New Limits on Fundamental Weak-Interaction Parameters from Superallowed β Decay

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A new survey of all world data on superallowed β decays provides demanding tests of, and tight constraints on, the weak interaction. In confirmation of the conserved vector current hypothesis, the vector coupling constant G_V is demonstrated to be constant to better than three parts in 10^4, and any induced scalar current is limited to $f_S \leq 0.0013$ in electron rest-mass units. Any possible fundamental scalar current is similarly limited to $|C_S/C_V| \leq 0.0013$. The superallowed data also determine the up-down element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix to be $V_{ud} = 0.9738(4)$. With Particle Data Group values for V_{us} and V_{ub}, the top-row test of CKM unitarity yields $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9966(14)$; if V_{us} comes instead from two recent results on K_{e3} decay, this sum becomes 0.9996(11). Either unitarity result can be used to constrain the possible existence of right-hand currents.

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Beta decay between nuclear analog states of spin parity, $J^\pi = 0^+$, and isospin, $T = 1$, has been a subject of continuous and often intense study for five decades. The strengths, or ft values, of such transitions are nearly independent of nuclear-structure ambiguities and depend uniquely on the vector part of the weak interaction. Thus, their measurement has given nuclear physicists access to clean tests of some of the fundamental precepts of weak-interaction theory, and, over the years, this strong motivation has led to very high precision being achieved in both the experiments and the theory required to interpret them.

As befits such an important issue, there have been periodic surveys of the relevant world data (see, for example, Refs. [1–4]). Because the last survey appeared in 1990 and a large amount of new data has appeared in the decade and a half since then, we have just completed a thorough new overview [5] in which we critically surveyed all relevant measurements, adjusted original data to take account of the most modern calibration standards, obtained statistically rigorous average results for each transition, and used updated and consistent calculations to extract weak-interaction parameters from those results. The outcome includes more exacting confirmation of the conserved vector current (CVC) hypothesis, a reduced limit on any possible scalar currents, and an improved value for the up-down quark-mixing element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, V_{ud}. The latter is an important component of the most demanding available test of the unitarity of that matrix and can help constrain or rule out the influence of right-hand currents.

For any superallowed $0^+ \to 0^+$ β decay between $T = 1$ analog states, the experimental ft value can be related to the vector coupling constant via an expression that includes several small ($\sim$1%) correction terms. It is convenient to combine some of these terms with the ft value and define a "corrected" $\mathcal{F}t$ value. Thus, we write [6]

$$\mathcal{F}t \equiv ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2G_V^2(1 + \Delta_R^V)}, \quad (1)$$

where $K/(\hbar c)^6 = 2\pi^3 \hbar \ln 2/(m_e c^2)^5 = 8120.271(12) \times 10^{-10}$ GeV^{-4} s, G_V is the vector coupling constant for semileptonic weak interactions, δ_C is the isospin-symmetry-breaking correction, and Δ_R^V is the transition-independent part of the radiative correction. The terms δ'_R and δ_{NS} compose the transition-dependent part of the radiative correction, the former being a function only of the electron's energy and the Z of the daughter nucleus, while the latter, like δ_C, depends in its evaluation on the details of nuclear structure. From this equation, it can be seen that each measured transition establishes an individual value for G_V and, if the CVC assertion is correct that G_V is not renormalized in the nuclear medium, all such values—and all the $\mathcal{F}t$ values themselves—should be identical within uncertainties, regardless of the specific nuclei involved.

The ft value that characterizes any β transition depends on three measured quantities: the total transition energy, Q_{EC}, the half-life, $t_{1/2}$, of the parent state, and the branching ratio, R, for the particular transition of interest. The Q_{EC} value is required to determine the statistical-rate function, f, while the half-life and branching ratio combine to yield the partial half-life, t. In our treatment [5] of the data, we considered all measurements formally published before November 2004 and those we knew to be in an advanced state of preparation for publication by that date. We scrutinized all the original experimental reports in detail and, where possible, adjusted those Q_{EC} values that were based on outdated calibration standards. If corrections to any measurements were evidently required but insufficient information was provided to make them, the results were rejected. Of the surviving results, only those with (updated) uncertainties that are within a factor of 10 of the most precise measurement for each quantity were retained for averaging. The statistical procedures we followed in analyzing the data were those used by the Particle Data Group in their periodic reviews of particle properties (e.g., [7]).

The final average ft values obtained from the survey [5], together with applied correction terms and the resulting $\mathcal{F}t$ values, are given in Table I for the 12 superallowed transitions that are now known to 0.4% precision or better; all but four are actually known to better than 0.1%. They cover a broad range of nuclear masses from $A = 10$ to 74. As anticipated by CVC [see Eq. (1)] the $\mathcal{F}t$ values are statistically consistent with one another, yielding an average value $\overline{\mathcal{F}t} = 3072.7(8)$ s, with a corresponding chi square per degree of freedom of $\chi^2/\nu = 0.42$. These results have many important outcomes. We deal with each, individually identified.

Nuclear-structure-dependent corrections.—As can be seen from Eq. (1), the $\mathcal{F}t$ values differ from their ft-value counterparts by the application of the δ_R' radiative correction and the nuclear-structure-dependent corrections, $\delta_{NS} - \delta_C$. To isolate and illustrate the effect of the latter corrections, we plot in the top panel of Fig. 1 the values of $ft(1 + \delta_R')$ for the 12 transitions listed in Table I and the corresponding values of $\mathcal{F}t$ in the bottom panel. They differ only by the inclusion of the structure-dependent corrections in the latter, and, obviously, these corrections act very well to remove the considerable "scatter" that is apparent in the top panel and is effectively absent in the bottom one. It is important to note that the calculations of δ_{NS} and δ_C [8] employ the best available shell-model wave functions, which are based on a wide range of spectroscopic data. They were further tuned to agree with measured binding energies, charge radii, and coefficients of the isobaric multiplet mass equation (except in the case of ^{74}Rb, where these properties are not yet measured). This means that the origins of the structure-dependent correction terms are completely independent of the superallowed decay data, so the consistency of the corrected $\mathcal{F}t$ values appearing in the bottom panel of the figure is a powerful validation of the calculated corrections used in their derivation.

CVC(1)–vector coupling constant.—The CVC hypothesis—that the weak vector current is just an isospin rotation of a conserved electromagnetic vector current—has several direct implications. The first is that G_V is indeed constant, independent of the nuclear medium. The fact that the $\mathcal{F}t$ values, listed in Table I and plotted in the bottom panel of Fig. 1, form a consistent set over a wide range of nuclei, $10 \le A \le 74$, verifies this expectation of CVC at the level of 3×10^{-4}, which is the fractional

TABLE I. Final average ft values, correction terms, and $\mathcal{F}t$ values [5] for the 12 best known superallowed transitions.

Parent nucleus	ft (s)	δ_R' (%)	$\delta_C - \delta_{NS}$ (%)	$\mathcal{F}t$ (s)
^{10}C	3039.5(47)	1.652(4)	0.540(39)	3073.0(49)
^{14}O	3043.3(19)	1.529(8)	0.570(56)	3071.9(26)
^{22}Mg	3052.4(72)	1.446(17)	0.505(24)	3080.9(74)
^{26m}Al	3036.7(12)	1.458(20)	0.261(24)	3072.9(15)
^{34}Cl	3050.5(47)	1.425(32)	0.720(39)	3071.7(19)
^{34}Ar	3059(12)	1.394(35)	0.825(44)	3076(12)
^{38m}K	3051.1(10)	1.423(39)	0.720(47)	3072.2(21)
^{42}Sc	3046.0(15)	1.437(47)	0.460(47)	3075.6(25)
^{46}V	3045.5(22)	1.429(54)	0.465(33)	3074.7(30)
^{50}Mn	3044.5(15)	1.429(62)	0.547(37)	3071.1(27)
^{54}Co	3047.4(15)	1.428(71)	0.639(43)	3071.2(28)
^{74}Rb	3084.3(80)	1.49(12)	1.50(41)	3083(15)
			Average, $\overline{\mathcal{F}t}$	3072.7(8)
			χ^2/ν	0.42

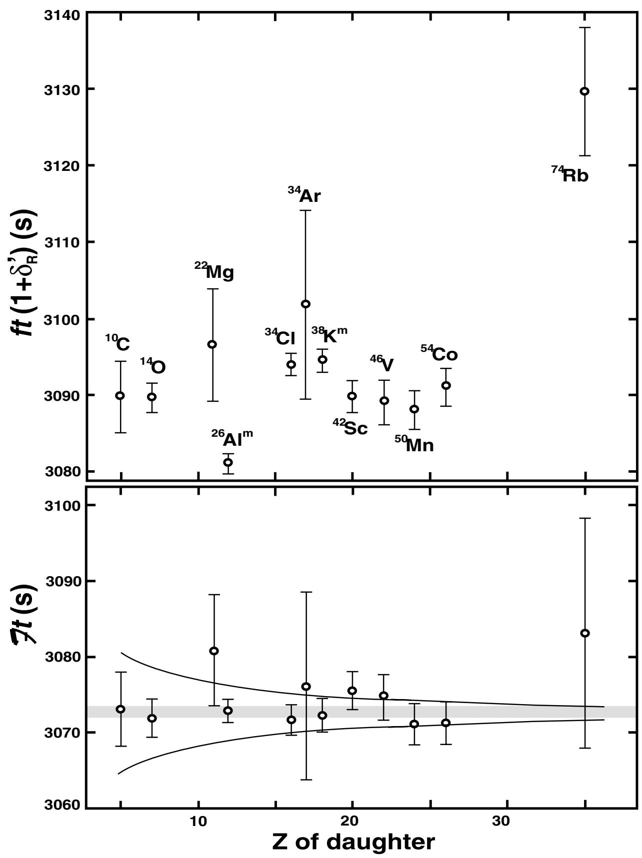

FIG. 1. In the top panel are plotted the experimental ft values corrected only for δ_R', those radiative effects that are independent of nuclear structure. In the bottom panel, the corresponding $\mathcal{F}t$ values are given; they differ from the top panel simply by the inclusion of the nuclear-structure-dependent corrections, δ_{NS} and δ_C. [See Eq. (1).] Note that the increased uncertainty for ^{74}Rb reflects the lack of experimental constraints on the nuclear shell model in this mass region. The horizontal gray band in the bottom panel indicates the average $\overline{\mathcal{F}t}$ value with its uncertainty. The curved lines represent the approximate loci the $\mathcal{F}t$ values would follow if an induced scalar current existed with values of $f_S = \pm 0.002$ in electron rest-mass units or a fundamental scalar current with $C_S/C_V = \pm 0.002$.

uncertainty we obtain for $\overline{\mathcal{F}t}$. This is a 30% improvement over the best previous value [4]—also obtained from superallowed beta decay—and can be attributed to improvements in the experimental data.

CVC(2)-induced scalar current.—A second implication of the CVC hypothesis is that the "induced" scalar term in the vector part of the weak interaction—as distinct from any possible "fundamental" scalar current—must be zero. An independent argument [9], that there be no second-class currents in the hadronic weak interaction, also requires it to vanish. The induced scalar term is characterized by the coupling constant f_S (using the notation of Behrens and Bühring [10]), the presence of which would affect the calculation of the statistical-rate function, f, via a term in the shape-correction function that is inversely proportional to the positron energy. Since the total superallowed-transition decay energy increases with Z, an induced scalar term would therefore have its greatest effect on the $\mathcal{F}t$ values at low Z, introducing curvature in that region. The bottom panel of Fig. 1 shows the loci of $\mathcal{F}t$ values that would be expected if $f_S = \pm 0.002$. Obviously, the $\mathcal{F}t$ values do not exhibit any such curvature and, from a least-squares fit to the data, we obtain $f_S = -0.000\,05(130)$ or, expressed as a limit, $|f_S| \leq 0.0013$, in electron rest-mass units.

Fundamental scalar current.—Although the weak interaction is normally described by an equal mix of vector and axial-vector interactions, which maximizes parity violation, there is no *a priori* reason to rule out other forms of fundamental couplings, notably scalar and tensor interactions. It turns out that a fundamental scalar current would have the same effect on the $\mathcal{F}t$ values as would an induced scalar term: it would introduce curvature in the loci of values at low Z as illustrated in Fig. 1. The superallowed data in this case yield the result $C_S/C_V = -0.000\,05(130)$, or a limit of $|C_S/C_V| \leq 0.0013$, as expressed in the conventional notation of Jackson, Treiman, and Wyld [11]. The corresponding result for the Fierz interference constant is $b_F = +0.0001(26)$. Though the standard deviation is only slightly reduced from our previously published result [6], the present result represents a factor of 30 reduction in the central value, due not to a single new measurement but rather to subtle changes in all the individual average ft values. Our limits on $|C_S/C_V|$ and $|b_F|$ are by far the most stringent ever obtained from nuclear beta decay.

The V_{ud} element of the CKM matrix.—With a mutually consistent set of $\mathcal{F}t$ values, we can now insert their average value [12], $\overline{\mathcal{F}t}$, into Eq. (1) and determine the vector coupling constant G_V using the value $\Delta_R^V = 2.40(8)\%$ calculated for the transition-independent radiation correction by Marciano and Sirlin [14]. The value of G_V itself is of little interest but, when combined with G_F, the weak-interaction constant for the purely leptonic muon decay, it yields a value for the element V_{ud} of the CKM matrix: $V_{ud} = G_V/G_F$. Taking the Particle Data Group (PDG) value [7] of $G_F/(\hbar c)^3 = 1.166\,39(1) \times 10^{-5}$ GeV^{-2}, we obtain $|V_{ud}| = 0.9738(4)$. Compared to our previously recommended value [6], this result differs by two units in the last digit quoted and has a reduced uncertainty. It also agrees with, but is considerably more precise than, the result obtained using the same statistical procedures from the world data for neutron decay [6]: viz. $|V_{ud}| = 0.9745(16)$. Note that, by more than an order of magnitude, V_{ud} is the most precisely determined element of the CKM matrix.

CKM unitarity–top row.—The CKM matrix transforms the basis of quark mass eigenstates to that of the quark weak-interaction eigenstates, and it must therefore be unitary in order that the basis remains orthonormal. An important test of the standard model is whether the experimentally determined matrix elements indeed satisfy a unitarity condition. Currently, the sum of the squares of the top-row elements, which should equal one, constitutes the most demanding such test available. With the value just obtained for $|V_{ud}|$ and the PDG's recommended values [7] of $|V_{us}| = 0.2200(26)$ and $|V_{ub}| = 0.00367(47)$, this test yields

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9966 \pm 0.0014, \qquad (2)$$

which fails unitarity by 2.4 standard deviations. Two recent measurements, one of the $K^+ \to \pi^0 e^+ \nu_e (K_{e3}^+)$ branching ratio from the Brookhaven E865 experiment [15] and the other of the $K_L \to \pi^\pm e^\mp \nu_e (K_{e3}^0)$ branching ratio from the Fermilab E832 experiment [16], together yield the average value $V_{us} = 0.2259 \pm 0.0018$. If this value alone were adopted for V_{us} rather than the PDG average of many experiments, the sum in Eq. (2) would equal 0.9996(11) and unitarity would be fully satisfied.

CKM unitarity–first column.—Though for now it is a less demanding test, the first column of the CKM matrix can also be used to test unitarity. With our value for V_{ud} and the PDG values for V_{cd} and V_{td}, the sum of squares of these three elements is 0.9985(54), in agreement with unitarity. The larger uncertainty is entirely due to V_{cd}.

Right-hand currents.—If parity violation were not maximal, then right-hand currents should be included in the analysis that yields a value for V_{ud}. Thus, we can set an upper limit on the possible role of right-hand currents by attributing any apparent nonunitarity of the CKM matrix entirely to that source. We express the result in terms of the left-right and left-left coupling constants, a_{LR} and a_{LL}, defined by Herczeg [17]. If we accept the unitarity test in Eq. (2), then we find $Re a_{LR}/a_{LL} = -0.0017(7)$. Within the context of the manifest left-right symmetric model, this result corresponds to a mixing angle of $\zeta = 0.0017(7)$. If, instead, we adopt the average of the E865 and E832 values for V_{us}, the result becomes $Re a_{LR}/a_{LL} = -0.0004(6)$.

The accumulated world data on superallowed $0^+ \to 0^+$ β decay comprise the results of over 100 measurements of comparable precision [5]. Virtually all the important experimental parameters used as input to the ft-value determinations have been measured in at least two, and often four or five, independent experiments. Obviously, just another measurement will not have much impact on the precision of the weak-interaction parameters quoted here. Nevertheless, it is still possible for well selected experiments with existing or currently foreseen techniques to make real improvements. For example, the validation of the nuclear-structure-dependent correction terms, δ_{NS} and δ_C, exemplified by the comparison of the two panels in Fig. 1, can be improved by the addition of new transitions selected from among those with large calculated corrections. If the ft values measured for cases with large calculated corrections also turn into corrected $\mathcal{F}t$ values that are consistent with the others, then this must verify the calculations' reliability for the existing cases, which have smaller corrections. In fact, the cases of ^{34}Ar and ^{74}Rb, which have only recently been measured, were chosen for this very reason and, although their precision does not yet equal that of the others, they do indicate that the corrections so far are living up to expectations. The precision of these new measurements can certainly be improved, and other new cases with large calculated corrections [8], such as ^{18}Ne, ^{30}S, and ^{62}Ga, would be valuable additions.

Another area of potential improvement is in the limit set on scalar currents. The bottom panel of Fig. 1 clearly illustrates the sensitivity of the low-Z cases to the possible presence of scalar currents. Reduced uncertainties, particularly on the decays of ^{10}C and ^{14}O, could thus reduce the scalar-current limit significantly.

Finally, it is important to point out that the biggest contribution to V_{ud} uncertainty—90% of it, in fact—comes from the calculation of the transition-independent part of the radiative correction [14], $\Delta_R^V = 2.40(8)\%$. Not only is the uncertainty quoted for Δ_R^V the principal limitation on the precision with which V_{ud} can be determined from nuclear superallowed β decay, but it will have a similar limiting effect on the determination of V_{ud} from neutron and pion decays as well. No significant improvement in the precision of the V_{ud} contribution to the CKM unitarity test can come from any source without a more definitive calculation of Δ_R^V.

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