Superfluidlike Motion of Vortices in Light Condensates

María J. Paz-Alonso and Humberto Michinel
Área de Óptica, Facultade de Ciencias de Ourense, Universidade de Vigo, As Lagoas s/n, Ourense, ES-32004 Spain
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We demonstrate, through numerical simulations, the generation of stable vortex lattices in light condensates. This can be achieved by propagating several concentric laser beams with nested vortices of different topological charges in an optical material with a cubic-quintic nonlinearity. We have considered several initial conditions, and in all the cases the net topological charges of the resulting lattice is equal to the topological charge of the initial outer vortex. The lattice exhibits rotation similar to vortex motion in superfluids. These vortex arrays could be used to implement all-optical photonic crystal fibers. Our results also apply to Bose-Einstein condensates in the presence of three-body elastic interactions.

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Vortices are waves characterized for possessing a phase singularity and a rotational flow around the singular point [1]. They appear in many branches of physics like fluid mechanics, Bose-Einstein condensation, light propagation or astrophysics [2], and in physical systems of different nature and scale, ranging from atmospheric tornadoes and water whirlpools to superfluids or superconductors [3].

In optics [4] they have attracted much attention because of possible applications in the optical transmission of information, or in guiding and trapping of particles [5]. Optical vortices [6] manifest themselves as isolated dark spots in the modal patterns of certain lasers [7]. The integer number of windings of the phase around the dislocation is called the topological charge l of the vortex [8], which plays the role of an angular momentum. These defects can appear spontaneously in light propagation through turbulent optical media or can be produced by appropriately shining a computer generated hologram [9].

Generation, propagation, and interaction of optical vortices in nonlinear media have also been subjects of extensive study. The first theoretical work analyzed their stability in Gaussian-like distributions propagating in Kerr materials [10]. In self-focusing saturable nonlinear media, it was found that a beam of finite size will always filament under the action of a phase dislocation [11]. Experimental observation of vortex states for self-defocusing materials was done both in the Kerr case for continuous background [12] and in the saturable case with finite size beams [13]. Propagation of vortex clusters in media with competing cubic and quintic (C-Q) nonlinearities has also been studied in recent works [14].

In the present work we study the possibility of generating stable vortex lattices in light condensates by propagating concentric vortex beams with different topological charges and almost square profiles in C-Q media. As we will show, the lattices exhibit rotation similar to vortex motion in superfluids [7,15]. Our results also apply to Bose-Einstein condensates (BECs), where experimental investigations on vortex lattices have been reported [16]. We must also stress the potential use of these light distributions as optically induced photonic crystal fibers [17] with guiding properties.

Physical model.—Paraxial propagation along z of a continuous linearly polarized laser beam in a C-Q nonlinear medium is given by a nonlinear Schrödinger equation of the form

$$2ikn_0\frac{\partial\Psi}{\partial z}+\nabla_\perp^2\Psi+2k^2n_0(n_2|\Psi|^2-n_4|\Psi|^4)\Psi=0, \quad (1)$$

where $k=2\pi/\lambda$ is the wave number in vacuum, n_0 is the linear refractive index, $|\Psi|^2$ is the intensity of the electromagnetic wave, and $\nabla_\perp^2 = r^{-2}\partial^2/\partial\theta^2 + r^{-1}\partial/\partial r + \partial^2/\partial r^2$ is the transverse Laplacian operator in cylindric coordinates (r, θ, z). The positive constants n_2 and n_4 characterize the dependence of the refractive index on the intensity of the beam. The C-Q nonlinearity is necessary to achieve the phase transition from a gas state to a liquid light state [18]. Physically, the combined effect of diffraction and the self-defocusing term of n_4 will balance the collapsing tendency induced by the Kerr effect, giving rise to a stable two-dimensional light distribution [19]. The search of C-Q materials has continuously progressed in the last years. Promising candidates are chalcogenide glasses [20] with $n_0 = 1.8$, $n_2 = 2\times10^{-4}$ cm^2/GW, and $n_4 = 2\times10^{-3}$ cm^4/GW^2, which need pulsed beams with peak powers in the range of GW/cm^2, that can be achieved with usual Nd-doped yttrium aluminum garnet (Nd:YAG) lasers with $\lambda = 1.064$ μm. However, for high powers absorption effects could play a significant role and cannot be neglected. On the other hand, quantum coherence techniques could be used to control the nonlinear refractive index for mW continuous lasers provided the sources are highly stabilized in frequency [21].

We have performed the numerical integration of Eq. (1) for stationary vortex states [22] of the form $\Psi(r,\theta,z) = \psi(r)e^{i(\beta z+l\theta)}$, $r=(x^2+y^2)^{1/2}$ being the radial dimension, β the nonlinear phase shift or propagation constant, and

$\theta = \tan^{-1}(y/x)$. For a given integer value of the topological charge l, a continuum of eigenstates can be obtained with β varying between zero and a critical value β_{cr} over which no stationary states can be found. For the particular case $n_2^2/n_4 = 1$, $\beta_{cr} = 0.1875$ for any value of l [18]. In this Letter we shall consider only large values of the beam power (β close to β_{cr}), for which the spatial light distributions show wide flattop profiles with sharp decaying edges [23], similar to hyper-Gaussian distributions. In this limiting case, it is possible to derive an analytical expression for the surface tension of these light condensates [18]. To do so, we express the increase of the system energy using the Hamiltonian $dH = [\frac{1}{2}|\nabla\Psi|^2 - \frac{1}{2} \times \frac{n_2}{n_4}|\Psi|^4 + \frac{1}{3}|\Psi|^6]2\pi r dr$, and we consider a thermodynamical model with $dH = \mu dN - \sigma dA$. By analogy with BECs in alkali gas, β plays the role of a chemical potential μ. Thus, σdA represents the work due to deformation against a surface tension σ. Considering a square eigenstate function of radius r, we obtain $\sigma = (9\pi/16) \times (n_2/n_4)^3 r$. Thus, the surface tension of the light condensates grows linearly with the radius, as in the case of usual liquid droplets. This adds extra support to the idea that light beams in C-Q nonlinear materials can undergo a phase transition from a photon gas to a liquid of light.

Generation of vortex lattices.—We will study the propagation of two concentric laser beams with nested vortices of different topological charges (TCs) l_1, l_2 and approximately square profiles in C-Q nonlinear media. The initial conditions are numerically calculated stationary states of Eq. (1). In the simplest case in which the inner beam is nodeless ($l_1 = 0$), a rotating lattice of $n = |l_2|$ vortices is created, being their TC $l = 1$ or $l = -1$, depending on the sign of l_2. Figure 1 shows three examples corresponding to $l_2 = 3$, 4, and 5. In all of them the vorticityless beam is much narrower than the vortex one and exactly covers the intensity deep in the center of the vortex beam. As can be seen in the interference patterns, they give rise to rotating lattices of three, four, and five single charged vortices, respectively. Note that the signature of a phase singularity is the characteristic fork defect in the fringe pattern.

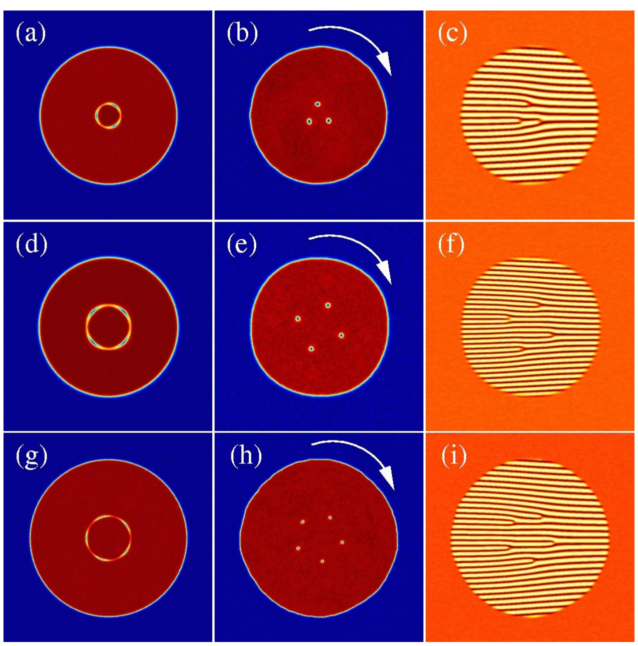

FIG. 1 (color online). Numerical simulation of the propagation of two concentric laser beams with nested vortices. The inner beam is nodeless and the TCs of the outer vortices are $l = 3$, 4, and 5, respectively. Propagation constants: $\beta = 0.99\beta_{cr}$; window widths (in μm): 550 (a)–(c), 450 (d)–(f), and 760 (g)–(i); propagation distances (in mm): $z = 0$ and 150. Images (c), (f), and (i) correspond to the interference patterns with a slightly tilted plane wave, at $z = 150$ mm.

The rotation of the whole structure is caused by the superposition of the phases of all vortices, which results in azimuthal phase gradient and nonzero total angular momentum. This motion is similar to vortex motion in superfluids, where any vortex line induces a velocity field given by the Biot-Savart formula (in complete analogy with a magnetic field around a filament with an electrical current), and the velocity of rotation around the fluid is given by the Kelvin law [15]

$$\frac{d\vec{r}_i}{dz} = \gamma \sum_{j \neq i} \frac{\vec{m}_j \wedge (\vec{r}_i - \vec{r}_j)}{|\vec{r}_i - \vec{r}_j|^2}, \tag{2}$$

where $\vec{r}_{i,j}$ are the spatial positions of the vortices in the (x, y) plane, $\vec{m}_j$ are vortex topological charge vectors in the direction perpendicular to the (x, y) plane, and γ is a scale coefficient with dimensions of a length.

Note that, if the distance between the vortices that compose the lattice does not vary during propagation, the angular velocity takes a constant value. This allows us to

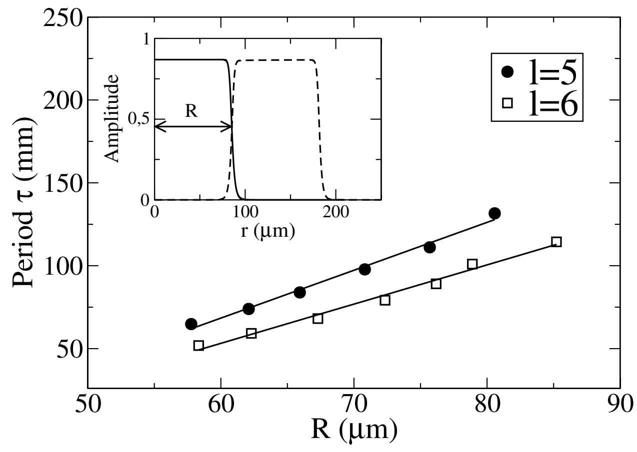

FIG. 2. Period of rotation of the vortex lattice versus internal radius of the outer beam R, considering the inner beam nodeless and the outer beam with TC $l = 5$ and $l = 6$, respectively. This period represents the propagation distance z at which the angle of rotation of the lattice is 2π rad. Inset: radial profiles of input beams with $\beta = 0.99\beta_{cr}$ and $l = 6$.

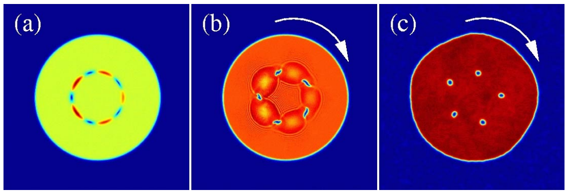

FIG. 3 (color online). Numerical simulation of the propagation of two laser beams with nested vortices of TCs $l = 0$ and 5, with propagation constants $\beta = 0.984\beta_{\rm cr}$ and $\beta = 0.99\beta_{\rm cr}$. The outer beam has been displaced 2 μm in the x axis. The window width is 500 μm. The propagation distances (in mm) are $z = 0$, 0.2, and 30.

study the period of rotation of the created vortex lattices. We have considered two initial concentric beams with nested vortices, the inner beam being nodeless, and varying the TC of the outer vortex. Figure 2 shows the period of rotation τ versus internal radius R of the outer vortex with TC $l = 5$ and $l = 6$, respectively. An example of radial profiles of input beams (for $l = 6$) can be seen in the inset. The period of rotation represents the propagation distance z at which the angle of rotation of the lattice is 2π rad. As can be appreciated, for a fixed value of l, τ increases linearly with R. This means that the velocity of rotation decreases when increasing the distance between the vortices in the lattice, in accordance to Eq. (2). Besides, for a fixed value of R, τ decreases with l. This result is evident taking into account that, in this case, the number of components of the lattice is given by the TC of the outer vortex. This implies that for higher values of l, the distance between the vortices in the lattice will be smaller.

Stability of the vortex lattices.—In order to test the stability of the created vortex lattices, we have considered the case in which one of the beams is slightly displaced in one direction. Figure 3 shows the propagation of two laser beams with nested vortices of TCs $l_1 = 0$ and $l_2 = 5$, with the outer vortex displaced 2 μm in the x axis. As can be appreciated, a lattice of five single charged vortices is created although the initial beams are not exactly concentric. We have also studied the propagation of the previous lattice in the presence of white noise, obtaining that this effect becomes important for a noise $\gtrsim 5\%$.

We will now study the propagation of two concentric laser beams with nested vortices of different TCs l_1 and l_2, with $l_1 \neq 0$. In this case, a rotating lattice of vortices is created, with the initial inner vortex in the center and $n = |l_2 - l_1|$ small vortices with TC $l = 1$ or $l = -1$, depending on the sign of l_2. Figures 4(a)–4(c) correspond to the case $l = 2$ and -8. As can be appreciated in the interference pattern, the resulting lattice is composed of the inner vortex with $l = 2$ in the center and ten small vortices of TC $l = -1$ around it. Considering propagation of three concentric laser beams with nested vortices of TCs l_1, l_2, and l_3, a rotating vortex lattice of two arrays is created, with their number of holes $n_1 = |l_2 - l_1|$ and $n_2 = |l_3 - l_2|$,

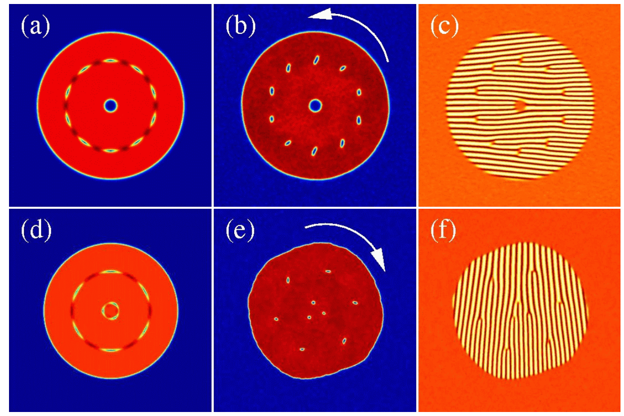

FIG. 4 (color online). Numerical simulation of the propagation of two (a)–(c) and three (d)–(f) laser beams with nested vortices of TCs $l = 2, -8$ and $l = 0, 3, 9$. Propagation constants $\beta/\beta_{\rm cr} = 0.99, 0.98$, and $\beta/\beta_{\rm cr} = 0.96, 0.99, 0.99$, respectively. Window widths (in μm): 500 (a)–(c) and 800 (d)–(f). Propagation distances (in mm): $z = 0$ and 40. Images (c) and (f) correspond to the interference patterns of (b) and (e) with a slightly tilted plane wave.

respectively. In order to preserve this rotating structure during propagation it is necessary that the initial vortices are wide enough. Figures 4(d)–4(f) correspond to the case $l = 0$, 3, and 9. As can be seen, a rotating lattice of three and six single charged vortices is created. Note that, in both cases, the net topological charge of the lattice is equal to the TC of the initial outer vortex. As we will see, this holds for any initial condition.

Stable propagation of the structures.—In the case of two concentric laser beams with nested vortices of opposite TCs, the resulting lattice does not exhibit rotation because

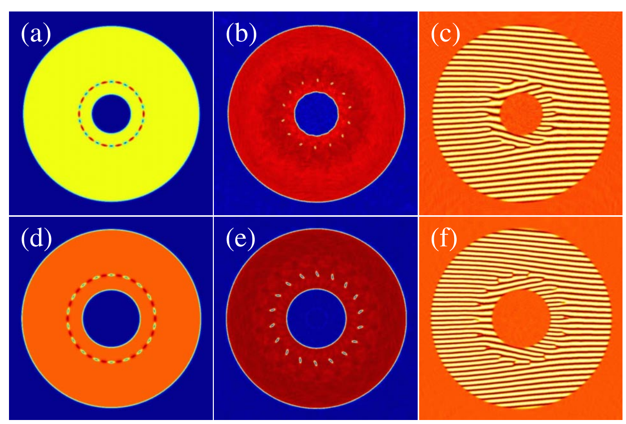

FIG. 5 (color online). Numerical simulation of the propagation of two concentric beams with nested vortices of opposite TCs $l = 7; -7$ (a) and $l = 9, -9$ (d); propagation constants $\beta/\beta_{\rm cr} =$ 0.98 and 0.99; window widths (in μm) 1100 (a)–(c) and 1000 (d)–(f); and propagation distances (in mm) $z = 0$ and 10. Images (c) and (f) correspond to the interference patterns of (b) and (e) with a slightly tilted plane wave.

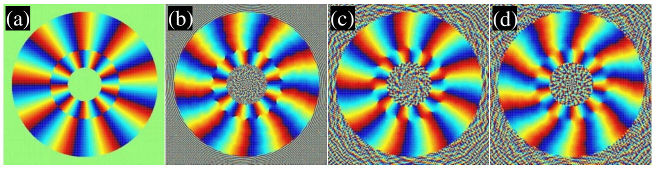

FIG. 6 (color online). Phase maps corresponding to Fig. 5(d). Propagation distances (in mm) are $z = 0$, 2, 8, and 10.

the initial total angular momentum is zero. The inner vortex does not disappear during propagation and $2l$ small vortices are created, with their TC $l = 1$ or $l = -1$, depending on the sign of l_2. Examples of Fig. 5 correspond to $l = 7, -7$ and $l = 9, -9$, respectively. As can be appreciated in the interference patterns 5(c) and 5(f) these lattices are composed of one central vortex of TC $l = 7$ or $l = 9$ and 14 5(b) or 18 5(e) small vortices of TC $l = -1$. Note that the net TC of the created lattice is equal to the topological charge of the outer vortex. The phase maps corresponding to the case of initial vortices with $l = 9, -9$ can be seen in Fig. 6.

It has been recently shown that the dynamic holographic optical tweezer technique [24] can be used to create optical vortices with TCs up to $l = 200$, which would allow us to generate our initial conditions.

We must notice that the different geometries exhibited by the vortex arrays are similar to the configurations of photonic crystal fibers, where a regular pattern of holes is altered by suppressing one of them. In our case, a similar structure is generated all optically creating an all-optical photonic crystal fiber with potential guiding properties.

Conclusions.—We have shown that propagation of concentric laser beams with nested vortices of different TCs in C-Q media yields to the formation of stable vortex lattices whose number of holes is given by the difference between the TCs of the initial vortices. The net topological charge of the lattice is equal to the TC of the initial outer vortex. If the initial TCs are of the same sign the lattice exhibits rotation similar to vortex motion in superfluids.

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