambda = 3(1 - P_q^2)$;

$$P_\Sigma = (4P_q - P_s - 3P_sP_q^2)/R_\Sigma, \quad R_\Sigma = 3 - 4P_qP_s + P_q^2;$$
$$P_\Xi = (4P_s - P_q - 3P_qP_s^2)/R_\Xi, \quad R_\Xi = 3 - 4P_qP_s + P_s^2;$$
$$P_\Omega = 2P_s(5 + P_s^2)/R_\Omega, \quad R_\Omega = 6(1 + P_s^2).$$

If $P_s \simeq P_q$ in the most likely case, we have the same $P_H = P_q$ with $R_H = 3(1 - P_q^2)$ for Λ, Σ, and Ξ.

It is difficult to estimate hyperon polarization from inclusive recombination. However, it could be the dominant process for the bulk hadron production, especially for large values of P_q. It is also required by entropy conservation. The polarization of produced hyperons would be smaller than in the exclusive recombination but should be proportional to P_q.

The extreme limit of inclusive recombination is fragmentation of polarized quarks, $q \to H + X$. Longitudinal polarization of hyperons in a similar process $e^+e^- \to Z^0 \to q\bar{q} \to \Lambda + X$ has been measured [14] and can be explained [15] by assuming that polarized hyperons contain the initial polarized leading quark in its SU(6) wave function. One can similarly calculate hyperon polarization from fragmentation of transversely polarized quarks and obtain

$$P_\Lambda = n_sP_s/(n_s + 2f_s),$$
$$P_\Sigma = (4f_sP_q - n_sP_s)/3(2f_s + n_s),$$
$$P_\Xi = (4n_sP_s - f_sP_q)/3(2n_s + f_s), \qquad P_\Omega = P_s/3,$$

where n_s and f_s are the strange quark abundances relative to up or down quarks in QGP and quark fragmentation, respectively. Including production of unpolarized hyperons in the process, the effective hyperon polarization from fragmentation of polarized quarks should increase with the fractional momentum $z_H = p_H/p_q$.

Because of the complication from hadronization, we cannot provide a model-independent estimate of the final hyperon polarization. If quark recombination is the dominant hadronization mechanism, hyperons' polarization will be determined by the quarks' polarization before hadronization. According to our estimate following Eq. (8), this would be in the order of tens of a percent and the hyperons' polarization would be around the same order. We can also provide other qualitative predictions of the global hyperon polarization P_H in noncentral heavy-ion collisions: (1) Hyperons and their antiparticles are similarly polarized along the same direction perpendicular to the reaction plane in noncentral heavy-ion collisions. (2) The global hyperon polarization P_H vanishes in central collisions and increases almost linearly with b in semicentral collisions. (3) It should have a finite value at small p_T and in the central rapidity region. It should increase with rapidity and eventually decreases and vanishes at large rapidities. (4) High $p_T \gg \Delta p_z \sim \mu L_0$ quarks should not be polarized by parton scattering in the medium. However, hyperon polarization should persist at moderate p_T where recombination of unpolarized high p_T quarks with polarized thermal quarks may still dominate. The polarization can be estimated within a recombination model. (5) Since hyperon's production planes are randomly oriented with respect to the reaction plane of heavy-ion collisions, the observed hyperon polarization in $p + A$ collisions should not contribute to the global polarization as we have defined here, except at large rapidity region where directed flow is observed [13]. In this region, the nonvanishing $\langle p_x \rangle$ can provide an average production plane $\vec{n}_H$ for hyperons. According to the observed polarization pattern in $p + A$ [1], the global P_H will be enhanced for Λ, Ξ, and $\bar{\Xi}$ and reduced for Σ in large rapidity region. In addition, one also expects $P_\Lambda > P_{\bar{\Lambda}}$ due to the directed flow.

In principle, hadronic interaction with a given direction of relative OAM can also lead to global hyperon polarization. Neglecting hyperon production in the hadronic phase, the dominant hadronic processes involving hyperons will be hyperon-π scattering. The scattering amplitude in general involves both scalar and vector channels [16]. One can show that global hyperon polarization from the vector channel is along while polarizations from the scalar channel and all the interferences are against the global quark polarization. Therefore, polarizations from different channels in hyperon-π scattering partially cancel each other. In addition, the transverse gradient of the longitudinal flow should be significantly reduced in the hadronic phase due to prior strong transverse expansion in the reaction plane as demonstrated by the large elliptic flow measured at RHIC. Therefore, the net effect of hadronic interaction should be small and should not change the final hyperon polarization significantly, in particular, at large p_T. We will leave detailed study to future work.

In summary, produced partons are shown to have large local relative OAM in noncentral heavy-ion collisions if quark-gluon plasma is formed. Parton scattering with given relative OAM is shown to polarize quarks along the same direction due to spin-orbital coupling. Such global quark polarization has many measurable consequences in high-energy heavy-ion collisions. Within different hadronization scenarios, we predict that hyperons will be polarized along the opposite direction of the reaction plane. Effects of hadronic interaction are expected to be small and would not change the qualitative feature of our prediction.

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