Temporally Heterogeneous Dynamics in Granular Flows

Leonardo E. Silbert*
James Franck Institute, University of Chicago, Chicago, Illinois 60637, USA
Department of Chemistry and Biochemistry, University of California Los Angeles, Los Angeles, California 90095, USA
(Received 10 November 2004; published 10 March 2005)

Granular simulations are used to probe the particle scale dynamics at short, intermediate, and long time scales for gravity-driven, dense granular flows down an inclined plane. On approach to the angle of repose, where motion ceases, the dynamics become intermittent over intermediate times, with strong temporal correlations between particle motions—*temporally heterogeneous dynamics*. This intermittency is characterized through large-scale structural events whereby the contact network periodically spans the system. A characteristic time scale associated with these processes increases as the stopped state is approached. These features are discussed in the context of the dynamics of supercooled liquids near the glass transition.

DOI: 10.1103/PhysRevLett.94.098002 PACS numbers: 81.05.Rm, 45.70.Mg, 61.20.Lc, 64.70.Pf

Granular materials persist at the forefront of contemporary research due to the extreme richness in their dynamics, either under shear [1], flowing out of a hopper [2], or driven by gravity [3]. Because thermodynamic temperature plays little role in determining these features, granular materials are widely recognized as a macroscopic analogue of athermal systems far from equilibrium [4]. The notion of *jamming* [5] provides a unifying framework to describe the behavior of a wide range of systems, from the molecular level (supercooled liquids and glasses), microscale (colloidal glasses), through to the macroscale (granular matter), near their jamming transition. The motivation for this work comes from the view that granular flows *can* be used as a macroscopically accessible analogue through which we can gain a better understanding of the critical slowing down of the dynamics in systems near their point of jamming. One of the features associated with the approach of the glass transition is the concept of spatially heterogeneous dynamics [6]. This work presents features of a driven granular material flowing down an inclined plane—chute flows—that exhibit large-scale, cooperative events on approach to the angle of repose where flow stops—its jamming point.

Gravity-driven flows continue to be extensively studied due to their ubiquity throughout nature—avalanches, debris flows, and sand dunes—and also because of their importance in materials handling throughout industry. There has been a recent focus of attention towards understanding the properties of inclined plane flows, or chute flows, because of the ability of obtaining well-defined and reproducible steady-state flows [7,8]. Such flows offer a relatively clean system in which to develop theoretical models for constitutive relations as well as describing the flow at the level of the grain size [9]. This is where the role of simulation has proved extremely useful.

Simulations of chute flows have captured features of the rheology with remarkable accuracy [10,11]. Three principal flow regimes, depicted in Fig. 1, are observed for chutes with rough bases, inclined at an angle θ: (i) no flow occurs below the angle of repose θ_r, (ii) steady state flow exists in the region $\theta_r < \theta < \theta_u$, and (iii) unstable flow occurs for $\theta > \theta_u$. The location of θ_r depends on the height of the flowing pile h [7,11]. Pouliquen introduced the quantity $h_{\mathrm{stop}}(\theta)$ that encodes all the (undetermined) information about the roughness and effective friction of the base to relate the dependency of θ_r on h. In the language of glasses and colloids, $h_{\mathrm{stop}}(\theta)$ represents the glass transition or yield stress line, respectively.

Inclined plane flows have the benefit that steady-state flow—either continuous or intermittent—is observed all the way down to the angle of repose. This is in contrast to discrete avalanches in heap flows [12]. This work characterizes the large-scale fluctuations, and spatial and temporal heterogeneous dynamics that emerge in the vicinity of θ_r. At an inclination angle θ_i, for $\theta_r < \theta \leq \theta_i < \theta_u$, over time scales intermediate between ballistic propagation, at short times, and diffusive motion, at long times, the particle dynamics become strongly intermittent. The mechanisms responsible for the change in the dynamics at θ_i involve the relaxation and reformation of mechanically stable clusters that periodically span the system. A characteristic time

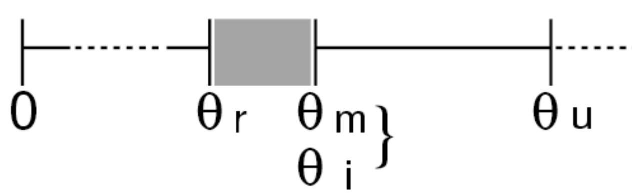

FIG. 1. Phase behavior of granular chute flow (schematic). θ_r is the angle of repose at which flow ceases. θ_m is the maximum angle of stability at which flow is initiated from rest. θ_u is the maximum angle for which stable flow is observed. The angle θ_i, at which intermittent dynamics emerges, the shaded region, coincides with $\theta_i \approx \theta_m$. For the simulations reported here, $\theta_r = 19.30° \pm 0.01°$, $\theta_m = 20.3° \pm 0.2°$, and $\theta_u \approx 26°$.

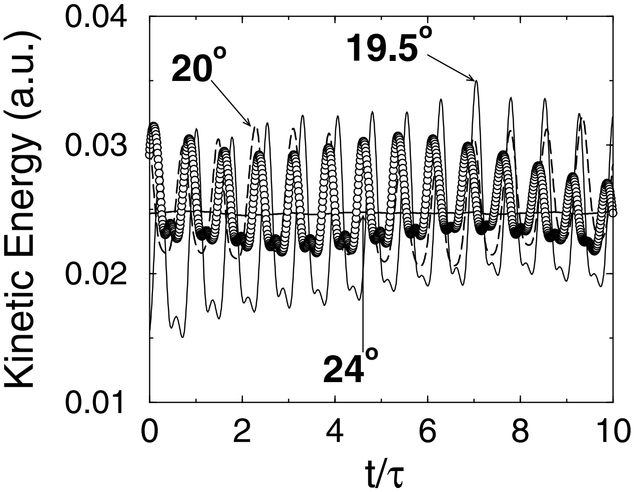

FIG. 2. Average kinetic energy per particle over an intermediate time window for $N = 8000$ and different inclination angles as indicated. The symbols are for $N = 160\,000$ at $\theta = 19.5\,°$.

scale τ_c, associated with these correlations, is found to increase as $(\theta - \theta_r) \rightarrow 0$.

The simulations were performed using the discrete element technique with interactions appropriate for granular materials [10]. Monodisperse (soft) spheres of diameter d and mass m flow down a roughened, inclined plane due to gravity g. Initially, θ is set to θ_u to remove any preparation history, then incrementally reduced in steps of $0.01\,° \leq \Delta\theta \leq 0.5\,°$. Particles interact only on contact (cohesionless), along directions normal and tangential (via static friction) to the vector joining their centers. Contacts are defined as two overlapping neighbors. All runs were performed with a particle friction coefficient $\mu = 0.5$ and coefficient of restitution $\epsilon = 0.88$. Most of the results presented are for $N = 8000$ particles flowing down a chute that is $L_x = 20d$ long, $L_y = 10d$ wide (the xy plane employs periodic boundary conditions), and an average flow height $h \approx 40d$ (with a free top surface). See Fig. 5. Massively, large-scale, parallel simulations were used for $N = 160\,000$, with $(L_x, L_y, h) = (100, 40, 40)d$, to demonstrate that the observed features are not a system size artifact. Length scales are nondimensionalized by the particle diameter d and time in units of $\tau = \sqrt{d/g}$.

The intermittent regime was first identified through measurements of the kinetic energy as shown in Fig. 2. In the continuous-flow, steady-state regime, the energy is approximately constant with small fluctuations about the mean value. As the inclination angle is incrementally decreased towards θ_r, changes in the time profile of the energy first become apparent for $\theta_i \approx 20.2\,°$.

Because of the convective nature of the flow, the mean-squared displacement (MSD), shown in Fig. 3, is measured normal to the inclined plane in the z direction: $\Delta z^2 \equiv \langle [z(t) - z(0)]^2 \rangle$. During continuous flow, there is a clear

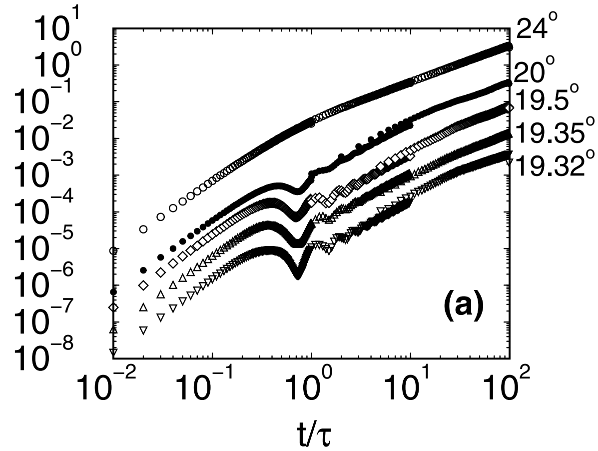

FIG. 3. The mean-squared displacement in the direction normal to the plane Δz^2, with decreasing angle from top to bottom. All curves have been shifted for clarity.

crossover between the ballistic regime, for $t/\tau < 10^{-1}$, and diffusive regime, $t/\tau > 10$. Closer to the jamming point, in the vicinity of θ_r, a novel dynamical regime emerges as signaled by oscillations in the MSD over intermediate times. The time profiles shown in Fig. 4 clearly reflect the temporally heterogeneous dynamics characteristic of this regime. The velocity autocorrelation function, $C(t) \equiv \langle v_z(t) v_z(0) \rangle$, where v_z is the velocity component in the direction normal to the plane, see Fig. 4(a), and the (dis-

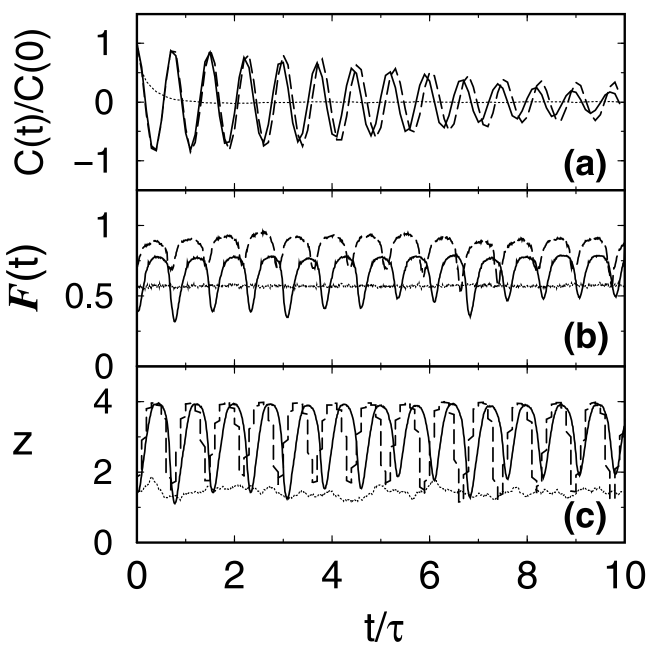

FIG. 4. Intermediate time window associated with the oscillations in the MSD, for $19.35\,°$ (dashed line), $19.5\,°$ (thick), and $24\,°$ (dotted): (a) velocity autocorrelation function $C(t)$, (b) force-force time correlation function $\mathcal{F}(t)$, and (c) coordination number z.

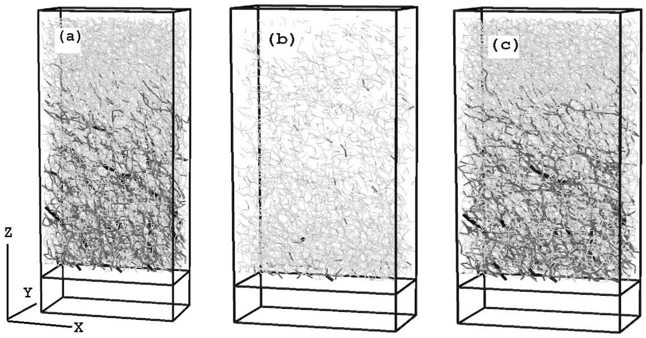

FIG. 5. The contact force network at $\theta = 20°$ for three successive times. (a) Snapshot corresponds to a peak in the coordination number, (b) a trough, and (c) the next peak. The shade and thickness of the lines represent the magnitudes of the normal forces between two particles in contact. The inclination has been removed in the figures and the frame is a guide to the eye. The simulation axes are indicated.

tinct) particle-particle contact force-force time correlation function $\mathcal{F}(t)$, see Fig. 4(b), both exhibit temporal fluctuations in comparison to the continuous-flow regime. The average coordination number z, Fig. 4(c), demonstrates that the structural processes associated with these correlations involve large-scale, cooperative events, indicating that the intermittent regime can be considered as temporally biphasic. During the *flowing* phase of the intermittent regime, z remains lower than the mechanically stable limit for frictional spheres $z_c = 4$ [13], whereas in the *static* phase, the system attains a value $z \approx z_c$, thus almost coming to rest. (There is always a residual creeping motion as flow does not completely stop until θ_r.) The physical picture of the intermittent regime is thus: over some characteristic time scale the system oscillates between almost total mechanical stability and fluidlike flow through time-dependent, system-spanning, structural relaxation events that involve the breakup and reformation of particle contacts. These processes are best viewed in the simulation snapshots of Fig. 5. A signature of this biphasic contact network is also seen in the probability distribution function of the normal contact forces $P(f)$. The partial distributions shown in Fig. 6 are computed for the high-z phase (averaged over configurations with $z > 3.5$) which resembles that of a static packing—exponential decay at high forces—and separately the low-z phase ($z < 2$)—the high-f tail decays more slowly. This suggests that it may be possible to distinguish between different types of flows by inspection of $P(f)$.

The intermittent regime is peculiar due to the complex interplay between different characteristic length and time scales. Identifying the structural entities of typical size ℓ remains a matter of debate [9,14]. However, the general picture is as follows: ℓ increases with decreasing θ or shear rate, such that at θ_r, $\ell \approx h$, resulting in the flowing-static transition. The development of an intermittent regime suggests another time scale, whereby $\ell \approx h$ can occur, but other mechanisms drive the system away from stability before the system permanently jams. θ_i seems to coincide with the point where the scaling of the flow velocity changes from that of continuous flow [11], and may indicate the onset of shear banding.

Progress can be made by identifying the characteristic time scales of the intermittent regime. One is the period-

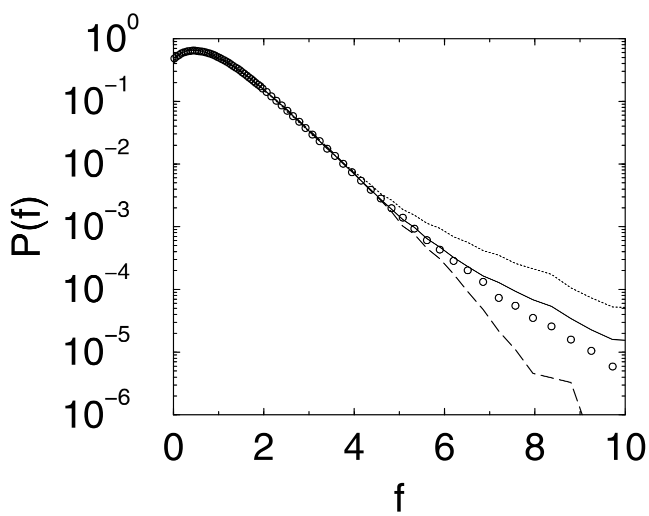

FIG. 6. Probability distribution function $P(f)$ of the normal contact forces (normalized so $\bar{f} = 1$) for $\theta = 20°$; total (solid line), the high-z phase (dashed), and the low-z phase (dotted). The full $P(f)$ for $\theta = 22°$ (symbols) is shown for comparison.

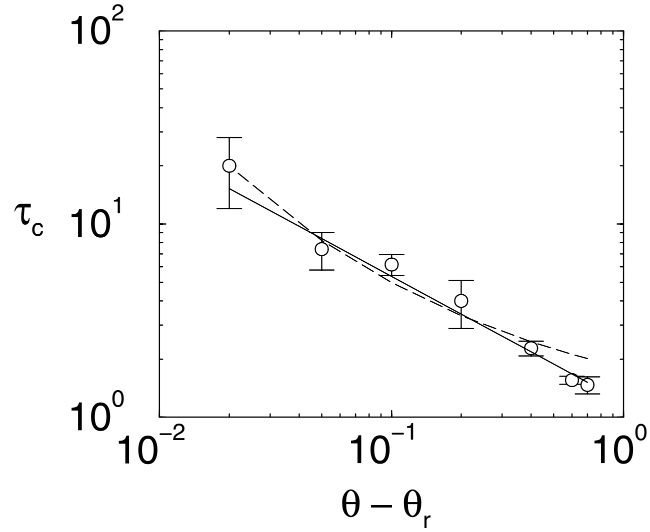

FIG. 7. Characteristic time scale τ_c associated with the intermittent structural events at intermediate times as a function of distance from the jamming point $(\theta - \theta_r)$. The solid line is a power-law fit, $\tau \sim (\theta - \theta_r)^{-\gamma}$, with $\gamma \approx 0.65$, whereas the dashed line is of the modified Vogel-Fulcher form, $\tau_c \sim \exp\{A(\theta - \theta_r)^{-p}\}$, with $A \approx 0.66$ and $p \approx 0.39$.

icity in the kinetic energy, correlation functions, and z. This time scale is rather insensitive to θ for intermittent flows. However, $C(t)$ has a decay envelope corresponding to the extent of the intermittency. From the initial decay of this envelope, a time scale τ_c is extracted and shown in Fig. 7. Interestingly, a power-law divergence, $\tau_c \sim (\theta - \theta_r)^{-\gamma}$, with $\gamma \approx 0.65$, captures the behavior quite well. For comparison, a modified Vogel-Fulcher form is also shown in Fig. 7, which is satisfactorily close to the jamming point but deviates further away. The available range for analysis is somewhat restricted through this definition of τ_c, so it remains unclear how this time scale might correspond to the more familiar definitions of relaxation times in supercooled fluids [15]. A more appropriate definition of τ_c is being investigated.

In conclusion, gravity-driven, dense, granular flows down an inclined plane exhibit intermittent dynamics over intermediate time scales in the vicinity of the angle of repose. The intermittent regime signals the onset of temporal heterogeneous dynamics, whereby the short time motion is ballistic, at long times diffusive, but at intermediate times a combination of creeping flow and traplike dynamics exists. The system visits temporally metastable states through large-scale cooperative events, with correlations that decay more slowly as the jamming point is approached. The behavior of the characteristic time scale τ_c shares similarities with the relaxation times of supercooled liquids in the vicinity of the glass transition. Such dynamical similarities were recently probed experimentally in vibrated bead packs [16]. In a typically heuristic fashion, an analogy with glassy dynamics theory can be made. θ_r marks the point where all dynamics cease and in this sense plays the role of the glass transition temperature T_g. Indeed, earlier work [17] showed that the dynamic-jamming transition resembles the way a model liquid approaches the glassy phase [18]. However, the way the dynamics change at θ_i is similar in spirit to the way the dynamics in a supercooled fluid are considered to change at the mode coupling temperature T_c [19]. With this view in mind, one can associate θ_i with T_c. These results therefore hint towards a unifying picture of dynamical heterogeneities in dense, amorphous systems. In particular, granular flows offer a convenient system to address questions often associated with glasses at the molecular and colloidal level. This emerging picture thus begs the question: can existing theories that are currently being applied to traditional glasses and colloids be extended to granular systems?

I thank Bulbul Chakraborty for insightful conversations and Gary S. Grest for critiquing the manuscript. Grant support from Grants No. NSF-DMR-0087349, No. DE-FG02-03ER46087, No. NSF-DMR-0089081, and No. DE-FG02-03ER46088 is gratefully acknowledged.

*Electronic address: lsilbert@uchicago.edu

[1] J.-C. Tsai, G. A. Voth, and J. P. Gollub, Phys. Rev. Lett. **91**, 064301 (2003); M. Toiya, J. Stambaugh, and W. Losert, Phys. Rev. Lett. **93**, 088001 (2004).
[2] E. Longhi, N. Easwar, and N. Menon, Phys. Rev. Lett. **89**, 045501 (2002); J. Choi, A. Kudrolli, R. R. Rosales, and M. Z. Bazant, Phys. Rev. Lett. **92**, 174301 (2004).
[3] N. Taberlet, P. Richard, A. Valance, W. Losert, J. M. Pasini, J. T. Jenkins, and R. Delannay, Phys. Rev. Lett. **91**, 264301 (2003); A. V. Orpe and D. V. Khakhar, Phys. Rev. Lett. **93**, 068001 (2004).
[4] L. P. Kadanoff, Rev. Mod. Phys. **71**, 435 (1999).
[5] A. J. Liu and S. R. Nagel, Nature (London) **396**, 21 (1998).
[6] M. D. Ediger, Annu. Rev. Phys. Chem. **51**, 99 (2000), and references therein.
[7] O. Pouliquen, Phys. Fluids **11**, 542 (1999).
[8] E. Azanza, F. Chevoir, and P. Moucheront, J. Fluid Mech. **400**, 199 (1999); C. Ancey, Phys. Rev. E **65**, 011304 (2002).
[9] P. Mills, D. Loggia, and M. Tixier, Europhys. Lett. **45**, 733 (1999); I. S. Aranson and L. S. Tsimring, Phys. Rev. E **64**, 020301 (2001); A. Lemaitre, Phys. Rev. Lett. **89**, 064303 (2002); D. Ertaş and T. C. Halsey, Europhys. Lett. **60**, 931 (2002); M. Y. Louge, Phys. Rev. E **67**, 061303 (2003).
[10] D. Ertaş, G. S. Grest, T. C. Halsey, D. Levine, and L. E. Silbert, Europhys. Lett. **56**, 214 (2001); L. E. Silbert, D. Ertaş, G. S. Grest, T. C. Halsey, D. Levine, and S. J. Plimpton, Phys. Rev. E **64**, 051302 (2001).
[11] L. E. Silbert, J. W. Landry, and G. S. Grest, Phys. Fluids **15**, 1 (2003).
[12] P.-A. Lemieux and D. J. Durian, Phys. Rev. Lett. **85**, 4273 (2000).
[13] S. F. Edwards, Physica A (Amsterdam) **249**, 226 (1998).
[14] L. E. Silbert (to be published).
[15] C. A. Angell, K. L. Ngai, G. B. McKenna, P. F. McMillan, and S. W. Martin, J. Appl. Phys. **88**, 3113 (2000).
[16] G. D'Anna and G. Gremaud, Nature (London) **413**, 407 (2001).
[17] L. E. Silbert, D. Ertaş, G. S. Grest, T. C. Halsey, and D. Levine, Phys. Rev. E **65**, 051307 (2002).
[18] C. S. O'Hern, S. A. Langer, A. J. Liu, and S. R. Nagel, Phys. Rev. Lett. **86**, 111 (2001).
[19] W. Götze and L. Sjögren, Rep. Prog. Phys. **55**, 241 (1992).