Vortex Sublattice Melting in a Two-Component Superconductor

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We consider the vortices in a superconductor with two individually conserved condensates in a finite magnetic field. The ground state is a lattice of cocentered vortices in both order parameters. We find two phase transitions: (i) a "vortex sublattice melting" transition where vortices in the field with lowest phase stiffness ("light vortices") lose cocentricity with the vortices with large phase stiffness ("heavy vortices"), entering a liquid state (the structure factor of the light vortices vanishes *continuously*; this transition is in the 3Dxy universality class); (ii) a first-order melting transition of the lattice of heavy vortices, *in a liquid of light vortices*.

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Theories with multicomponent bosonic scalar matter fields minimally coupled to a gauge field are of interest in a variety of condensed matter systems and beyond. This includes superconducting low-temperature phases of light atoms [1–6] under extreme enough pressures to produce liquid metallic states, easy-plane quantum antiferromagnets [7], as well as other multiple-component superconductors [2,3]. It also has applications in particle physics [8]. The projected liquid metallic states of hydrogen (LMH) [1] may soon be realized in high pressure experiments [9–11]. At low temperatures, compressed liquid hydrogen is particularly interesting since it features prominent quantum fluctuations which lead to the possibility of a new state of matter, a near ground state liquid metal [1]. Its superconducting counterpart involves Cooper pairs of electrons and protons [1], whence symmetry precludes Josephson coupling between different condensate species. Resolving what happens to such a system in a magnetic field is now a matter of some urgency, due to new and detailed first principles calculations predicting LMH under extreme pressures of order 400 GPa [10]. This is not far from experimentally achieved pressures of 320 GPa [9], where hints of a maximum in the melting temperature versus pressure are evident. Magnetic-field experiments may very likely be exclusive probes to provide confirmation of LMH. A first study of the phase diagram of the projected LMH in magnetic fields has been presented, unveiling a phase diagram with rich structure [5]. This raises issues of interest also in the broader domain of physics concerning the order and universality classes of possible phase transitions separating phases of partially broken symmetries in quantum fluids. We report a study of this using a confluence of exact topological arguments and large scale Monte Carlo (MC) simulations.

For a general number of components N, the Ginzburg-Landau model is defined by the Lagrangian

$$\mathcal{L} = \sum_{\alpha=1}^{N} \frac{|\mathbf{D}\Psi_0^{(\alpha)}(\mathbf{r})|^2}{2M^{(\alpha)}} + V[\{\Psi_0^{(\alpha)}(\mathbf{r})\}] + \frac{1}{2}[\nabla \times \mathbf{A}(\mathbf{r})]^2. \quad (1)$$

Here, $\{\Psi_0^{(\alpha)}(\mathbf{r}) | \alpha = 1, \ldots, N\}$ are complex scalar fields, $M^{(\alpha)}$ is the mass of the condensate species α, and $\mathbf{D} = \nabla - ie\mathbf{A}(\mathbf{r})$. In LMH *each individual condensate is conserved*, consequently $V[\{\Psi_0^{(\alpha)}(\mathbf{r})\}]$ is only a function of $|\Psi_0^{(\alpha)}(\mathbf{r})|^2$. The model is studied in the phase-only approximation $\Psi_0^{(\alpha)}(\mathbf{r}) = |\Psi_0^{(\alpha)}| \exp[i\theta^{(\alpha)}(\mathbf{r})]$, where $|\Psi_0^{(\alpha)}| =$ const. Then, the V term is a constant which may be omitted from the action [5].

For the discussions in this paper, another form of the action is useful. Introducing $|\psi^{(\alpha)}|^2 = |\Psi_0^{(\alpha)}|^2/M^{(\alpha)}$ and $\Psi^2 \equiv \sum_{\alpha=1}^{N} |\psi^{(\alpha)}|^2$, Eq. (1) may be rewritten [6] in terms of *one* charged and $N-1$ neutral modes,

$$\mathcal{L} = \frac{1}{2\Psi^2}\left(\sum_{\alpha=1}^{N} |\psi^{(\alpha)}|^2 \nabla\theta^{(\alpha)} - e\Psi^2\mathbf{A}\right)^2 + \frac{1}{2}(\nabla \times \mathbf{A})^2 + \frac{1}{4\Psi^2}\sum_{\alpha,\beta=1}^{N} |\psi^{(\alpha)}|^2 |\psi^{(\beta)}|^2 [\nabla(\theta^{(\alpha)} - \theta^{(\beta)})]^2. \quad (2)$$

While Eq. (1) is convenient for MC simulations, Eq. (2) has advantages for analytical considerations, since the neutral and the charged modes are explicitly identified. Moreover, Eq. (2) is convenient for identifying various states of partially broken symmetry, emerging when an N-component system is subjected to an external magnetic field [5,6].

We now focus on the case $N = 2$. In the case of LMH, $\Psi_0^{(1)}$ and $\Psi_0^{(2)}$ will denote protonic and electronic superconducting condensates, respectively, and hence $|\psi^{(1)}|^2 \ll |\psi^{(2)}|^2$. In zero external magnetic field, this system features a low-temperature phase transition in the 3Dxy universality class at T_{c1} where superfluidity is lost, followed at higher temperatures by an inverted 3Dxy transition at T_{c2} where superconductivity and the Higgs mass of $\mathbf{A}$ (Meissner effect) is lost [4]. Here, we will consider the system in finite magnetic field at temperatures below T_{c2} [5].

We define a type-α vortex as a topological defect in $\Psi_0^{(\alpha)}$ associated with a nontrivial phase winding $\Delta\theta^{(\alpha)} = \pm 2\pi$, whereas a *composite vortex* is a topological defect where type-1 and type-2 vortices coincide in space. At low temperatures the formation of a vortex lattice (VL) of *non-composite vortices* is forbidden because these vortices have a logarithmically divergent energy, whereas *composite* vortices have finite energy [3,4]. In a type-II 2-component superconductor, therefore, a VL of composite type-1 and type-2 vortices is formed, illustrated in Fig. 1(A). At elevated temperatures, the 2-component system in a magnetic field will exhibit thermal excitations in the form of fractional-flux vortex loops similar to the case of zero magnetic field $\mathbf{B} = \nabla \times \mathbf{A} = 0$ [3,4]. Since the field-induced vortices are logarithmically bound states of constituent (elementary) vortices, the thermal fluctuations will induce a *local splitting* of composite vortices in the form of two half loops connected to a straight line [5], as shown in Fig. 1(B).

Consider now the processes illustrated in Fig. 1(B) for the case $|\psi^{(1)}|^2 \ll |\psi^{(2)}|^2$, upon increasing the temperature beyond the low-temperature regime. We may view this process as a type-1 closed vortex loop superposed on a VL of (slightly) fluctuating composite vortices. An important point to notice is that a type-α vortex does not interact with a composite vortex by means of a neutral mode [6]. This follows from a topological argument that two split branches will feature nontrivial winding in the composite neutral field $\theta^{(1)}$–$\theta^{(2)}$, while a composite vortex line does not. Hence, the splitting transition may be viewed as a *type-1 vortex-loop proliferation in a neutral superfluid*. This is illustrated in Fig. 2. Thus, we may utilize the well known results for the critical properties of the 3Dxy model for neutral superfluids described as a vortex-loop proliferation [12–14]. This "vortex sublattice melting" phase transition is therefore in the 3Dxy universality class [12–14], not a first-order melting transition. The resulting phase is one where superfluidity is lost and longitudinal superconductivity is retained in the component $\Psi_0^{(2)}$ [5], illustrated in Fig. 1(C).

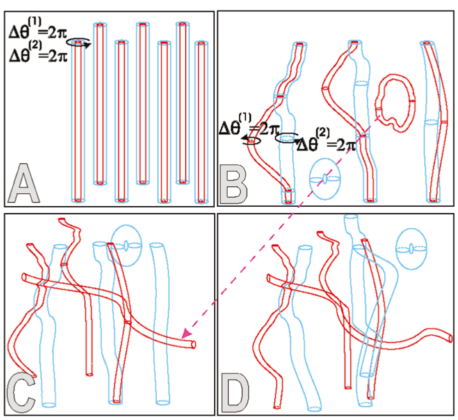

FIG. 1 (color). (A) A type-II, $N = 2$ superconductor at zero temperature in a magnetic field forms a lattice of composite vortices, i.e., cocentered type-1 (red) and type-2 (blue) vortices. (B) Low-temperature fluctuations in the composite vortex lattice (VL) generate closed loops of type-1 vortices and *local* splitting of field-induced composite vortices. This phase features superfluidity, as well as longitudinal superconductivity. (C) There is a temperature region in low magnetic field when type-1 vortices form a vortex liquid and the corresponding vortex loops are proliferated, while type-2 vortices form a VL. Superfluidity is lost, longitudinal superconductivity is retained. The arrow from (B) to (C) illustrates type-1 loop proliferation. (D) A vortex liquid of type-1 and type-2 vortices. Superfluidity and longitudinal superconductivity is lost, i.e., the normal phase [5]. There is no type-2 loop proliferation going from (C) to (D), since this is a first-order melting transition of the type-2 VL *in the background of a liquid of line-tension-less type*-1 *vortices*.

Apart from the sublattice melting transition, thermal fluctuations will produce a melting transition of the type-2 VL at a higher temperature. It is well known that sufficiently strong thermal fluctuations drive *a first-order* melting transition of the Abrikosov lattice [14] in $N = 1$ superconductors. Because of the interplay with the proliferated type-1 vortices, a counterpart to this effect for the case $N = 2$ when $|\psi^{(1)}|^2 \neq |\psi^{(2)}|^2$ is more complex. The melting temperature $T_{\mathrm{M}}(B)$ of the type-2 Abrikosov lattice is suppressed with increasing magnetic field [14]. At low enough magnetic fields, upon heating the system, the 3Dxy type-1 vortex-loop transition at $T_{c1}(B)$ will be encountered before the melting transition of type-2 vortices at $T_{\mathrm{M}}(B)$. Above $T_{\mathrm{M}}(B)$, longitudinal superconductivity is also lost, whence we may infer that the vortex-liquid mixture of liberated type-1 and type-2 vortices is the normal metallic phase [5], depicted in Fig. 1(D).

The above is borne out in MC simulations. We consider the model based on Eq. (1) for $N = 2$ on an L^3 lattice (with L up to 96) with periodic boundary conditions for coupling constants $|\psi^{(1)}|^2 = 0.2$, $|\psi^{(2)}|^2 = 2$, and $e^2 = 1/10$. The ratio $|\psi^{(2)}|^2/|\psi^{(1)}|^2 = 10$ brings out one second order phase transition at $T_{c1}(B)$ in the 3Dxy universality class well below the melting temperature $T_{\mathrm{M}}(B)$ of the VL. In

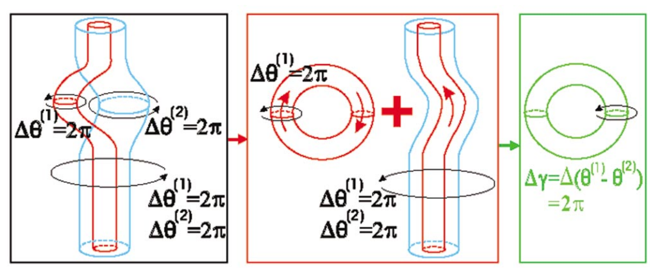

FIG. 2 (color). Detailed illustration of the low-temperature thermal fluctuations in a VL of composite vortices. A local excursion of a type-1 vortex away from the composite VL may be viewed as a type-1 bound vortex-loop superposed on the composite VL. The composite vortex line does not interact with a vortex with nontrivial winding in $\Delta\gamma = \Delta(\theta^{(1)}–\theta^{(2)})$ [6]. A splitting of the composite VL, illustrated in going from (B) to (C) in Fig. 1, may be viewed as a *zero-field* vortex-loop proliferation of type-1 vortices; a 3Dxy phase transition universality [12–14].

LMH $|\psi^{(2)}|^2/|\psi^{(1)}|^2 \approx 2000$, but the physical picture remains. For real estimates of $T_{c1}(B)$ and $T_M(B)$ in LMH, see Ref. [1]. The Metropolis algorithm with local updating is used in combination with Ferrenberg-Swendsen reweighting [15]. The external magnetic field $\mathbf{B}$ studied is $B^x = B^y = 0$, $B^z = 2\pi/32$, thus there are 32 plaquettes in the (x, y) plane per flux quantum. This is imposed by splitting the gauge field into a static part $\mathbf{A}_0$ and a fluctuating part $\mathbf{A}_{\rm fluct}$. The former is kept fixed to $(A_0^x, A_0^y(\mathbf{r}), A_0^z) = (0, 2\pi x f, 0)$, where $f = 1/32$ is the magnetic filling fraction, on top of which the latter field is free to fluctuate. Together with periodic boundary conditions on $\mathbf{A}_{\rm fluct}$, the constraint $\oint_C(\mathbf{A}_0 + \mathbf{A}_{\rm fluct})d\boldsymbol{l} = 2\pi f L^2$, where C is a contour enclosing the system in the (x, y) plane, is ensured. It is imperative to fluctuate $\mathbf{A}$, otherwise type-1 and type-2 vortices do not interact [3,4,6]. To investigate the transition at T_{c1}, we have performed finite size scaling (FSS) of the third moment of the action. The simulations are done by using vortices directly [4], but with a finite magnetic induction $B^z = 2\pi/32$.

We compute the specific heat C_V and the third moment of the action $S = \beta\int d\mathbf{r}\mathcal{L}$, defined as $M_3 = (\langle S^3\rangle - \langle S\rangle^3)/L^3$. Here, β is inverse temperature. The peak to peak value of M_3 scales with system size as $L^{(1+\alpha)/\nu}$ and the width between the peaks scales as $L^{-1/\nu}$ [16]. To probe the structural order of the vortex system, we compute the planar structure function $S^{(\alpha)}(\mathbf{k}_\perp)$ of the *local vorticity* $\mathbf{n}^{(\alpha)}(\mathbf{r}) = (\nabla \times [\nabla\theta^{(\alpha)} - e\mathbf{A}])/2\pi$, given by

$$S^{(\alpha)}(\mathbf{k}_\perp) = \frac{1}{(fL^3)^2}\left\langle \left| \sum_{\mathbf{r}} n_z^{(\alpha)}(\mathbf{r}) e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp} \right|^2 \right\rangle, \qquad (3)$$

where $\mathbf{r}$ runs over dual lattice sites and $\mathbf{k}_\perp$ is perpendicular to $\mathbf{B}$. This function will exhibit sharp peaks for the characteristic Bragg vectors $\mathbf{K}$ of the type-α VL and will feature a ring structure in its corresponding liquid of type-α vortices. The signature of vortex sublattice melting will be a transition from a sixfold symmetric Bragg-peak structure to a ring structure in $S^{(1)}(\mathbf{K})$ while the peak structure remains intact in $S^{(2)}(\mathbf{K})$. Furthermore, we compute the *vortex cocentricity* $N_{\rm co}$ of type-1 and type-2 vortices, defined as $N_{\rm co} \equiv N_{\rm co}^+ - N_{\rm co}^-$, where

$$N_{\rm co}^{\pm} \equiv \frac{\sum_{\mathbf{r}} |n_z^{(2)}(\mathbf{r})| \delta_{n_z^{(1)}(\mathbf{r}), \pm n_z^{(2)}(\mathbf{r})}}{\sum_{\mathbf{r}} |n_z^{(2)}(\mathbf{r})|}, \qquad (4)$$

where $\delta_{i,j}$ is the Kronecker delta. The reason for considering $N_{\rm co}$ is that we then eliminate the effect of random overlap of vortices in the high-temperature phase $T > T_{c1}$ due to vortex-loop proliferation, and focus on the *compositeness* of field-induced vortices. The quantity $N_{\rm co}$ is the fraction of type-2 vortex segments that are cocentered with type-1 vortices, providing a measure of the extent to which vortices of type-1 and type-2 form a *composite* vortex system. Hence, it probes the splitting processes visualized in Fig. 1(B) and in Fig. 2. The results are shown in Fig. 3.

At T_{c1}, C_V has a pronounced peak associated with the 3Dxy transition, and a broader less pronounced peak which is the finite field remnant of the zero field inverted 3Dxy transition [13]. Scaling of M_3 at T_{c1} shown in Fig. 3(c) yields the critical exponents $\alpha = -0.02 \pm 0.05$ and $\nu = 0.67 \pm 0.03$ in agreement with the 3Dxy universality class. A novel result is that $S^{(1)}(\mathbf{K})$ vanishes *continuously* as the temperature approaches T_{c1} from below, precisely the hallmark of the decomposition transition that separates the two types of vortex states depicted in Fig. 1(B) and 1(C). A related feature is the *vanishing* of $N_{\rm co}$ at T_{c1} as a function of temperature, discussed in detail below. The first-order melting transition takes place at T_M, where $S^{(2)}(\mathbf{K})$ vanishes discontinuously. This is the temperature at which the translational invariance is restored through melting of the type-2 VL. The resulting translationally invariant high-temperature phase is depicted in Fig. 1(D). In the tempera-

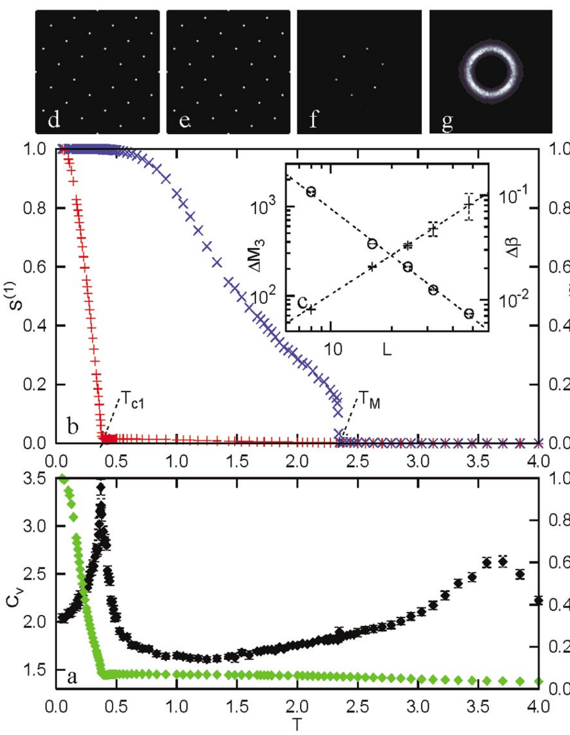

FIG. 3 (color). MC results for $N = 2$ $|\psi^{(1)}|^2 = 0.2$, $|\psi^{(2)}|^2 = 2$, and $e = 1/\sqrt{10}$. (a) C_V (black) and $N_{\rm co}$ (green). The C_V anomaly at $T_{c1} = 0.37$, where type-1 vortices proliferate, matches the point at which $N_{\rm co}$ drops to zero. Thus, type-1 vortices are torn off type-2 vortices. The remnant of the zero-field anomaly in C_V is seen as a hump at $T \sim 3.6$. (b) $S^{(1)}(\mathbf{K})$ (red) and $S^{(2)}(\mathbf{K})$ (blue) for the particular Bragg vector $\mathbf{K} = (\pi/4, -\pi/4)$. $S^{(1)}(\mathbf{K})$ vanishes continuously at T_{c1}, while $S^{(2)}(\mathbf{K})$ vanishes discontinuously at $T_M = 2.34$. (c) FSS plots of the M3 from which the exponents $\alpha = -0.02 \pm 0.05$ and $\nu = 0.67 \pm 0.03$ are extracted, showing that the sublattice melting is a 3Dxy phase transition. (d)–(g) Plots of $S^{(2)}(\mathbf{k}_\perp)$ for the temperatures $T_{\rm d} = 0.35$, $T_{\rm e} = 0.4$, $T_{\rm f} = 1.66$, and $T_{\rm g} = 2.85$, respectively. At $T_{\rm d}$, $T_{\rm e}$, and $T_{\rm f}$, the VL remains intact. The VL melts at T_M to give a vortex-liquid ring pattern at $T_{\rm g}$.

ture interval $T < T_{c1}$, the system features superconductivity and superfluidity simultaneously [5], since there is long-range order both in the charged and the neutral vortex modes. In the temperature interval $T_{c1} < T < T_M$, long-range order in the neutral mode is destroyed by loop proliferation of type-1 vortices, thus superfluidity is lost [5]. However, longitudinal one-component superconductivity is retained along the direction of the external magnetic field. For $T > T_M$ superconductivity is also lost; hence, this is the normal metallic state, which is a two-component vortex liquid.

The most unusual and surprising feature is the continuous variation of $S^{(1)}(\mathbf{K})$ with temperature, even at T_{c1} where it vanishes. The explanation for this is the proliferation of type-1 vortices (which destroys the neutral superfluid mode) in the background of a composite VL, which the type-1 vortices essentially do not see; cf. Fig. 2. As far as the composite neutral Bose field $\theta^{(1)}$–$\theta^{(2)}$ is concerned, *it is precisely as if the composite VL were not present at all*. Hence, $S^{(1)}(\mathbf{K})$ vanishes for a completely different reason than $S^{(2)}(\mathbf{K})$, namely, due to *critical fluctuations, i.e., vortex-loop proliferation* in the condensate component with lowest bare stiffness. Such a phase transition does not completely restore broken translational invariance associated with a VL, since for the type-2 vortices *quite remarkably, the VL order survives the decomposition transition*, due to interaction between heavy vortices mediated by charged modes. The vanishing of N_{co} is particularly interesting, and finds a natural explanation within the framework of the above discussion. That is, for $T \ll T_{c1}$, we have $N_{co} \approx 1$, so the vortex system consists practically exclusively of composite vortices. As the temperature increases, thermal fluctuations induce excursions such as those illustrated in Fig. 1(B) and Fig. 2, which reduces N_{co}^{+} from its low-temperature value, reaching a *minimum* at T_{c1}, and then *increase* for $T > T_{c1}$. Conversely, N_{co}^{-} remains essentially zero until T_{c1}, thereafter increasing monotonically. For temperatures above, but close to T_{c1}, fluctuations in vortices originating in $\Delta\theta^{(2)}$ are still small, so the variations in $N_{co} = N_{co}^{+} - N_{co}^{-}$ reflect thermal fluctuations in vortices originating in $\Delta\theta^{(1)}$. The increase of $N_{co}^{\pm}$ means that type-1 vortex loops are thermally generated, and thus tend to *randomly* overlap more with the moderately fluctuating type-2 vortices. At their first-order melting transition, type-2 vortices fluctuate only slightly. *Thus, the vanishing of* N_{co} *above* T_{c1} *reflects the increase in the density of thermally generated type*-1 *vortex loops in the background of a slightly fluctuating type*-2 *VL*; cf. Fig. 1(C).

We have investigated the character of thermally driven phase transitions in a two-component vortex system in a magnetic field. We find (i) a 3D*xy* phase transition associated with melting of the VL originating in the phases with lowest stiffness, which may be viewed as vortex-loop proliferation taking place in the background of a composite VL (the structure function for type-1 vortices vanishes *continuously* at the transition, and this has no counterpart in a one-component superconductor); (ii) a first-order VL melting of the type-2 vortex system. The corresponding structure function vanishes *discontinuously* at the transition; this takes place in the background of a *liquid of line-tension-less type*-1 *vortices.* This sets the type-2 VL melting apart from the corresponding phenomenon in one-component superconductors.

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